Large-Scale Matrix Factorization with Distributed Stochastic Gradient Descent

Rainer Gemulla

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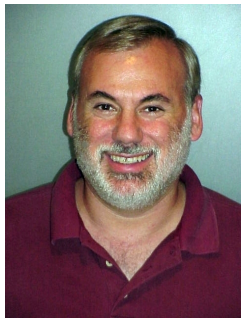
Peter J. Haas Yannis Sismanis Christina Teflioudi Faraz Makari




Outline

Outline

Matrix completion visualized

Original image

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Partially observed image

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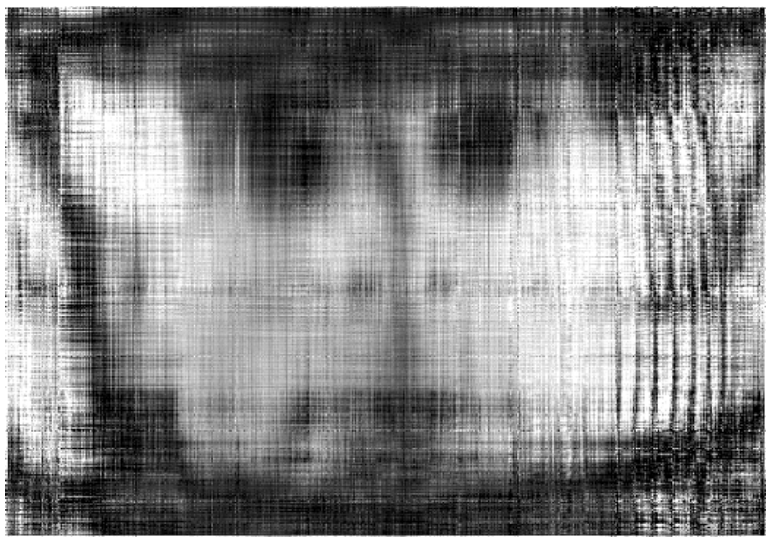

Reconstructed image

Matrix completion for recommender systems

- Discover latent factors ($r = 1$)

	Avatar	**The Matrix**	**Up**
Alice		**4**	**2**
Bob	**3**	**2**	
Charlie	**5**		**3**

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- Minimum loss

$$\min_{\mathbf{W},\mathbf{H}} \sum_{(i,j)\in Z} (\mathbf{V}_{ij} - [\mathbf{WH}]_{ij})^2$$

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$$\min_{\mathbf{W},\mathbf{H},\mathbf{u},\mathbf{m}} \sum_{(i,j)\in Z} (\mathbf{V}_{ij} - \mu - \mathbf{u}_i - \mathbf{m}_j - [\mathbf{WH}]_{ij})^2$$

- Bias

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- Bias, regularization

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$$\min_{\mathbf{W},\mathbf{H},\mathbf{u},\mathbf{m}} \sum_{(i,j,t)\in Z_t} (\mathbf{V}_{ij} - \mu - \mathbf{u}_i(t) - \mathbf{m}_j(t) - [\mathbf{W}(t)\mathbf{H}]_{ij})^2 + \lambda\left(\|\mathbf{W}(t)\| + \|\mathbf{H}\| + \|\mathbf{u}(t)\| + \|\mathbf{m}(t)\|\right)$$

- Bias, regularization, time, ...

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- A general machine learning problem
  - Recommender systems, text indexing, face recognition, ...

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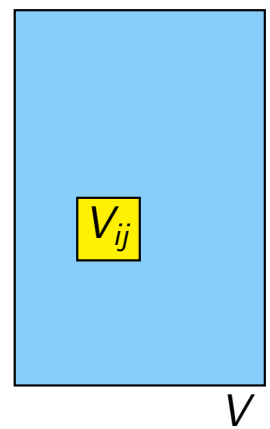

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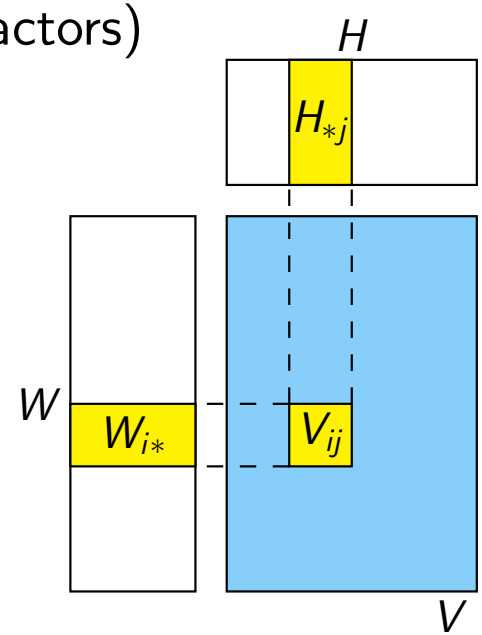

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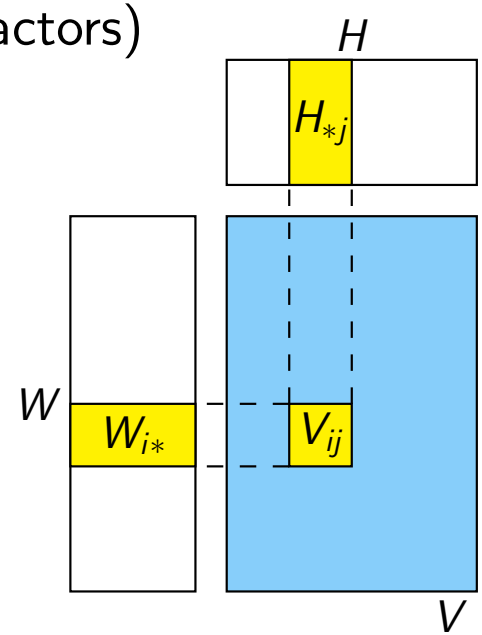
- Model
  - $L_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$: loss at element (i, j)
  - Includes prediction error, regularization, auxiliary information, ...
  - Constraints (e.g., non-negativity)



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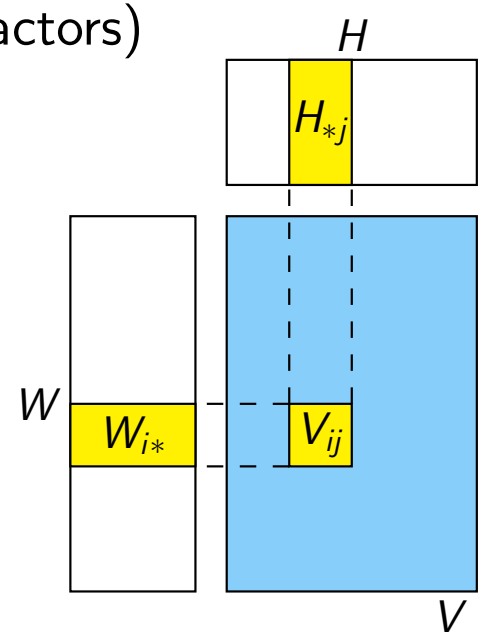
- Find best model

$$\underset{\mathbf{W},\mathbf{H}}{\operatorname{argmin}} \sum_{(i,j)\in Z} L_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$



