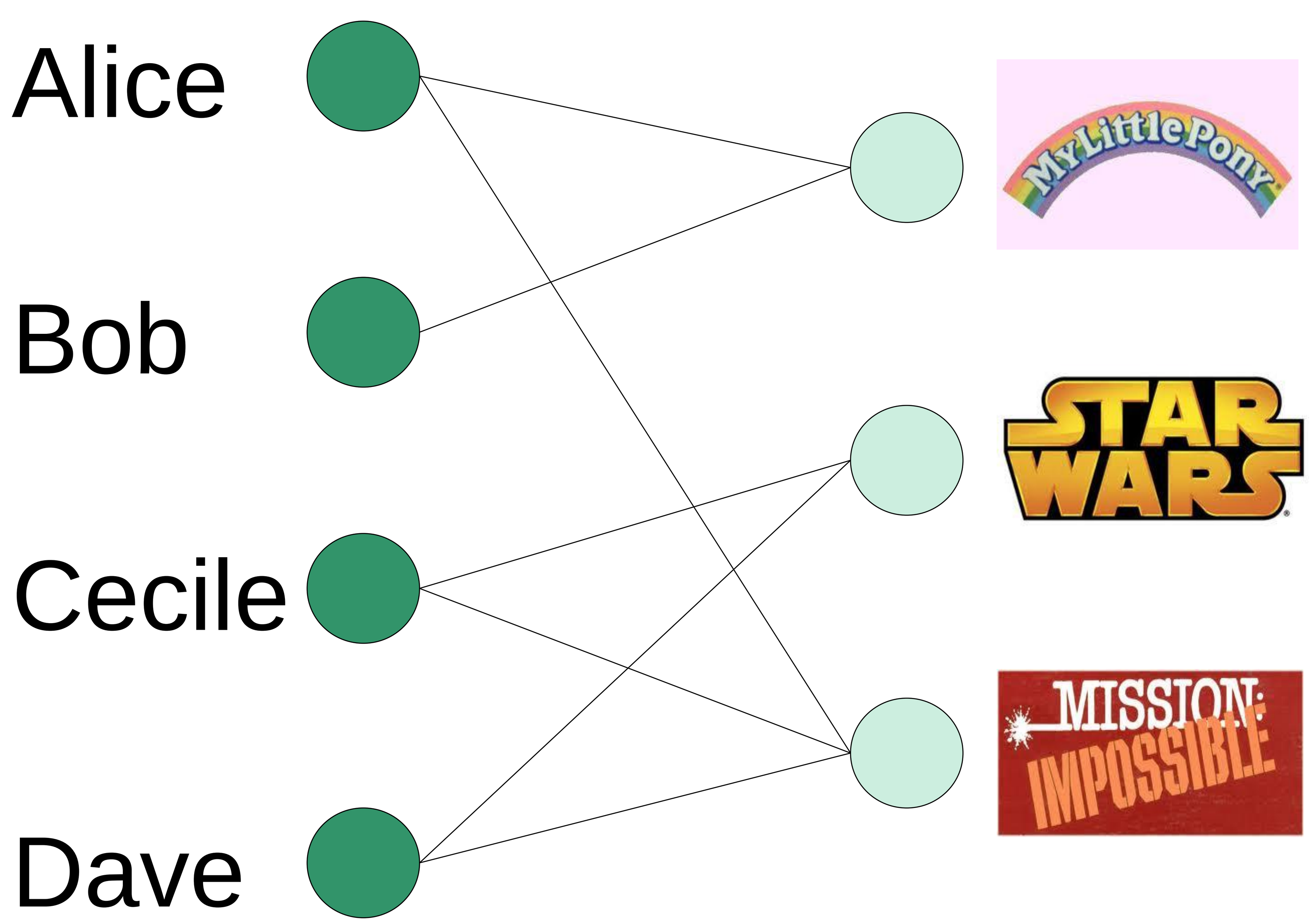
# Reconstructing Graphs from Neighborhood Data

Dora Erdos, Rainer Gemulla, Evimaria Terzi


Alice
Bob
Cecile
Dave
My Little Pony
STAR WARS
MISSION: IMPOSSIBLE

| | | | |
|---|---|---|---|
| Alice | **1** | **0** | **1** |
| Bob | 1 | 0 | 0 |
| Cecile | 0 | 1 | 1 |
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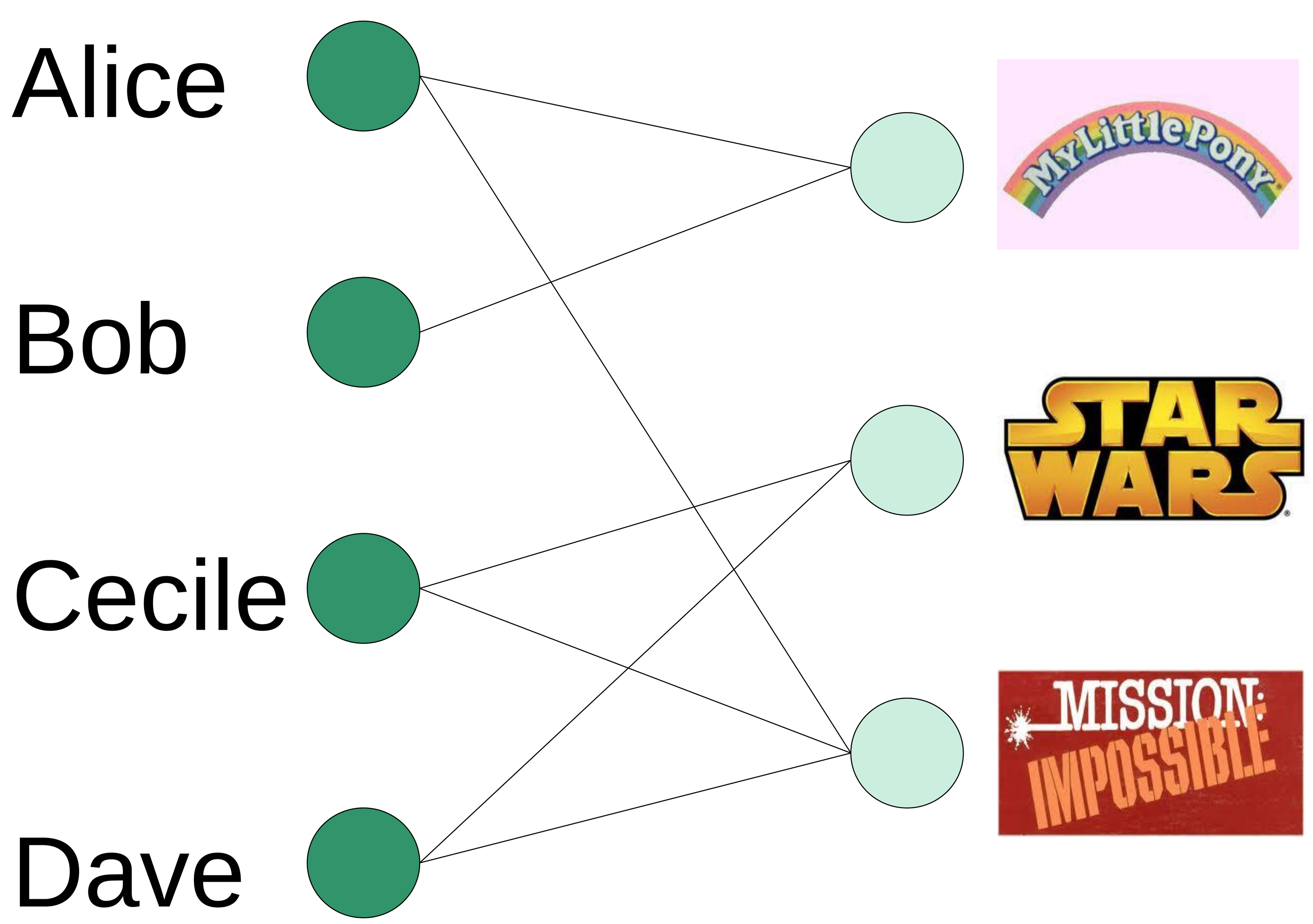
M



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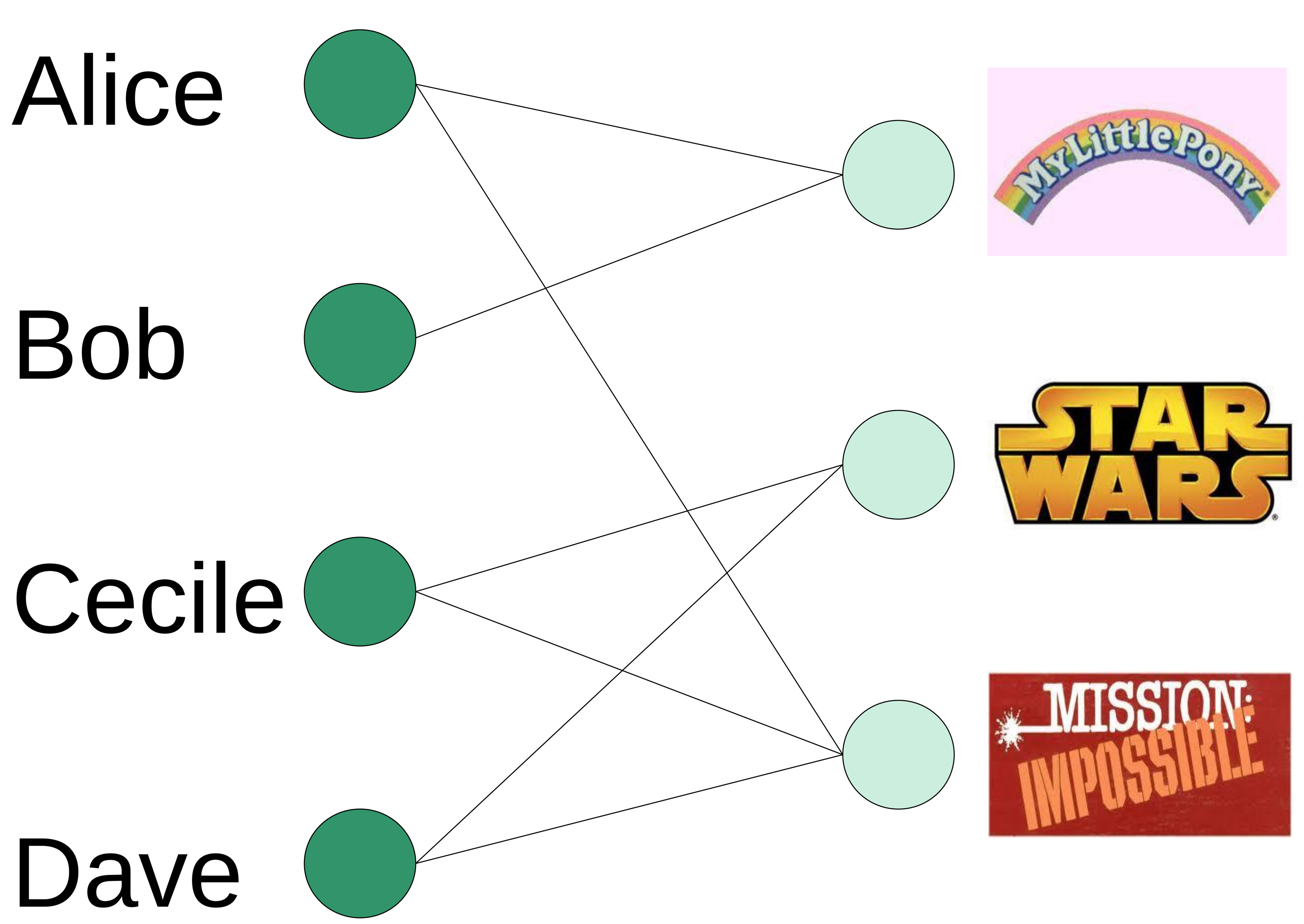


Left neighborhood information of M.

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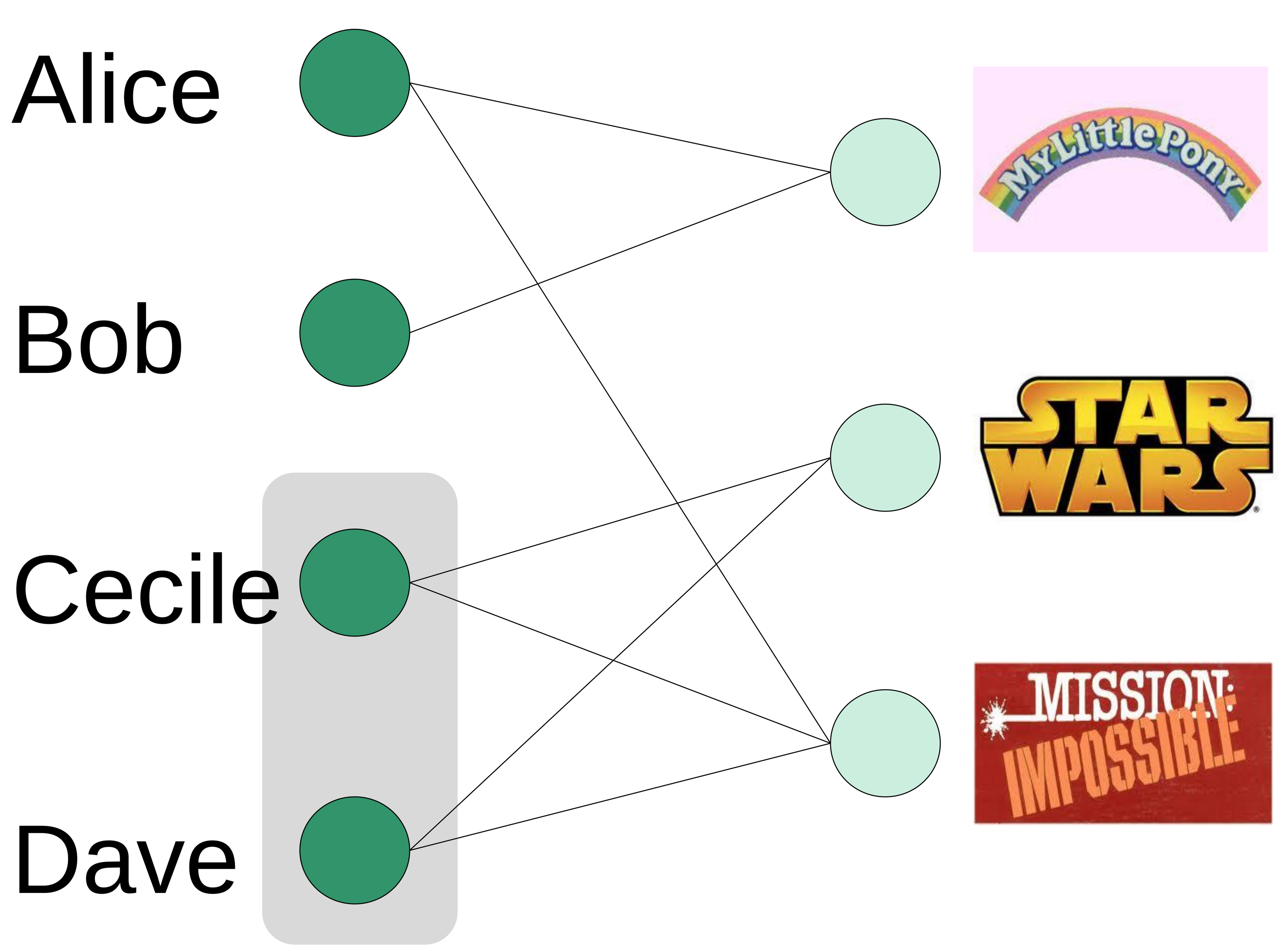


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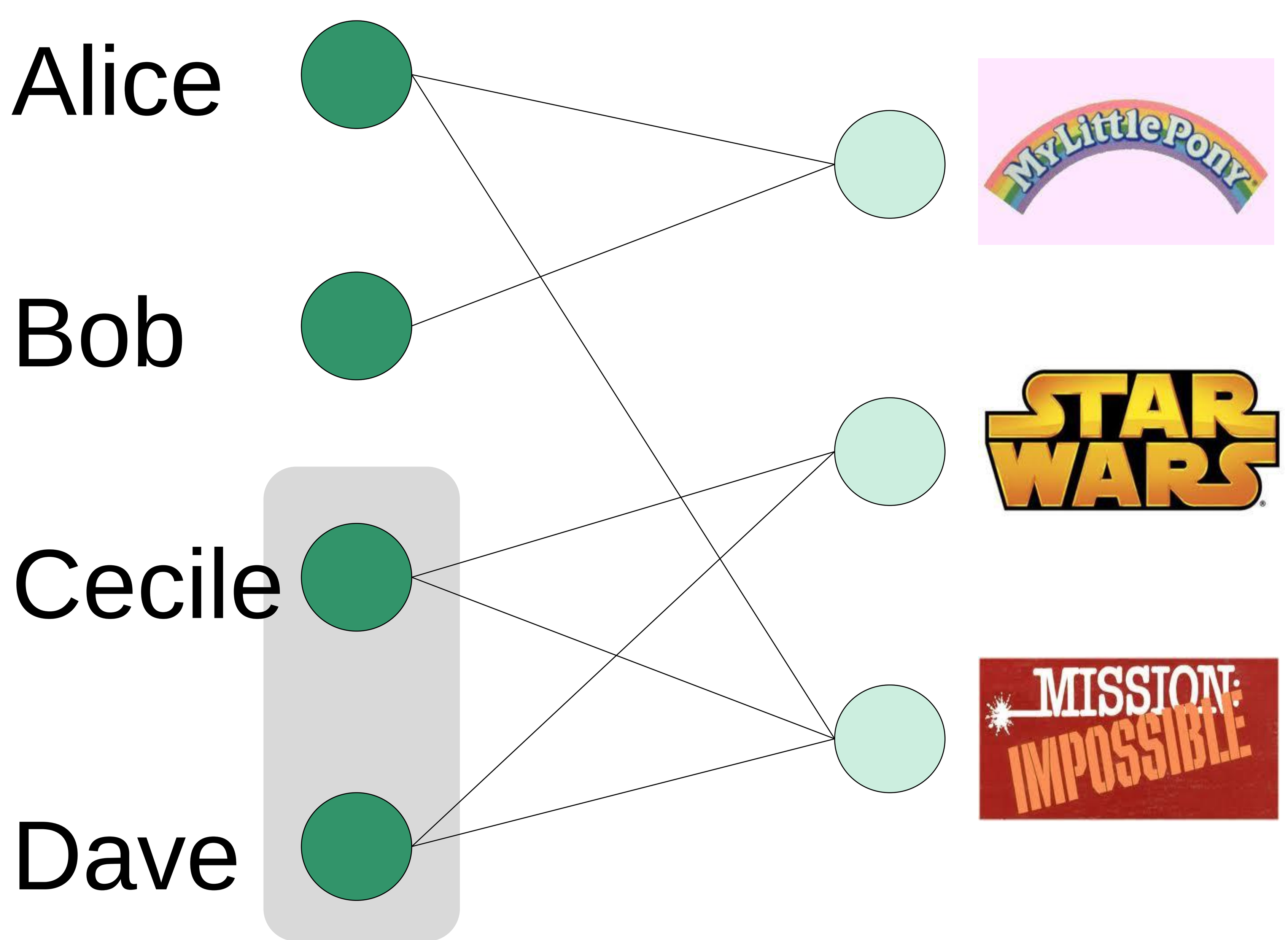