Successful Applications

- Movie recommendation (Netflix, competition papers)
  - $>$12M users, $>$20k movies, 2.4B ratings (projected)
  - 36GB data, 9.2GB model (projected)
  - Latent factor model
- Website recommendation (Microsoft, WWW10)
  - 51M users, 15M URLs, 1.2B clicks
  - 17.8GB data, 161GB metadata, 49GB model
  - Gaussian non-negative matrix factorization
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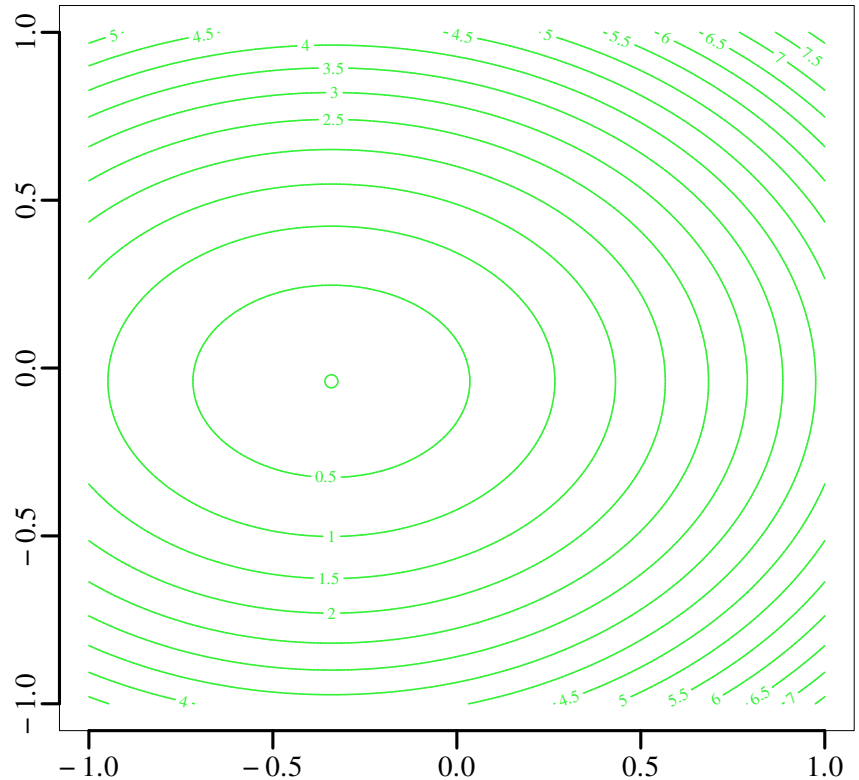
Distributed processing is necessary!

- Big data
- Large models
- Expensive computations

Outline

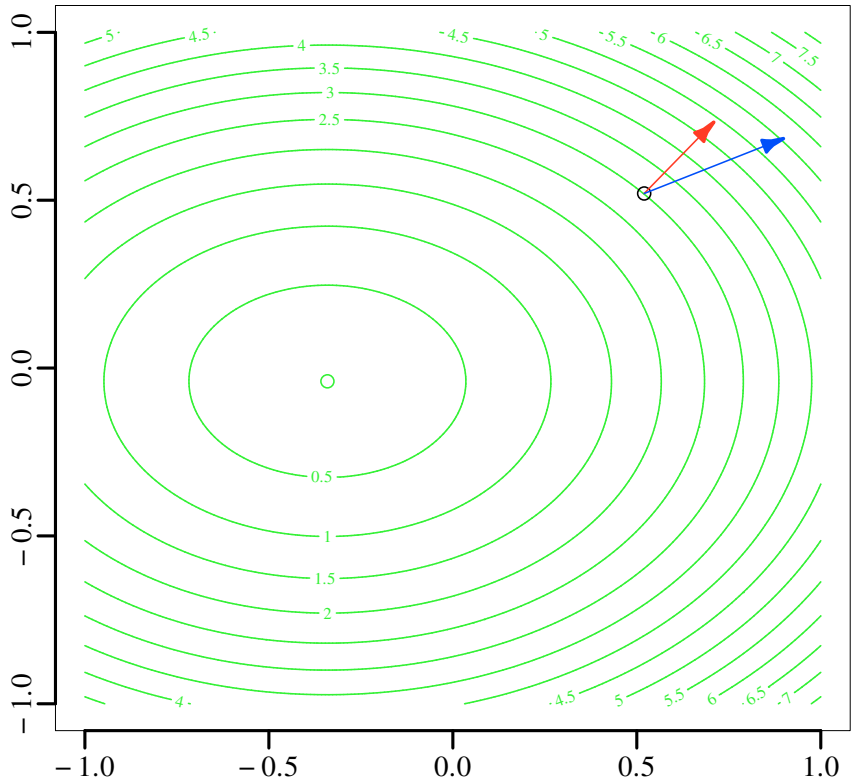
Stochastic Gradient Descent

- Find minimum θ^* of function L



Stochastic Gradient Descent

- Find minimum θ^* of function L
- Pick a starting point θ_0
- Approximate gradient $\hat{L}'(\theta_0)$



Stochastic Gradient Descent

- Find minimum θ^* of function L
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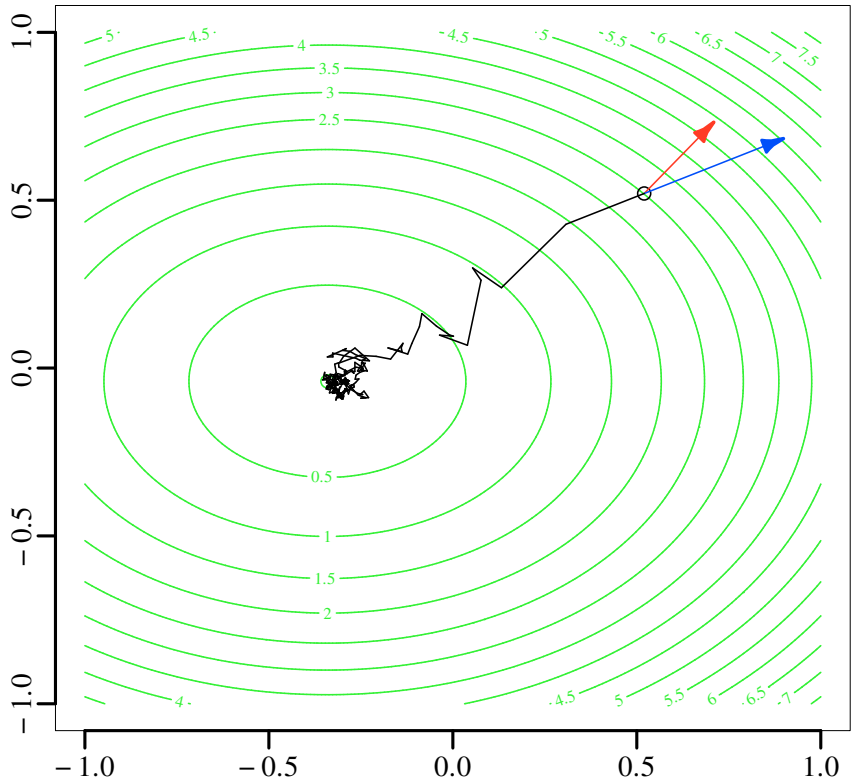
- Move "approximately" downhill