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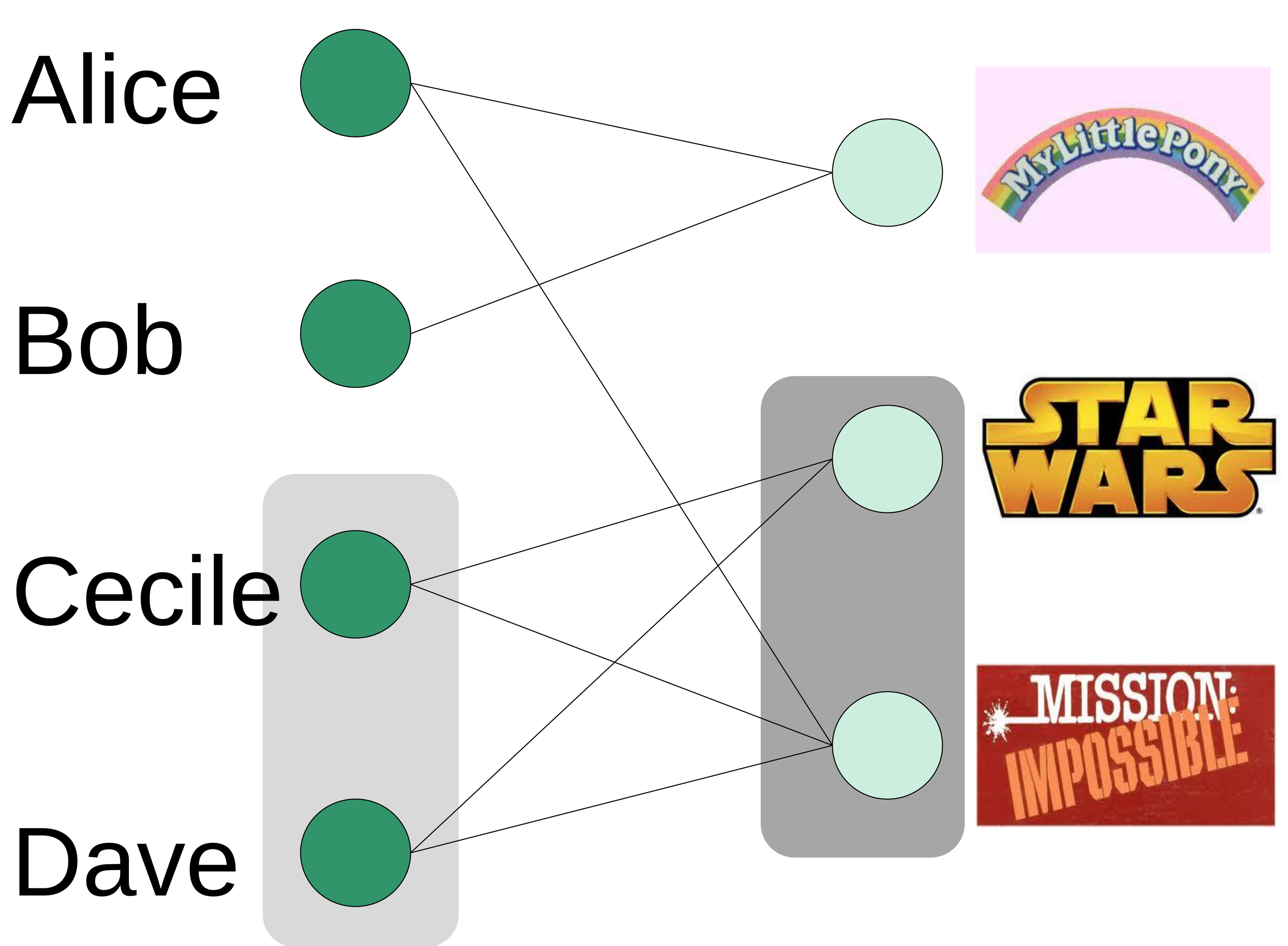


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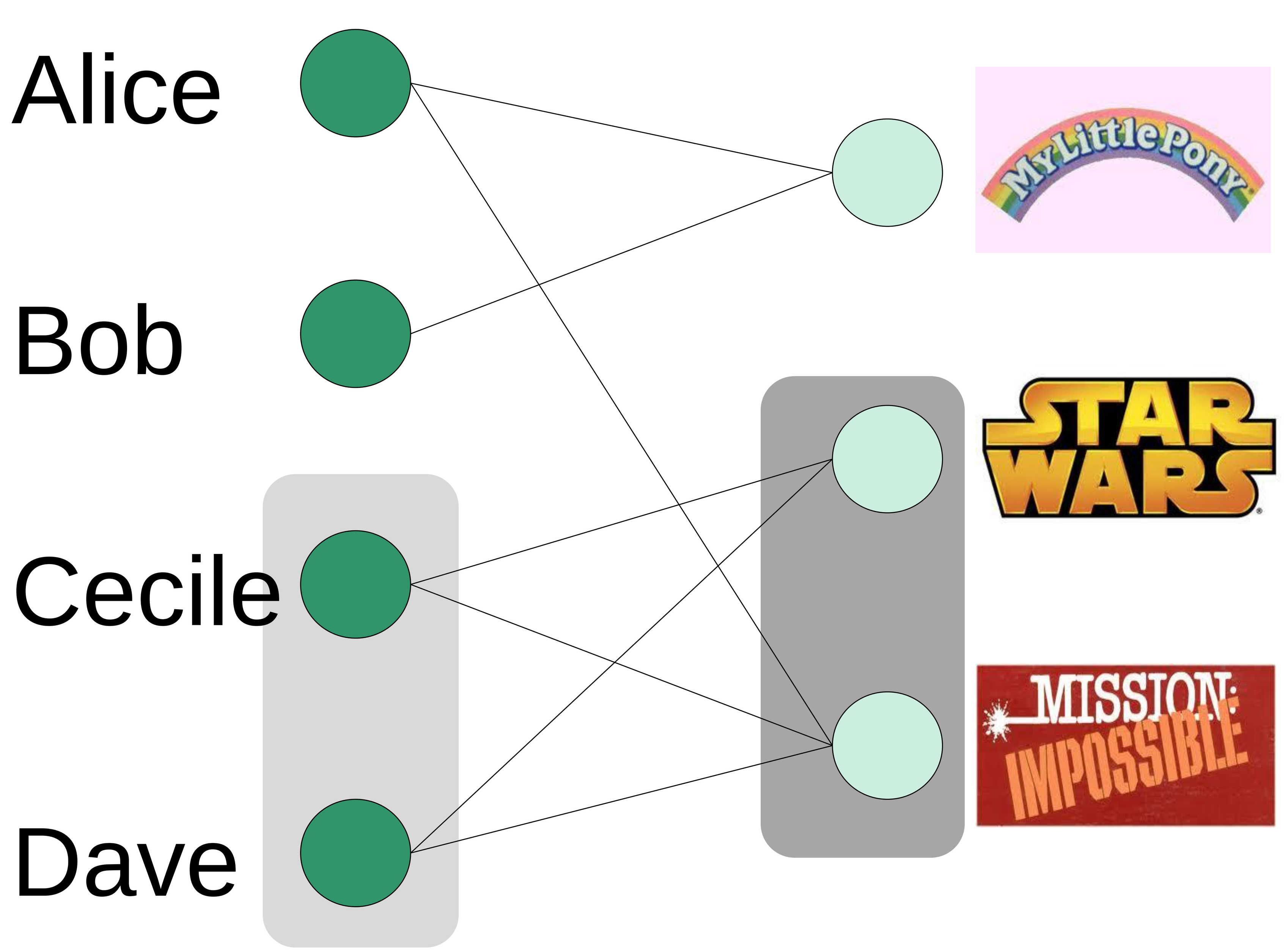


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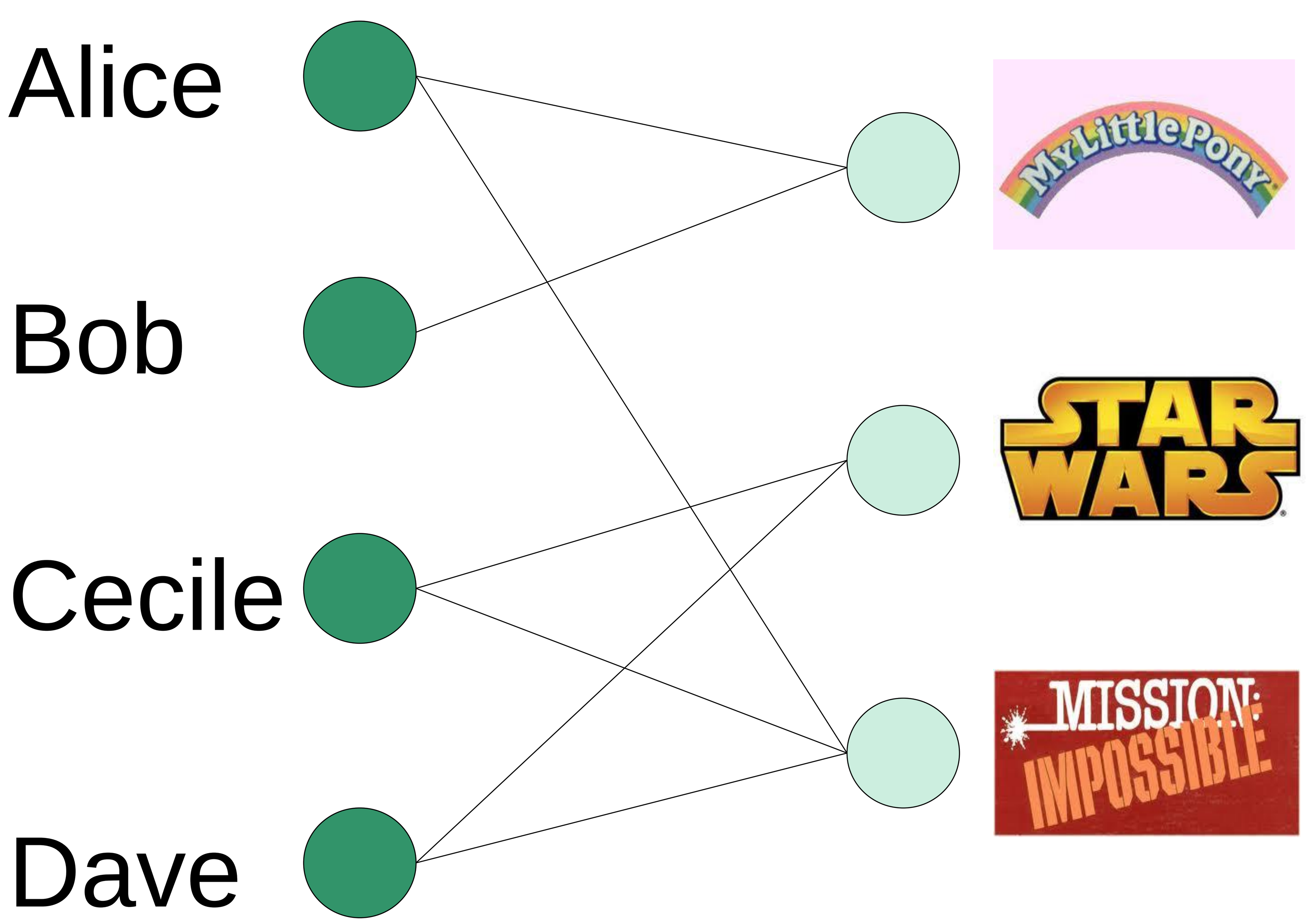


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L is a similarity matrix between people

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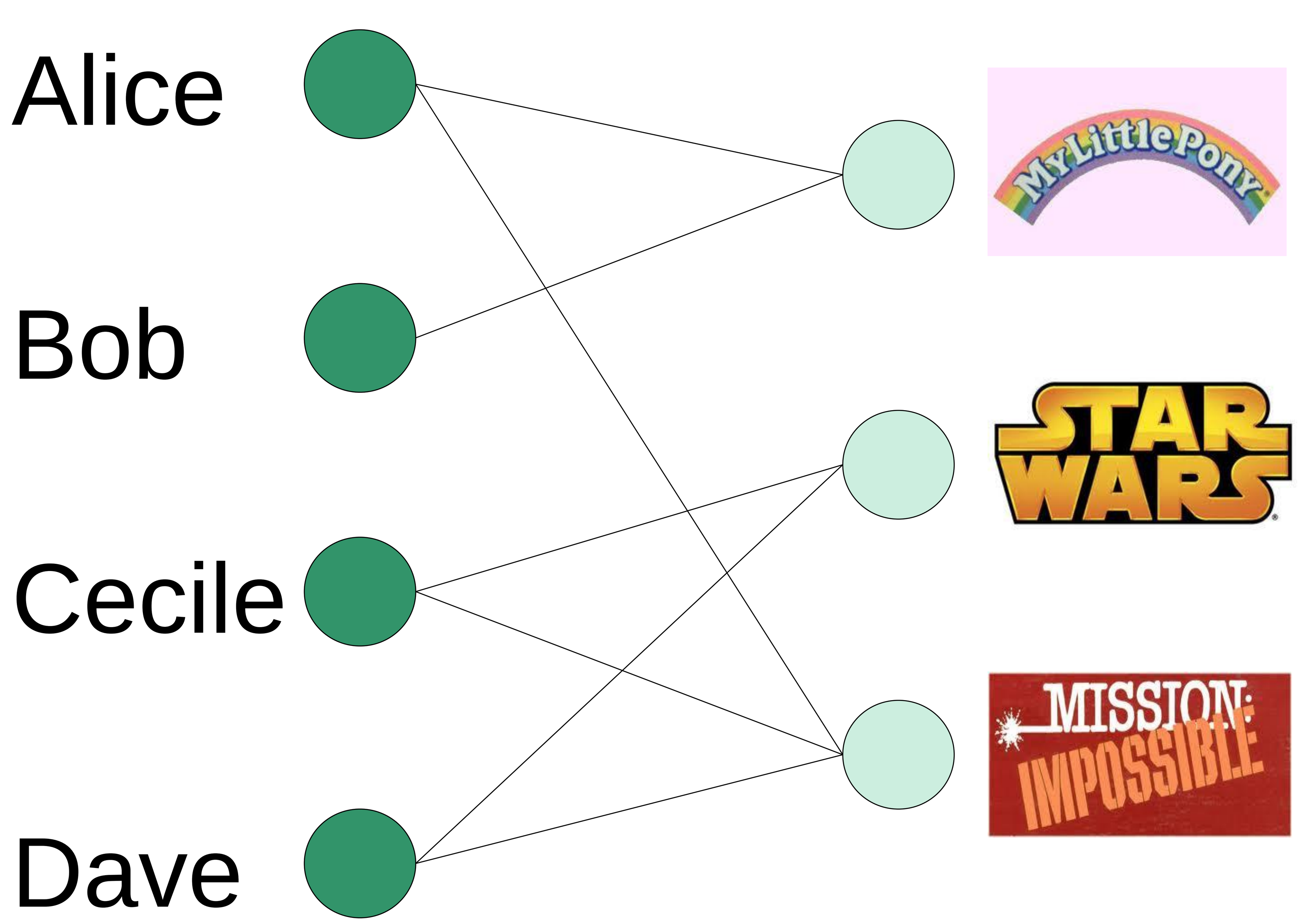


Right neighborhood information of M.

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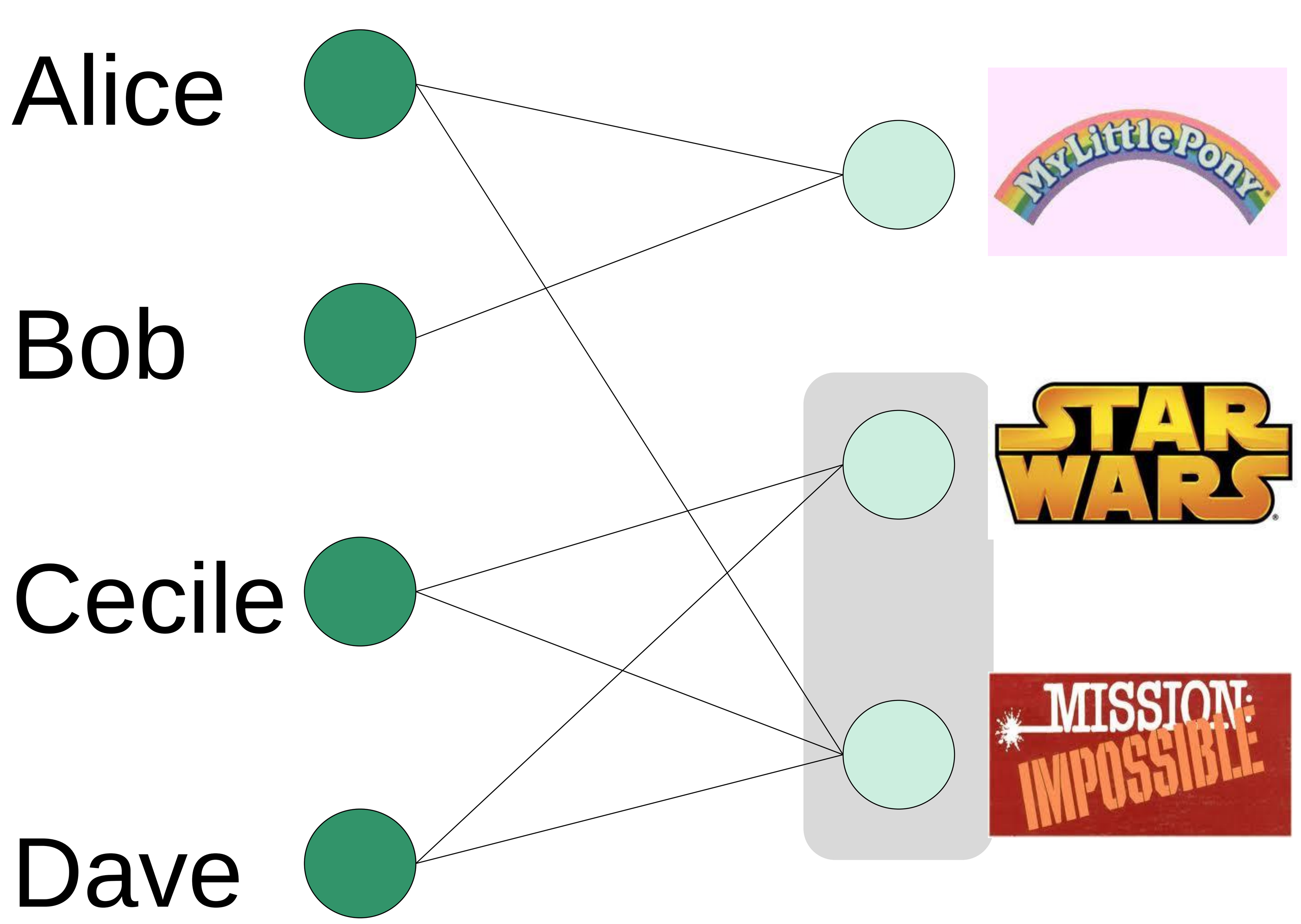