Stochastic Gradient Descent

- Find minimum θ^* of function L
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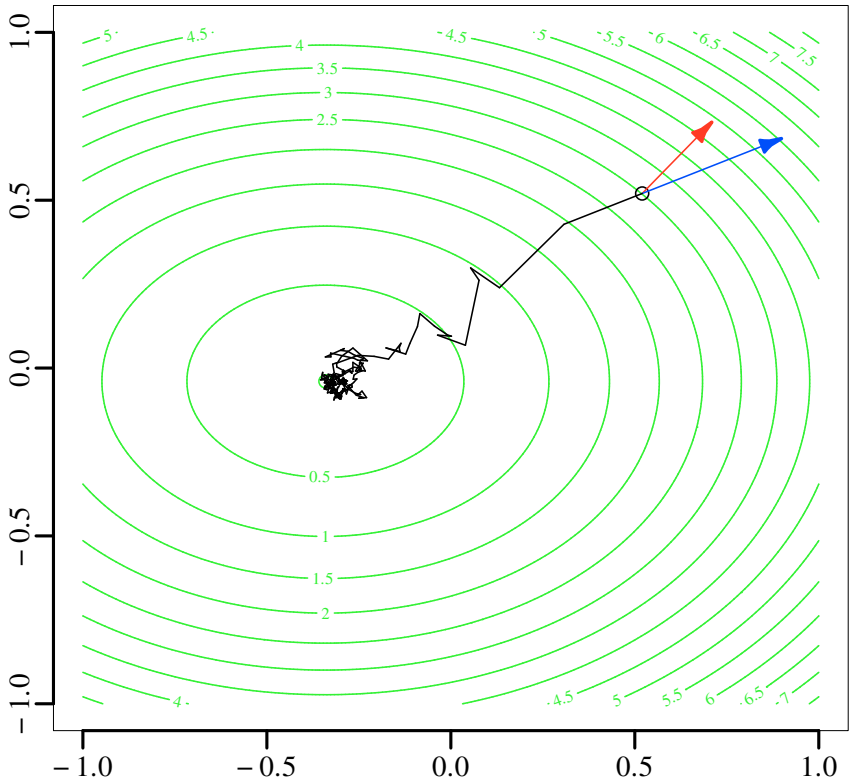
- Stochastic difference equation

$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$



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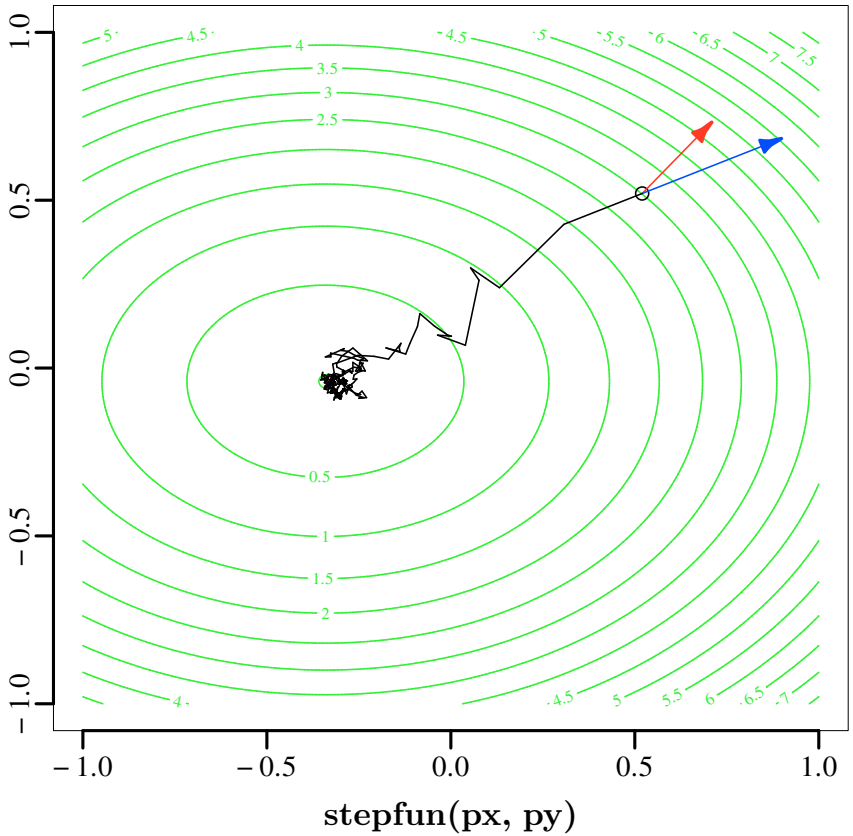
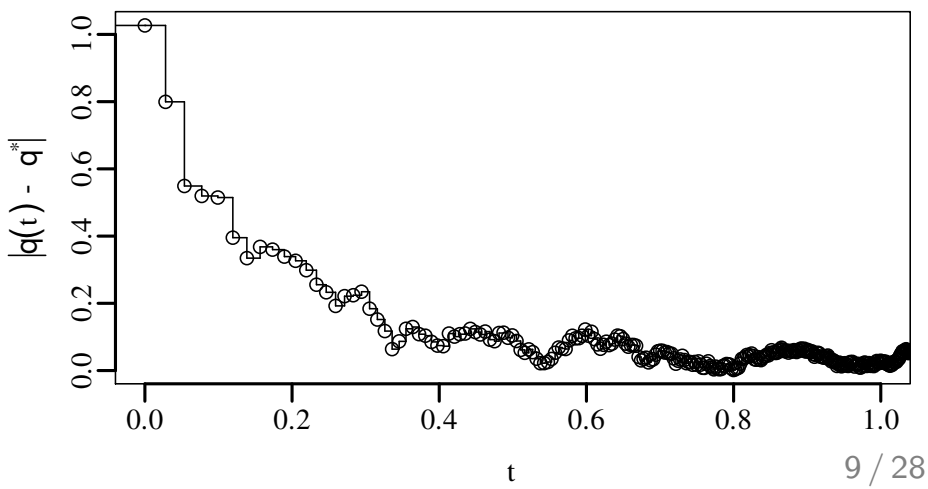
- Under certain conditions, asymptotically approximates (continuous) gradient descent

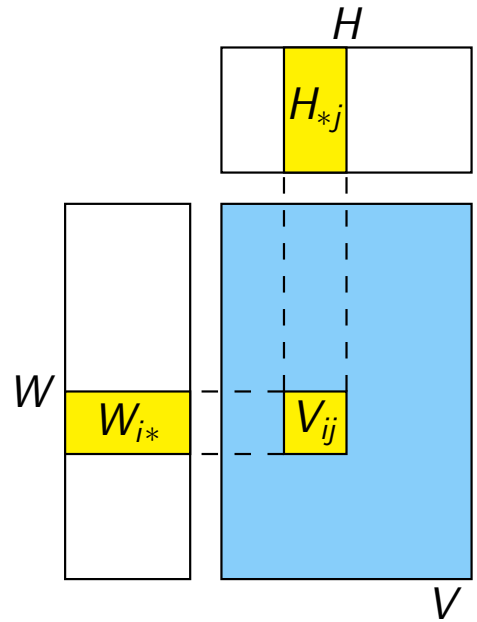




Stochastic Gradient Descent for Matrix Factorization

- Set $\theta = (\mathbf{W}, \mathbf{H})$ and use

$$L(\theta) = \sum_{(i,j) \in Z} L_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$



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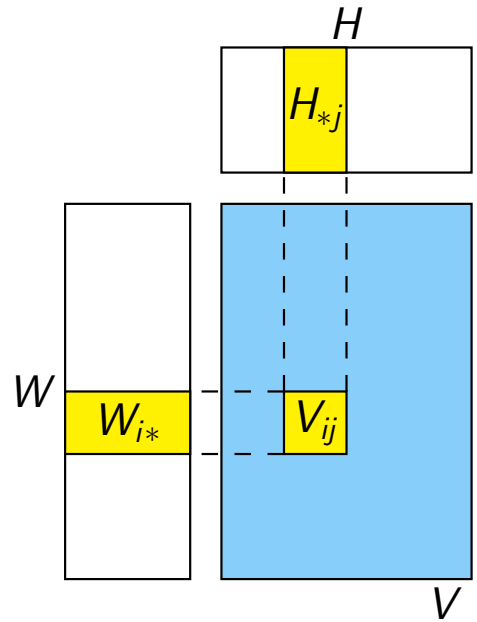

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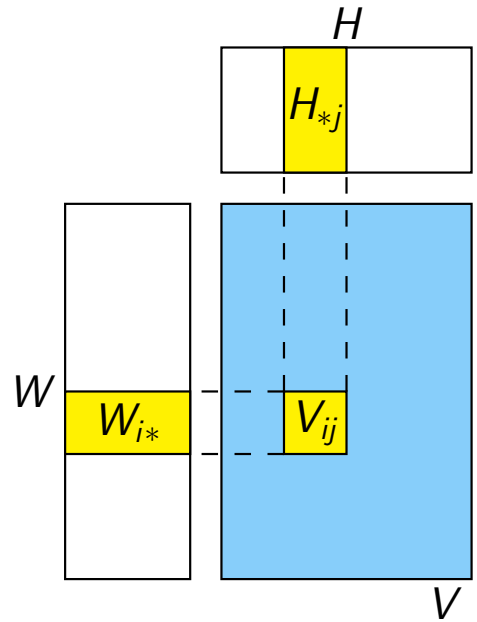
$$\hat{L}'(\theta, z) = N L'_{i_z j_z}(\mathbf{W}_{i_z *}, \mathbf{H}_{*j_z}),$$

where $N = |Z|$



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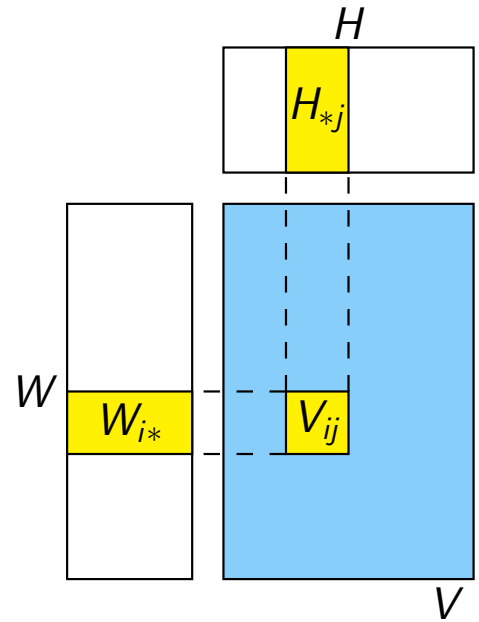
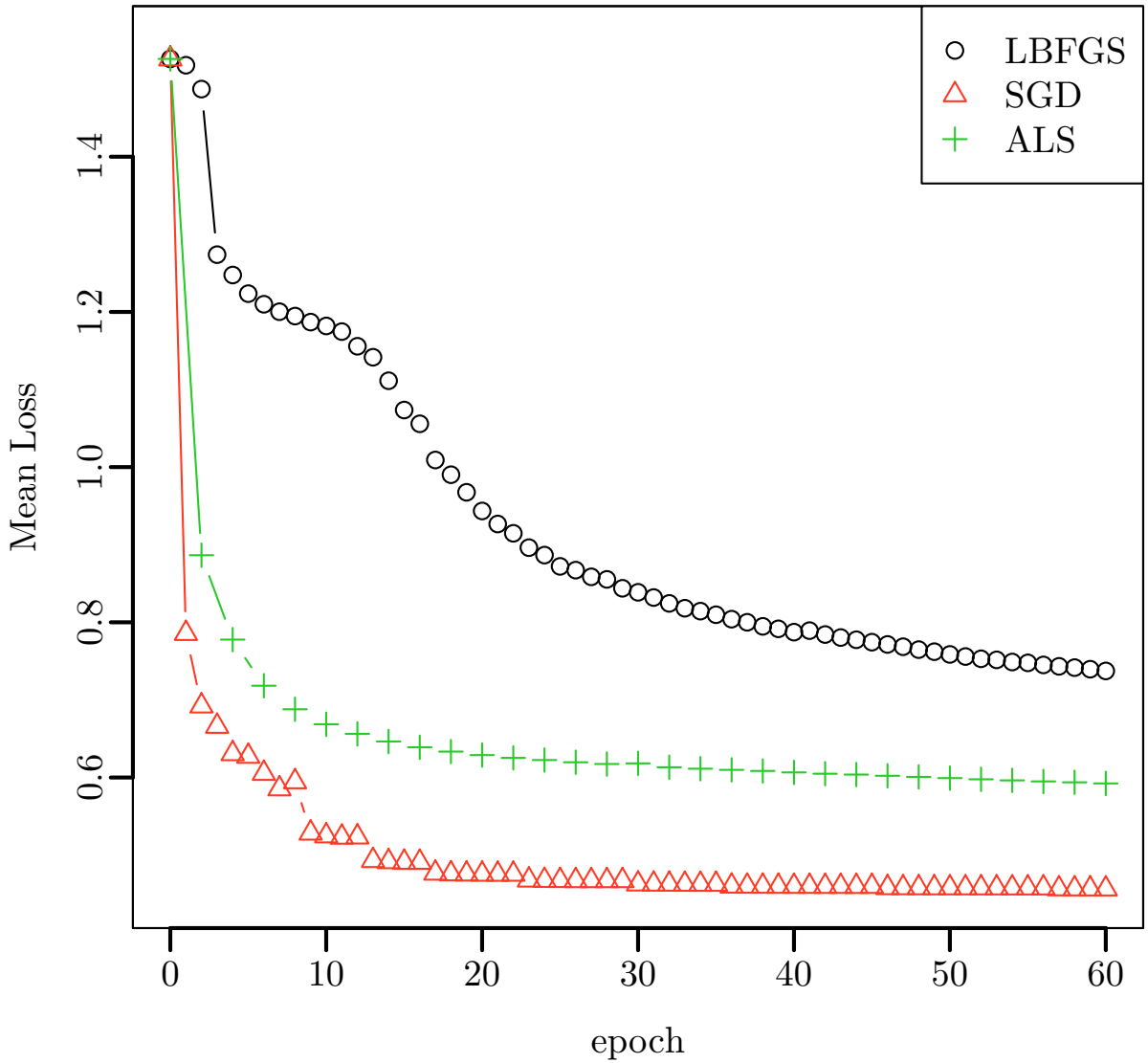


- SGD epoch
  1. Pick a random entry $z \in Z$
  2. Compute approximate gradient $\hat{L}'(\theta, z)$
  3. Update parameters
$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n, z)$$
  4. Repeat N times

Stochastic Gradient Descent on Netflix Data



Outline

Averaging Techniques

- SGD steps depend on each other

$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

How to distribute?