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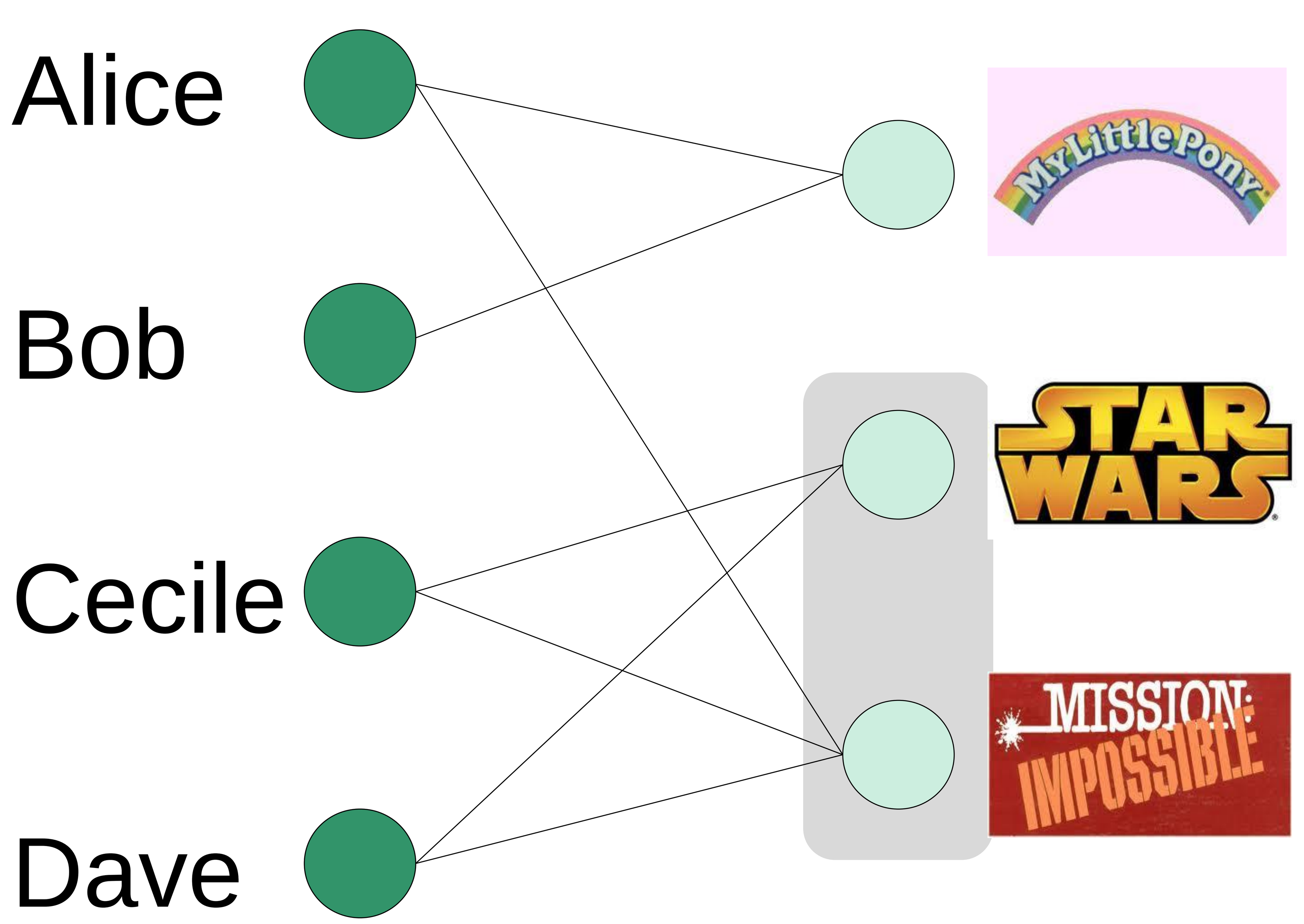


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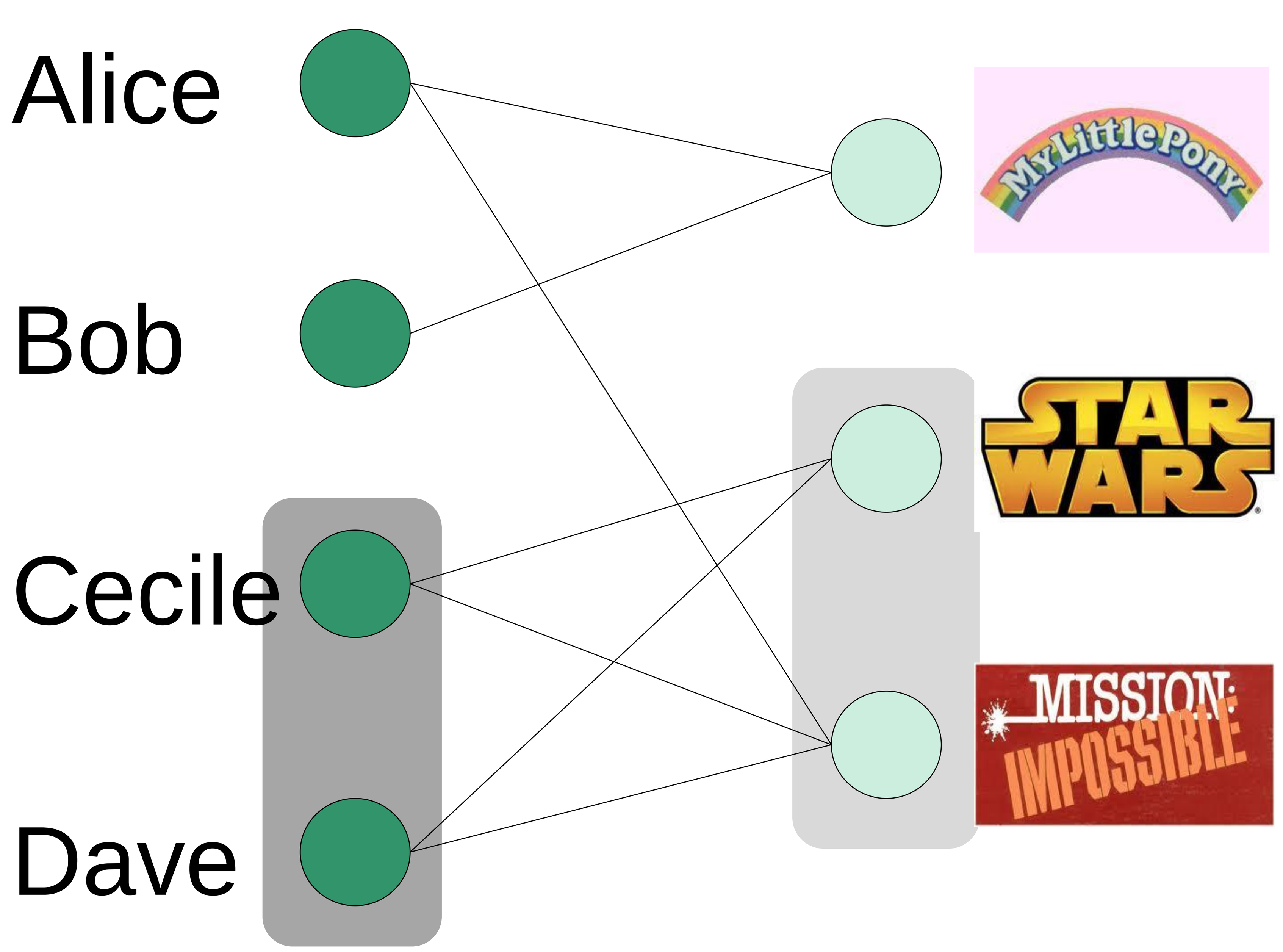


Right neighborhood information of M.

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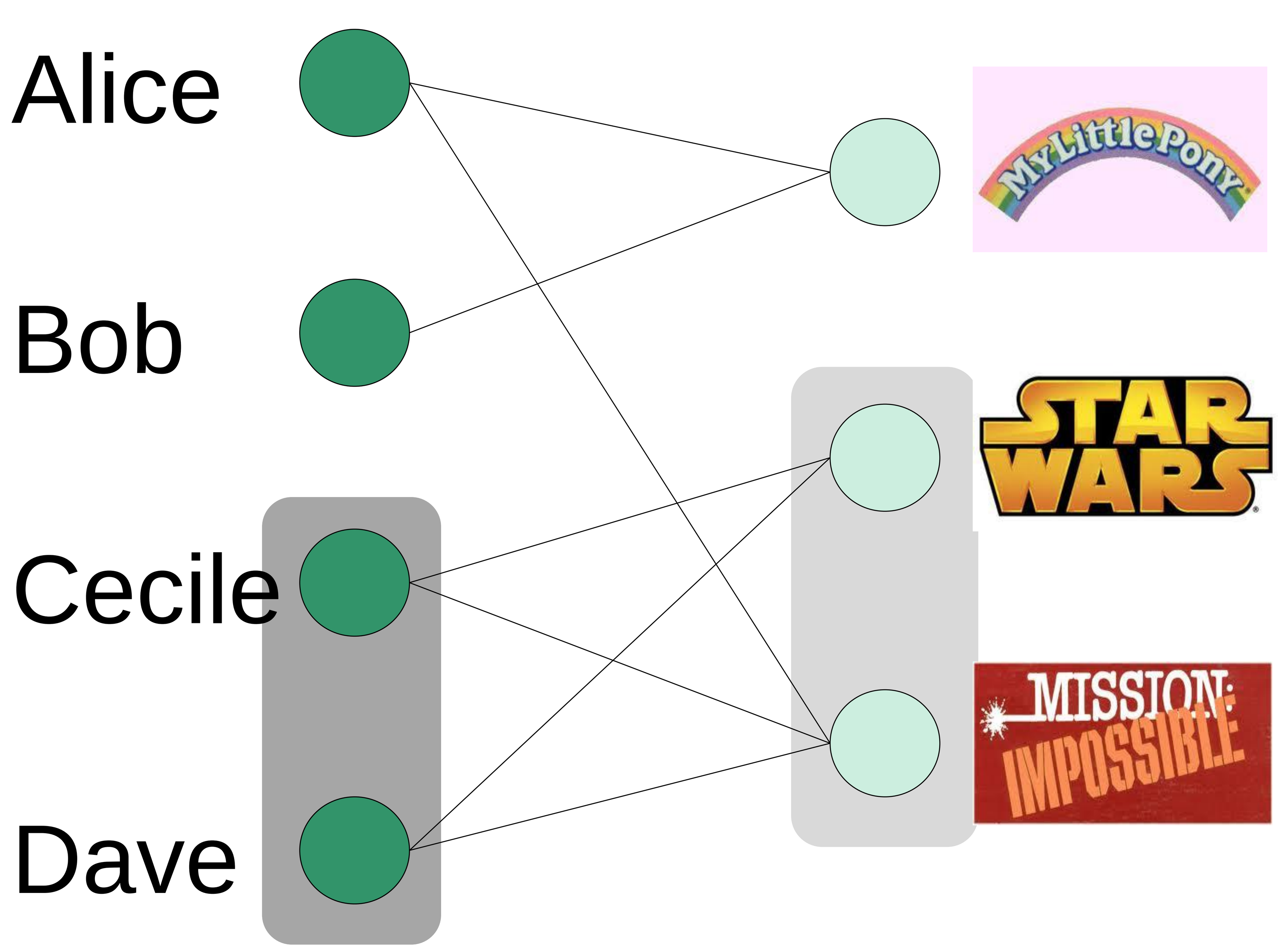


Right neighborhood information of M.

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M

L is a similarity matrix between people

R is a similarity matrix between movies

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R

NETFLIX Recommend movies to users

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L

L is a similarity matrix between people

R is a similarity matrix between movies

NETFLIX Recommend movies to users

## Recommendation systems:

1. Compute similarity matrices L and R from M

$$L = MM^T$$
$$R = M^T M$$

2. Apply some algorithm to L and R to obtain recommendation

   collaborative filtering

   nearest neighbor algorithm

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R

## Recommendation systems:

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1. Compute similarity matrices L and R from M

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2. Apply some algorithm to L and R to obtain recommendation
   - collaborative filtering
   - nearest neighbor algorithm

What do we do if M is hidden?

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What do we do if M is hidden?

Reconstruction problem:

Given L and R reconstruct M.

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R

# Outline

Problem definition

Connection to SVD

Greedy SVD reconstruction

Experiments

Discussion

# Outline

- Problem definition
- Connection to SVD
- Greedy SVD reconstruction
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M

Given the neighborhood information L and R how would you reconstruct M?

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$$R = M^T M$$

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R

# Reconstruction problem:

Given neighborhood matrices L and R construct binary matrix $\hat{M}$

Such that $F_L(\hat{M}) + F_R(\hat{M})$ is minimized

$$F_L(\hat{M}) = \left\| \hat{M}\hat{M}^T - L \right\|$$

$$F_R(\hat{M}) = \left\| \hat{M}^T\hat{M} - R \right\|$$

Measures the distance between L, R and the neighborhood matrices of $\hat{M}$

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# Connection to SVD

Idea: Reconstruct M by obtaining its SVD decomposition

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Eigen decomposition of L: $L = U \Lambda U^T$ Remember: $L = MM^T$

# Connection to SVD

Idea: Reconstruct M by obtaining its SVD decomposition

SVD decomposition of M: $M = U\Sigma V^T$

Eigen decomposition of L: $L = U\Lambda U^T$ Remember: $L = MM^T$

Eigen decomposition of R: $R = V\Lambda V^T$ $R = M^T M$

# Connection to SVD

Idea: Reconstruct M by obtaining its SVD decomposition

SVD decomposition of M: $M = U\Sigma V^T$

Eigen decomposition of L: $L = U\Lambda U^T$ Remember: $L = MM^T$

Eigen decomposition of R: $R = V\Lambda V^T$ $R = M^T M$

Observe $\Lambda = \Sigma^2$

# Connection to SVD

Eigen decomposition of L and R: $L = U\Lambda U^T \quad R = V\Lambda V^T \quad \Lambda = \Sigma^2$

As a result we have all components of the SVD representation of $M$

$U, V$ and $\Sigma = \sqrt{\Lambda}$

# Connection to SVD

Eigen decomposition of L and R: $L = U\Lambda U^T$ $R = V\Lambda V^T$ $\Lambda = \Sigma^2$

As a result we have all components of the SVD representation of $M$

$U, V$ and $\Sigma = \sqrt{\Lambda}$

Compute $\hat{M} = U\sqrt{\Lambda}V^T$

# Connection to SVD

Eigen decomposition of L and R: $L = U \Lambda U^T$ $R = V \Lambda V^T$ $\Lambda = \Sigma^2$

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Done?

# Connection to SVD

Eigen decomposition of L and R: $L = U\Lambda U^T$ $R = V\Lambda V^T$ $\Lambda = \Sigma^2$

As a result we have all components of the SVD representation of $M$

$U, V$ and $\Sigma = \sqrt{\Lambda}$

Compute $\hat{M} = U\sqrt{\Lambda}V^T$

Done?

We don't know what sign to pick for the singular values

$$\hat{\Sigma}_i = \pm\sqrt{\Lambda}_i$$

We don't know whether the resulting $\hat{M}$ is binary.

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# Greedy SVD reconstruction

1. Compute eigenvalue decompositions of L and R to obtain $U, V, \Lambda$

Set $\hat{\Sigma}_i = \pm\sqrt{\Lambda_i}$

2. Fix signs of singular values in decreasing order of magnitude

$$|\Sigma_1| \geq |\Sigma_2| \geq \ldots |\Sigma_n|$$

# Greedy SVD reconstruction

1. Compute eigenvalue decompositions of L and R to obtain $U, V, \Lambda$

Set $\hat{\Sigma}_i = \pm\sqrt{\Lambda_i}$

2. Fix signs of singular values in decreasing order of magnitude

$$|\Sigma_1| \geq |\Sigma_2| \geq \ldots |\Sigma_n|$$

Iteration *i*:

$$M_i^+ = M_{i-1} + U_i \cdot |\Sigma_i| \cdot V_i^T$$

$$M_i^- = M_{i-1} - U_i \cdot |\Sigma_i| \cdot V_i^T$$

# Greedy SVD reconstruction

1. Compute eigenvalue decompositions of L and R to obtain $U, V, \Lambda$

Set $\hat{\Sigma}_i = \pm\sqrt{\Lambda}$

2. Fix signs of singular values in decreasing order of magnitude

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Compute binary matrix by rounding:

$$BM_i^+ = (M_i^+ \geq t)$$

$$BM_i^- = (M_i^- \geq t)$$

# Greedy SVD reconstruction

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$$BM_i^+ = (M_i^+ \geq t)$$

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Choose binary matrix that is closest to its real version

# Greedy SVD reconstruction

$t = 0.5$ Closest binary matrix

$t = 0.1$ Predicts mostly 1s

$t = 0.9$ Predicts mostly 0s

Iteration *i*:

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## Flickr dataset[1]:

2000 users x 1989 groups

Density 5%

Users exhibit power law degree distribution

Groups have exponential degree distribution

[1] Zheleva *et al. WWW* ‘09

# Experiments

Flickr dataset[1]: 2000 users x 1989 groups

Density 5%

Data is very sparse

estimating M to be all 0 would yield very small error

Relative absolute error

$$\frac{\left\| B\hat{M} - M \right\|}{\left\| M \right\|}$$

[1] Zheleva *et al. WWW* ‘09

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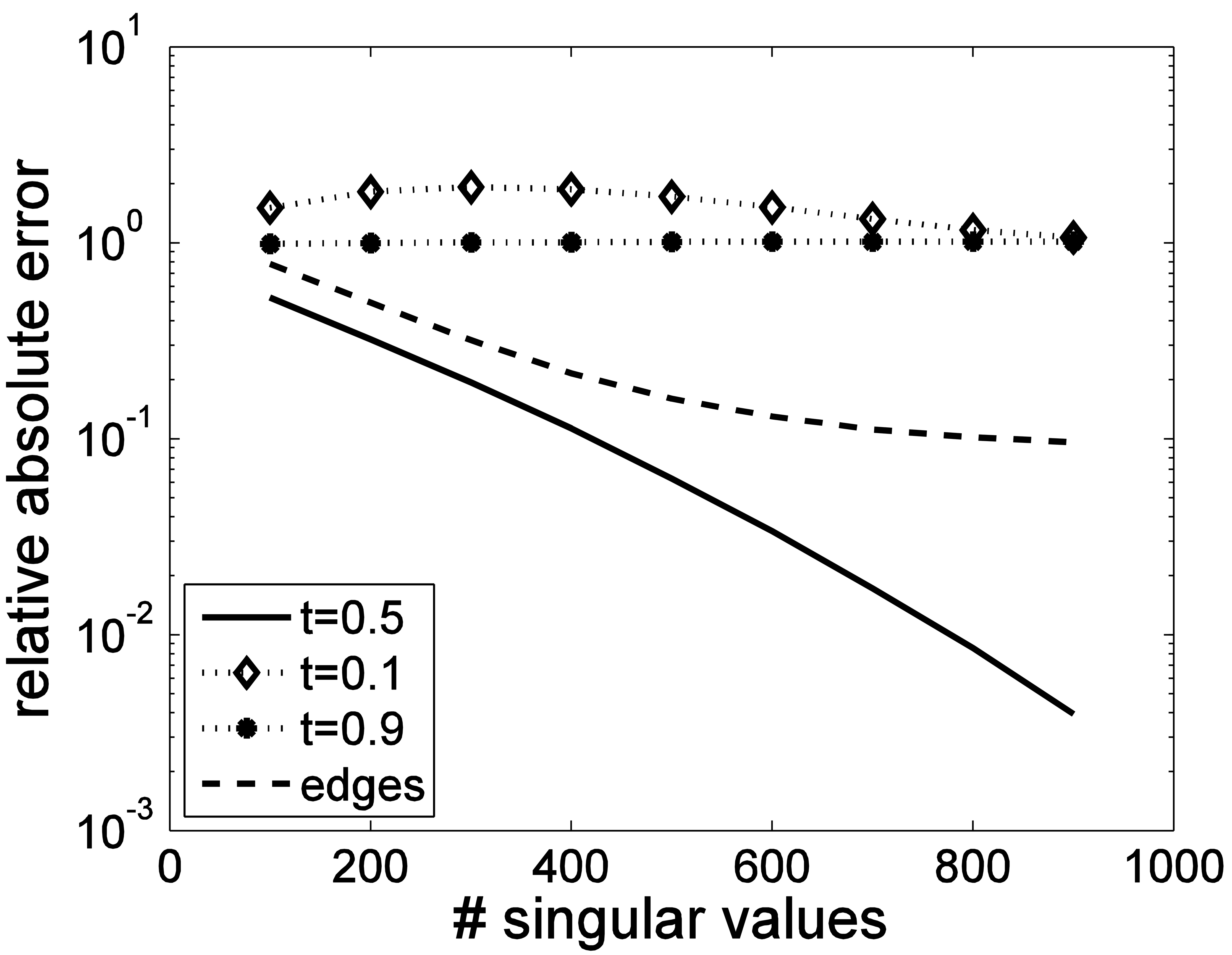


X axis: number k of highest magnitude singular values

Y axis: relative absolute error (log scale)

relative absolute error: $$\frac{\left\| B\hat{M} - M \right\|}{\left\| M \right\|}$$