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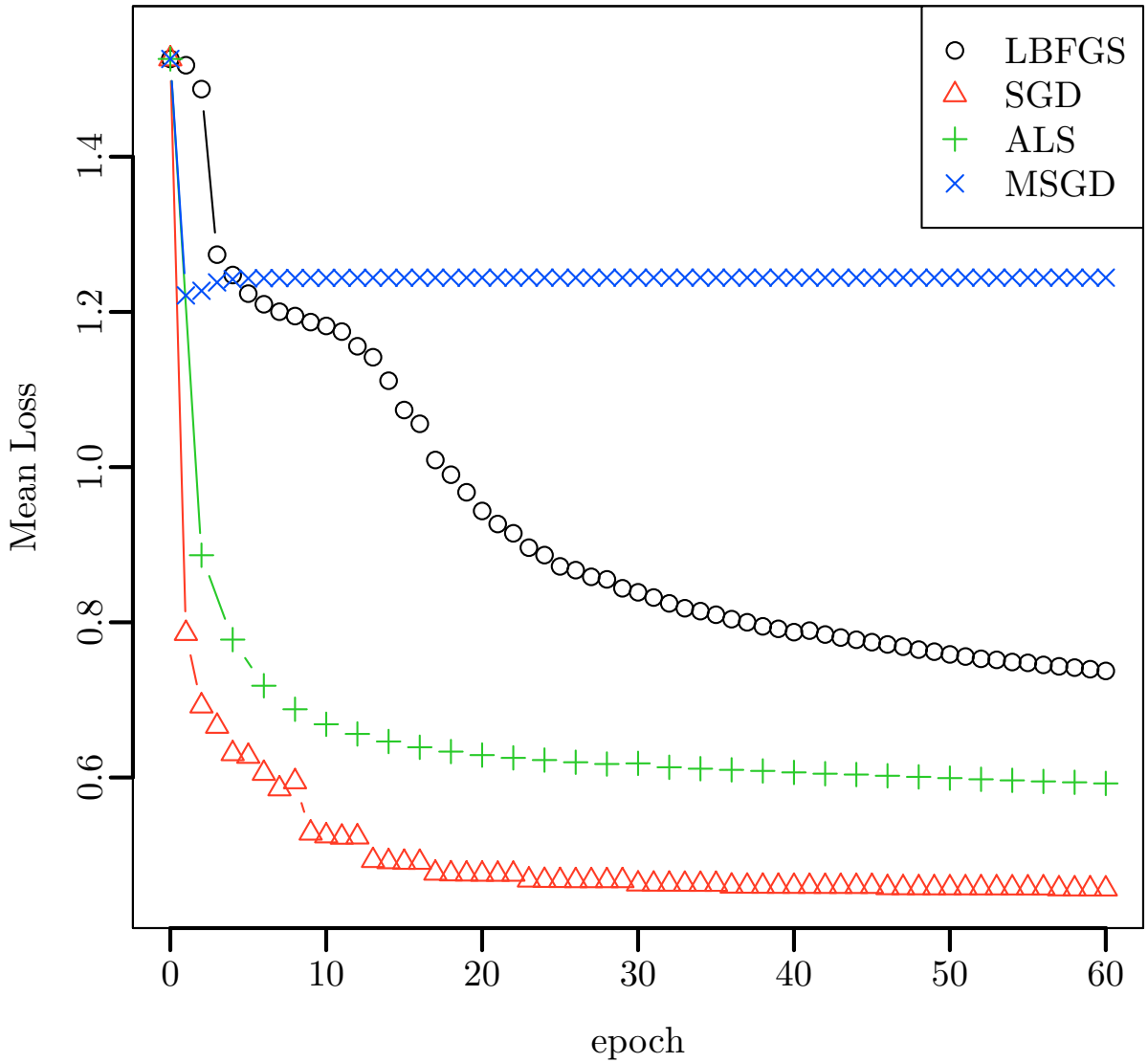

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- Iterative Parameter mixing (ISGD)
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  - Repeat

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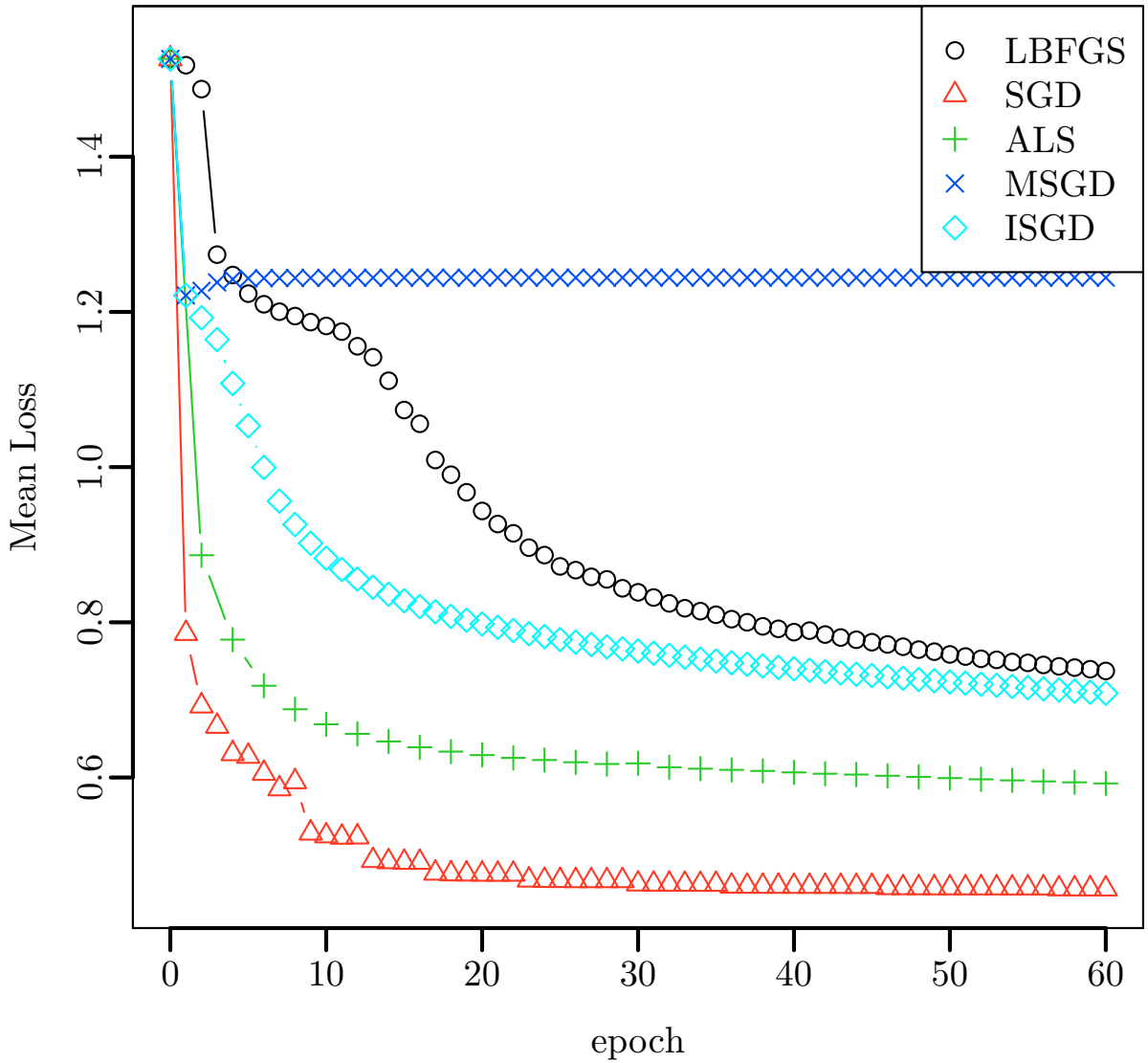

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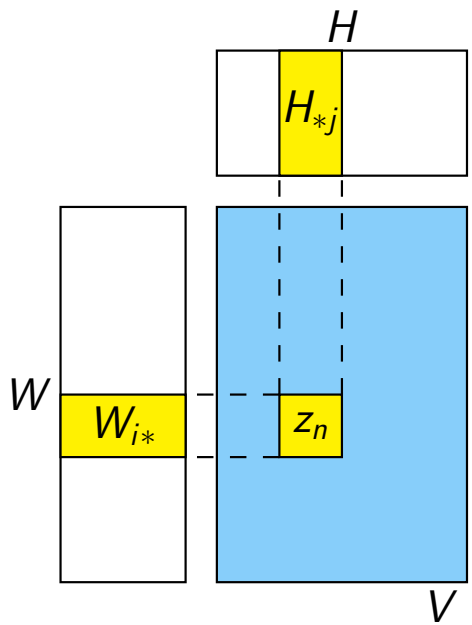
  - Repeat
  - Converges slowly!