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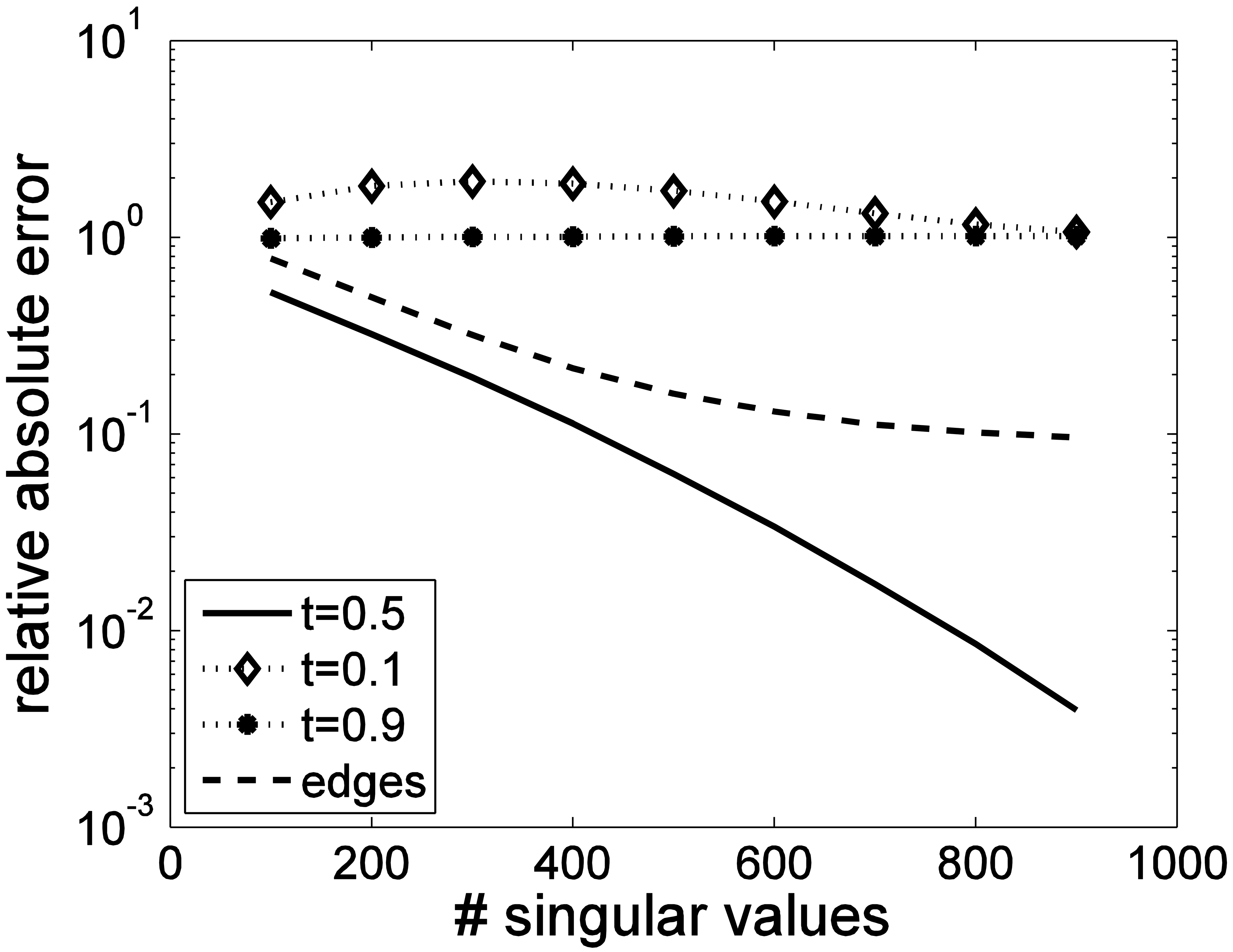


X axis: number k of highest magnitude singular values

Y axis: relative absolute error (log scale)

Matrix M has rank 1989, but with only 900 singular values we can achieve almost perfect reconstruction.

relative absolute error: $$\frac{\left\| B\hat{M} - M \right\|}{\left\| M \right\|}$$

[1]Zheleva *et al. WWW* '09

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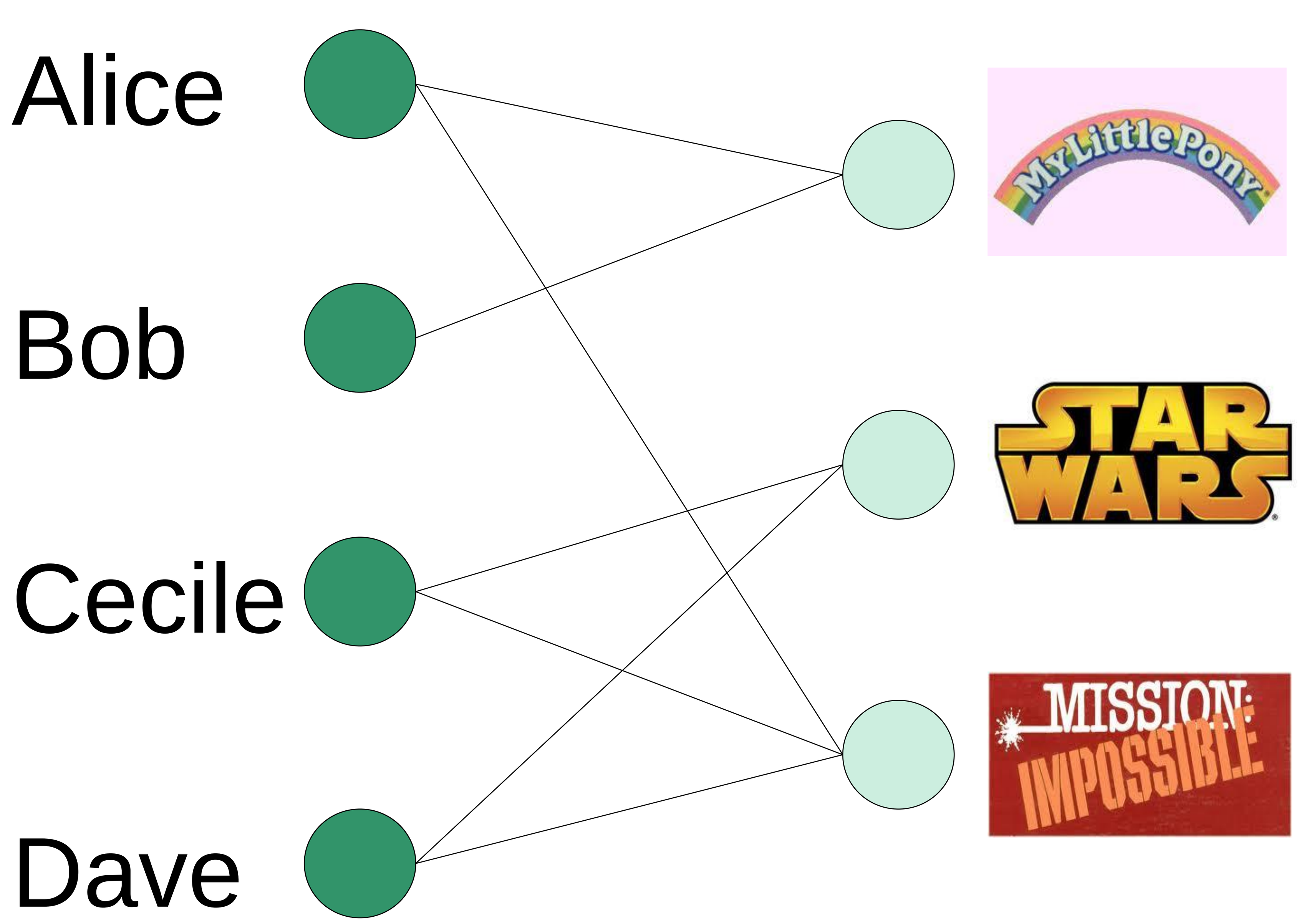


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|---|---|---|---|
| | **2** | **0** | **1** |
| | 0 | 2 | 2 |
| | 1 | 2 | 3 |