Problem Structure

- SGD steps depend on each other

$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

- An SGD step on example $z \in Z$ ...
  1. Reads $W_{i_z *}$ and $H_{* j_z}$
  2. Performs gradient computation $L'_{ij}(W_{i_z *}, H_{* j_z})$
  3. Updates $W_{i_z *}$ and $H_{* j_z}$



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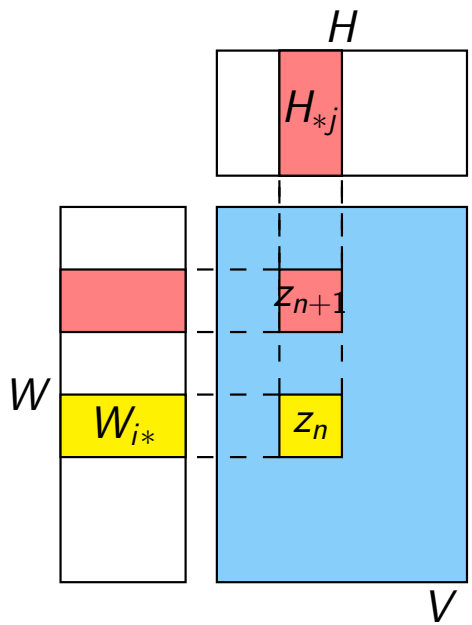

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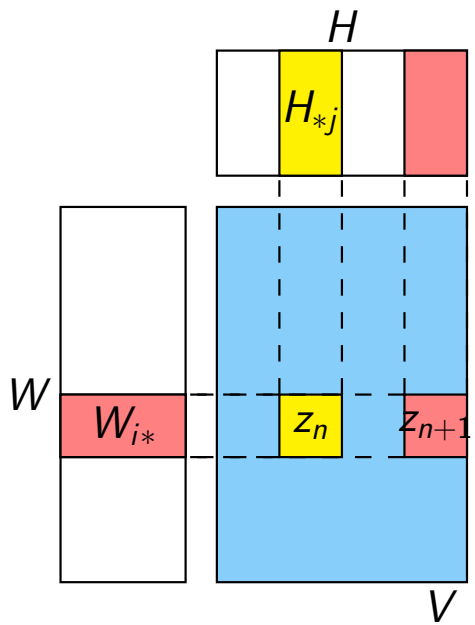

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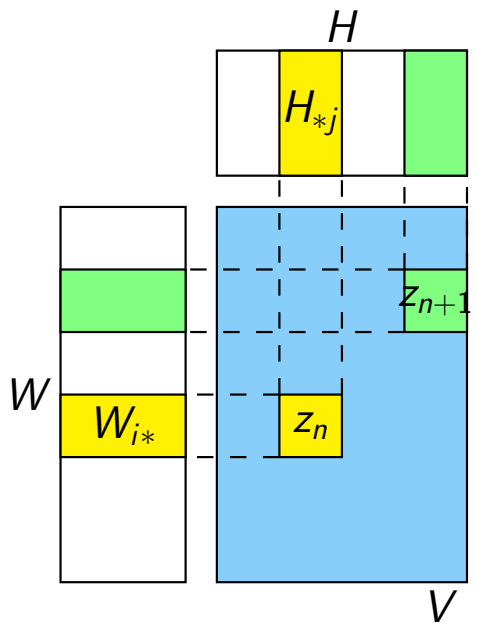
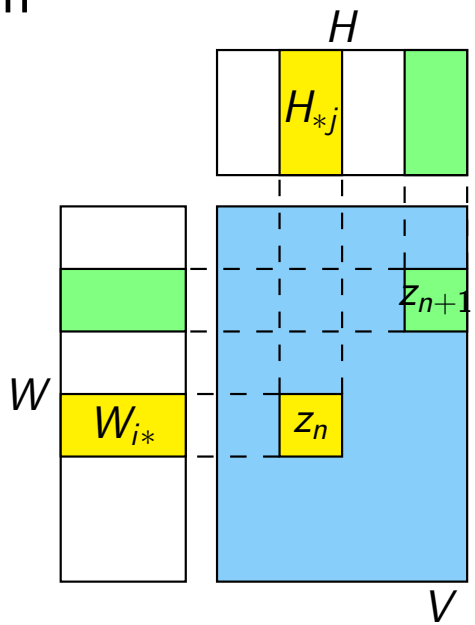
- Not all steps are dependent



Interchangeability

- Two elements $z_1, z_2 \in Z$ are *interchangeable* if they share neither row nor column



- When z_n and z_{n+1} are interchangeable, the SGD steps

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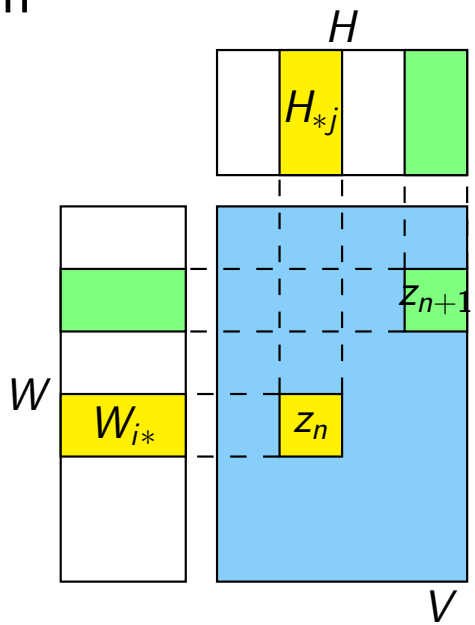

- When z_n and z_{n+1} are interchangeable, the SGD steps

$$\theta_{n+2} = \theta_n - \epsilon \hat{L}'(\theta_n, z_n) - \epsilon \hat{L}'(\theta_{n+1}, z_{n+1})$$

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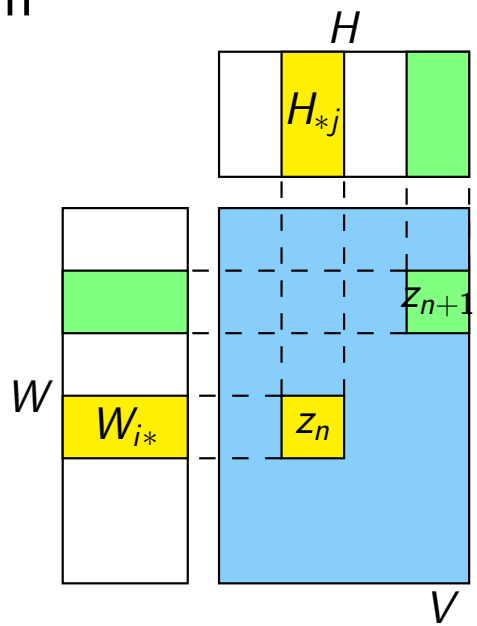


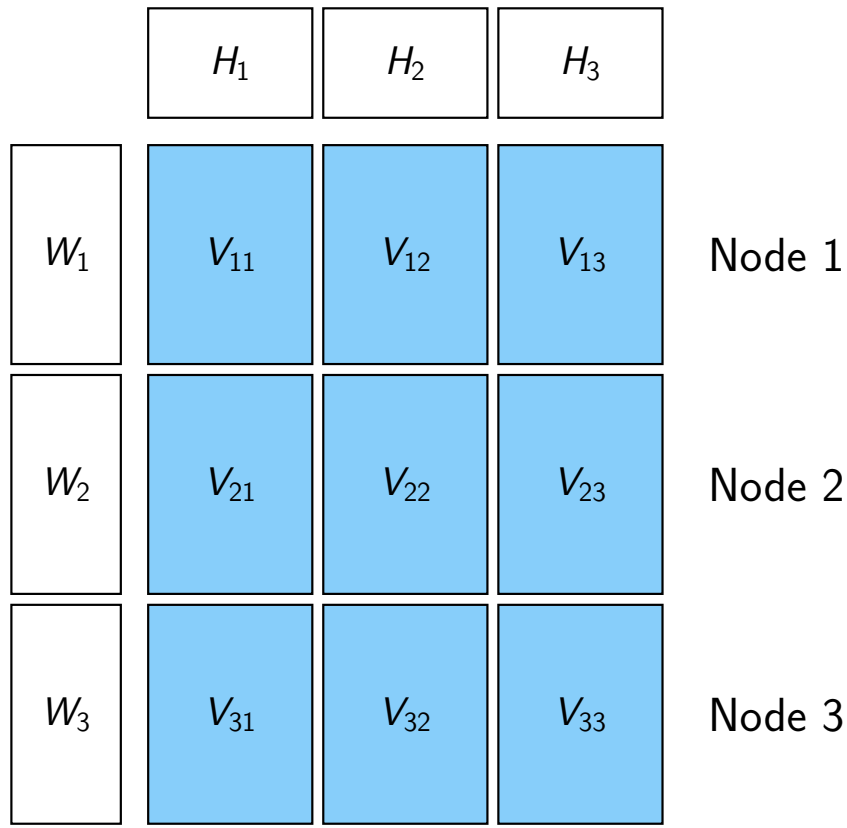
- When z_n and z_{n+1} are interchangeable, the SGD steps

$$\begin{aligned}\theta_{n+2} &= \theta_n - \epsilon\hat{L}'(\theta_n, z_n) - \epsilon\hat{L}'(\theta_{n+1}, z_{n+1}) \\ &= \theta_n - \epsilon\hat{L}'(\theta_n, z_n) - \epsilon\hat{L}'(\theta_n, z_{n+1}),\end{aligned}$$

become parallelizable!

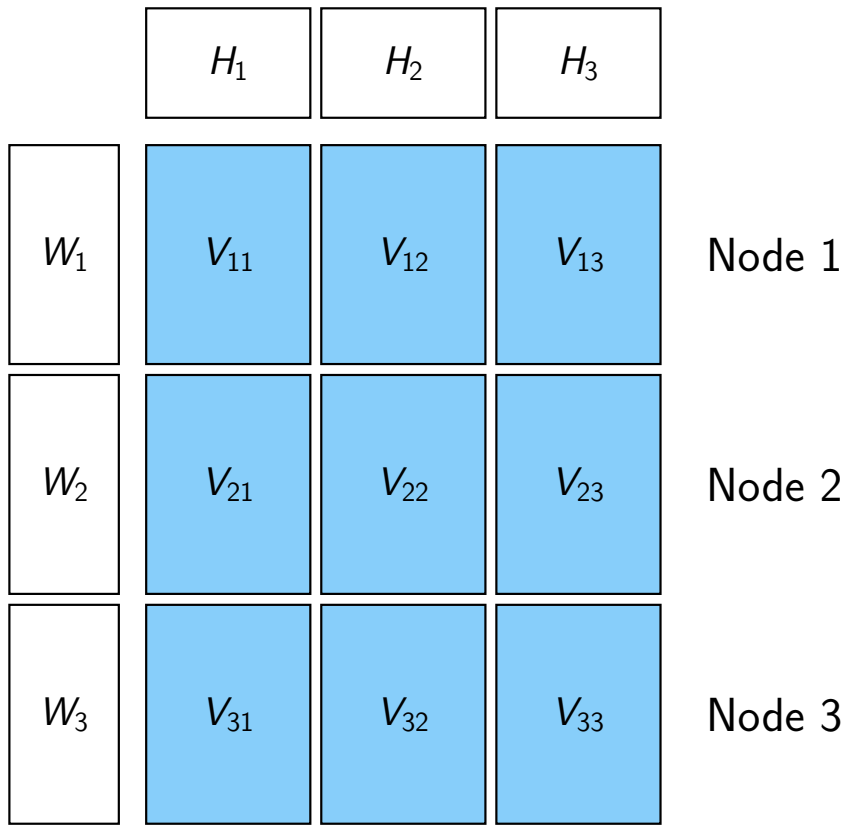
Exploitation

- Block and distribute the input matrix $\mathbf{V}$



Exploitation

- Block and distribute the input matrix **V**
- High-level approach (Map only)
  1. Pick a "diagonal"
  2. Run SGD on the diagonal (in parallel)
  3. Merge the results
  4. Move on to next "diagonal"
     - Steps 1–3 form a *cycle*



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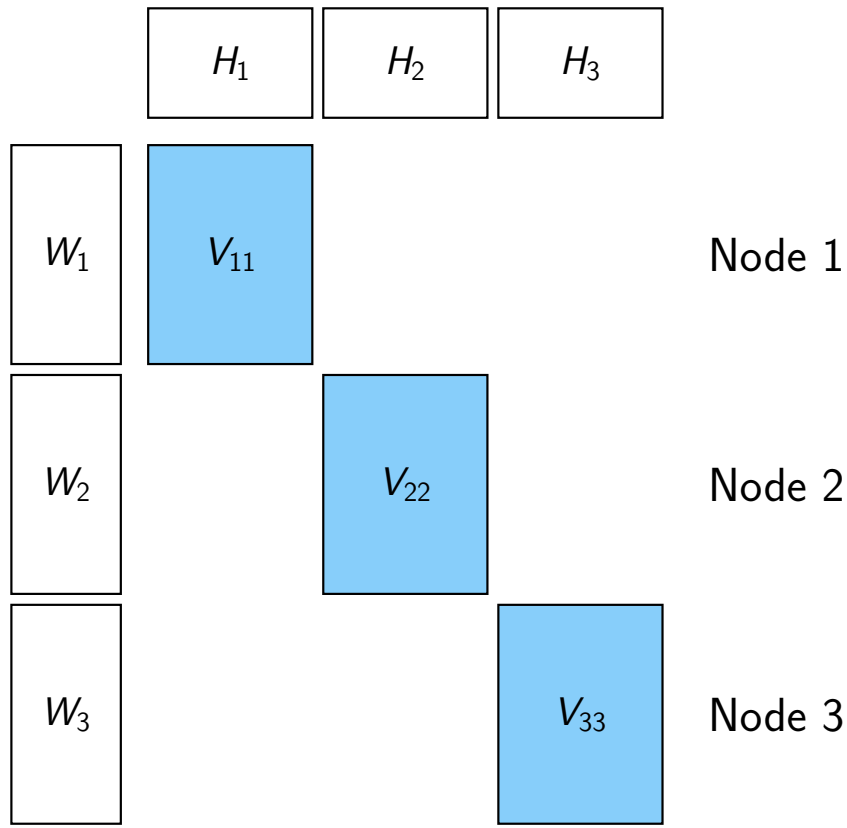

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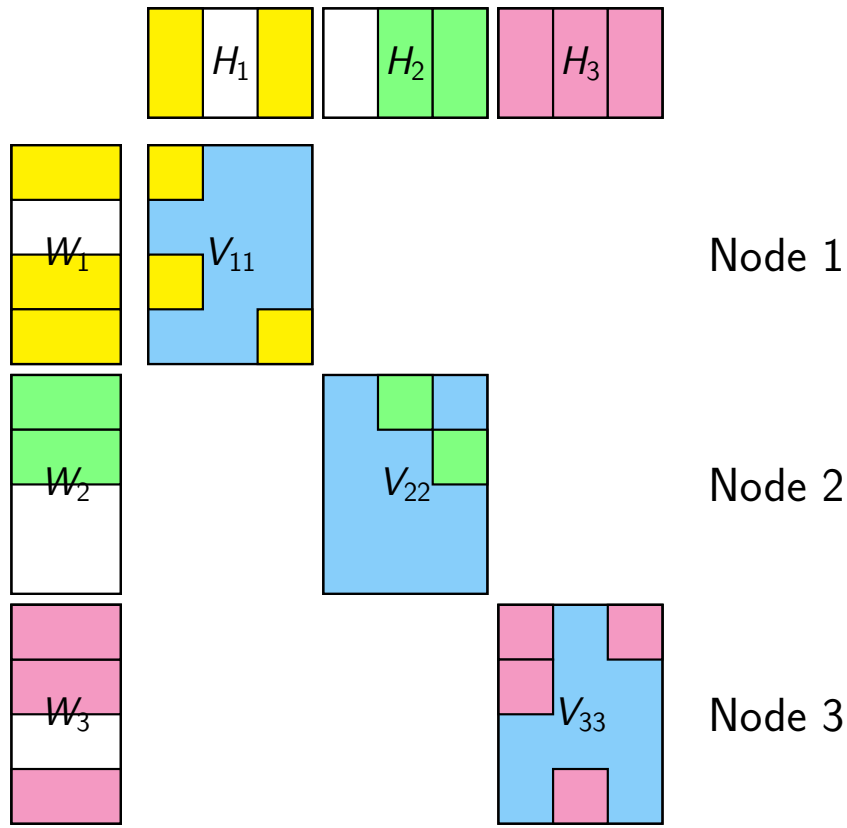
- Step 2:
  Simulate sequential SGD
  - Interchangeable blocks
  - Throw dice of how many iterations per block
  - Throw dice of which step sizes per block