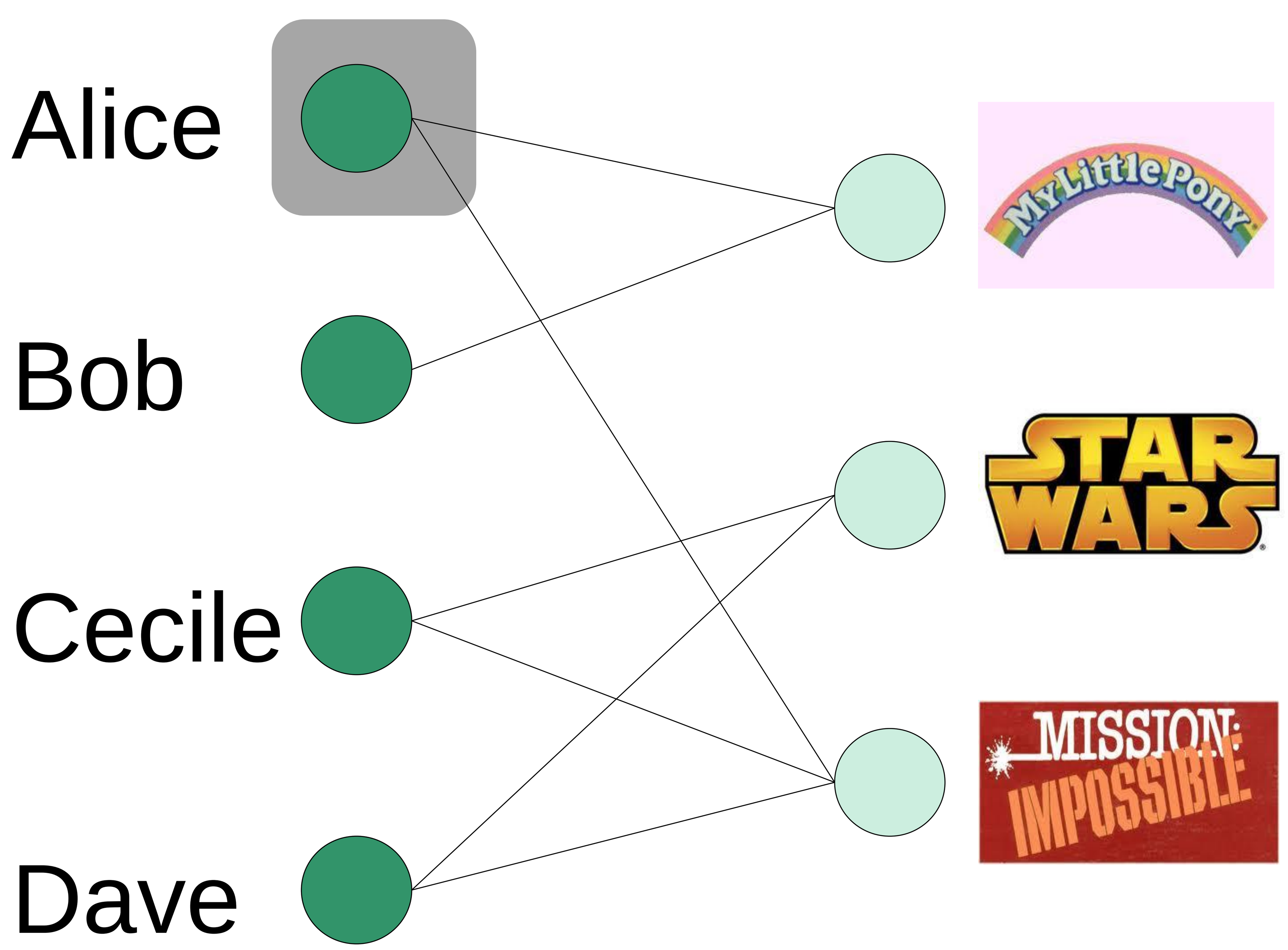
R

| | Alice | Bob | Cecile | Dave |
|---|---|---|---|---|
| Alice | **2** | **1** | **1** | **1** |
| Bob | 1 | 1 | 0 | 0 |
| Cecile | 1 | 0 | 2 | 2 |
| Dave | 1 | 0 | 2 | 2 |

L

| | My Little Pony | Star Wars | Mission: Impossible |
|---|---|---|---|
| Alice | **1** | **0** | **1** |
| Bob | 1 | 0 | 0 |
| Cecile | 0 | 1 | 1 |
| Dave | 0 | 1 | 1 |

M




| | My Little Pony | Star Wars | Mission: Impossible |
|---|---|---|---|
| My Little Pony | **2** | **0** | **1** |
| Star Wars | 0 | 2 | 2 |
| Mission: Impossible | 1 | 2 | 3 |

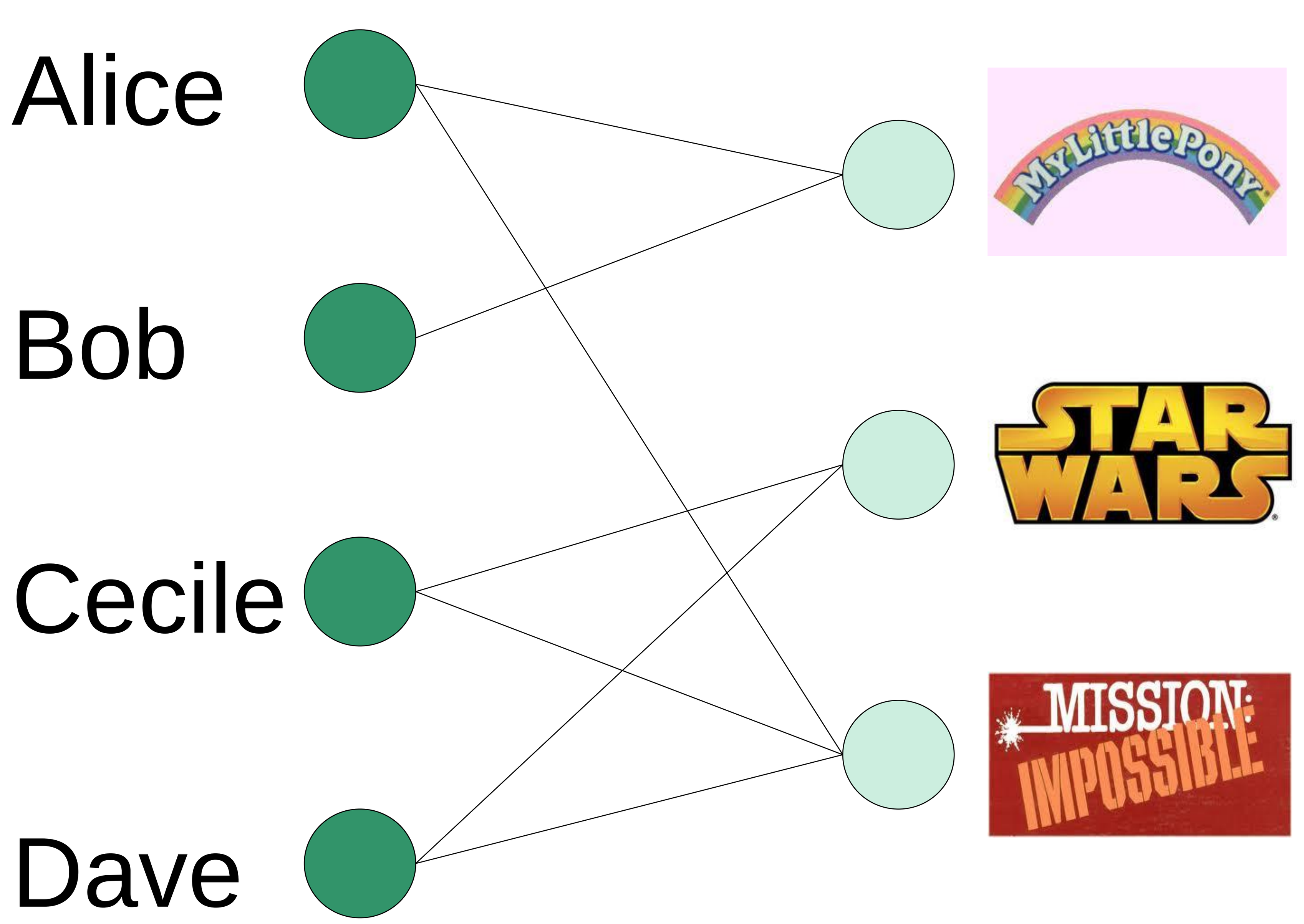
R

| | Alice | Bob | Cecile | Dave |
|---|---|---|---|---|
| Alice | **2** | **1** | **1** | **1** |
| Bob | 1 | 1 | 0 | 0 |
| Cecile | 1 | 0 | 2 | 2 |
| Dave | 1 | 0 | 2 | 2 |

L

| | My Little Pony | Star Wars | Mission: Impossible |
|---|---|---|---|
| Alice | **1** | **0** | **1** |
| Bob | 1 | 0 | 0 |
| Cecile | 0 | 1 | 1 |
| Dave | 0 | 1 | 1 |

M




| | My Little Pony | Star Wars | Mission: Impossible |
|---|---|---|---|
| My Little Pony | **2** | **0** | **1** |
| Star Wars | 0 | 2 | 2 |
| Mission: Impossible | 1 | 2 | 3 |

R

| | Alice | Bob | Cecile | Dave |
|---|---|---|---|---|
| Alice | **2** | **1** | **1** | **1** |
| Bob | 1 | 1 | 0 | 0 |
| Cecile | 1 | 0 | 2 | 2 |
| Dave | 1 | 0 | 2 | 2 |

L

| | My Little Pony | STAR WARS | MISSION: IMPOSSIBLE |
|---|---|---|---|
| Alice | **1** | **0** | **1** |
| Bob | 1 | 0 | 0 |
| Cecile | 0 | 1 | 1 |
| Dave | 0 | 1 | 1 |

M

Knowing degrees of nodes carries a lot of extra information

| | My Little Pony | STAR WARS | MISSION: IMPOSSIBLE |
|---|---|---|---|
| My Little Pony | **2** | **0** | **1** |
| STAR WARS | 0 | 2 | 2 |
| MISSION: IMPOSSIBLE | 1 | 2 | 3 |

R

| | Alice | Bob | Cecile | Dave |
|---|---|---|---|---|
| Alice | **0** | **1** | **1** | **1** |
| Bob | 1 | 0 | 0 | 0 |
| Cecile | 1 | 0 | 0 | 2 |
| Dave | 1 | 0 | 2 | 0 |

L'

| | My Little Pony | STAR WARS | MISSION: IMPOSSIBLE |
|---|---|---|---|
| Alice | **1** | **0** | **1** |
| Bob | 1 | 0 | 0 |
| Cecile | 0 | 1 | 1 |
| Dave | 0 | 1 | 1 |

M

Knowing degrees of nodes carries a lot of extra information

Obtain L' and R' by setting the main diagonals to 0.

| | My Little Pony | STAR WARS | MISSION: IMPOSSIBLE |
|---|---|---|---|
| My Little Pony | **0** | **0** | **1** |
| STAR WARS | 0 | 0 | 2 |
| MISSION: IMPOSSIBLE | 1 | 2 | 0 |

R'

| | Alice | Bob | Cecile | Dave |
|---|---|---|---|---|
| Alice | **0** | **1** | **1** | **1** |
| Bob | 1 | 0 | 0 | 0 |
| Cecile | 1 | 0 | 0 | 2 |
| Dave | 1 | 0 | 2 | 0 |

L’

Knowing degrees of nodes carries a lot of extra information

Reconstruction problem:

Given L’ and R’ reconstruct M.

| | My Little Pony | Star Wars | Mission: Impossible |
|---|---|---|---|
| My Little Pony | **0** | **0** | **1** |
| Star Wars | 0 | 0 | 2 |
| Mission: Impossible | 1 | 2 | 0 |

R’

# Conclusions

(Bipartite) graphs can be reconstructed from neighborhood data with quite high accuracy.

Often a smaller than rank(M) number of singular values is sufficient.

Thank You!