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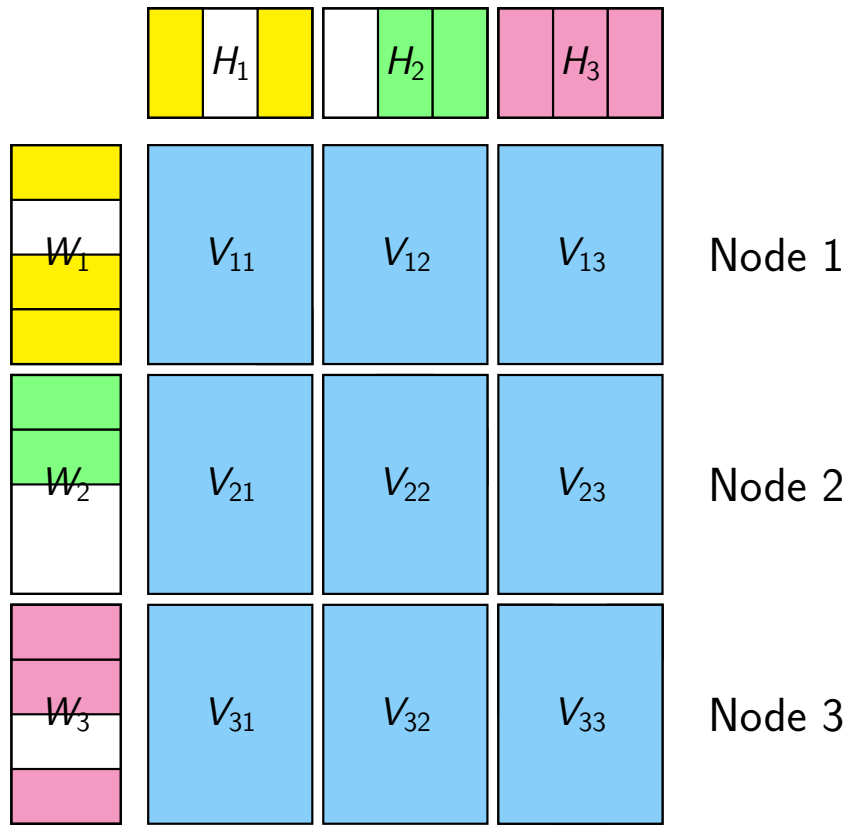

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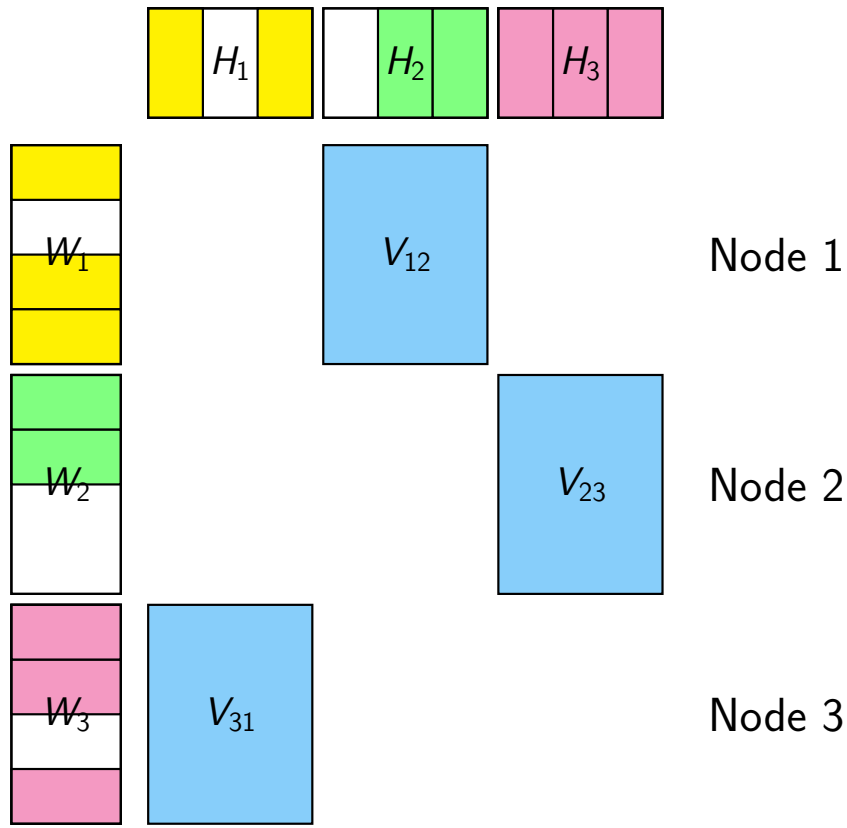

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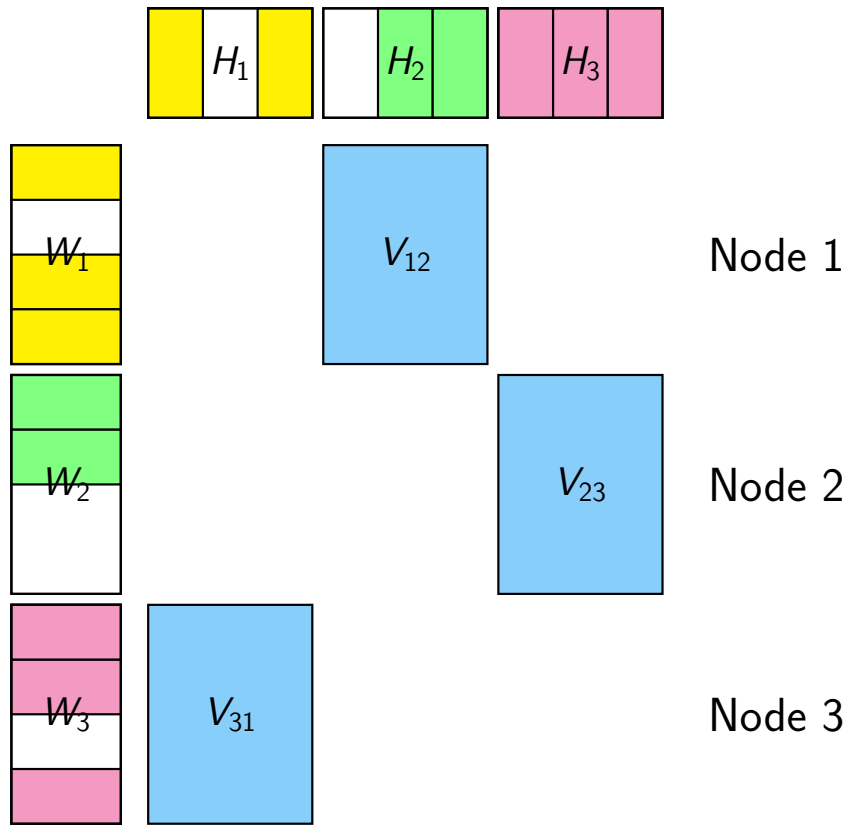
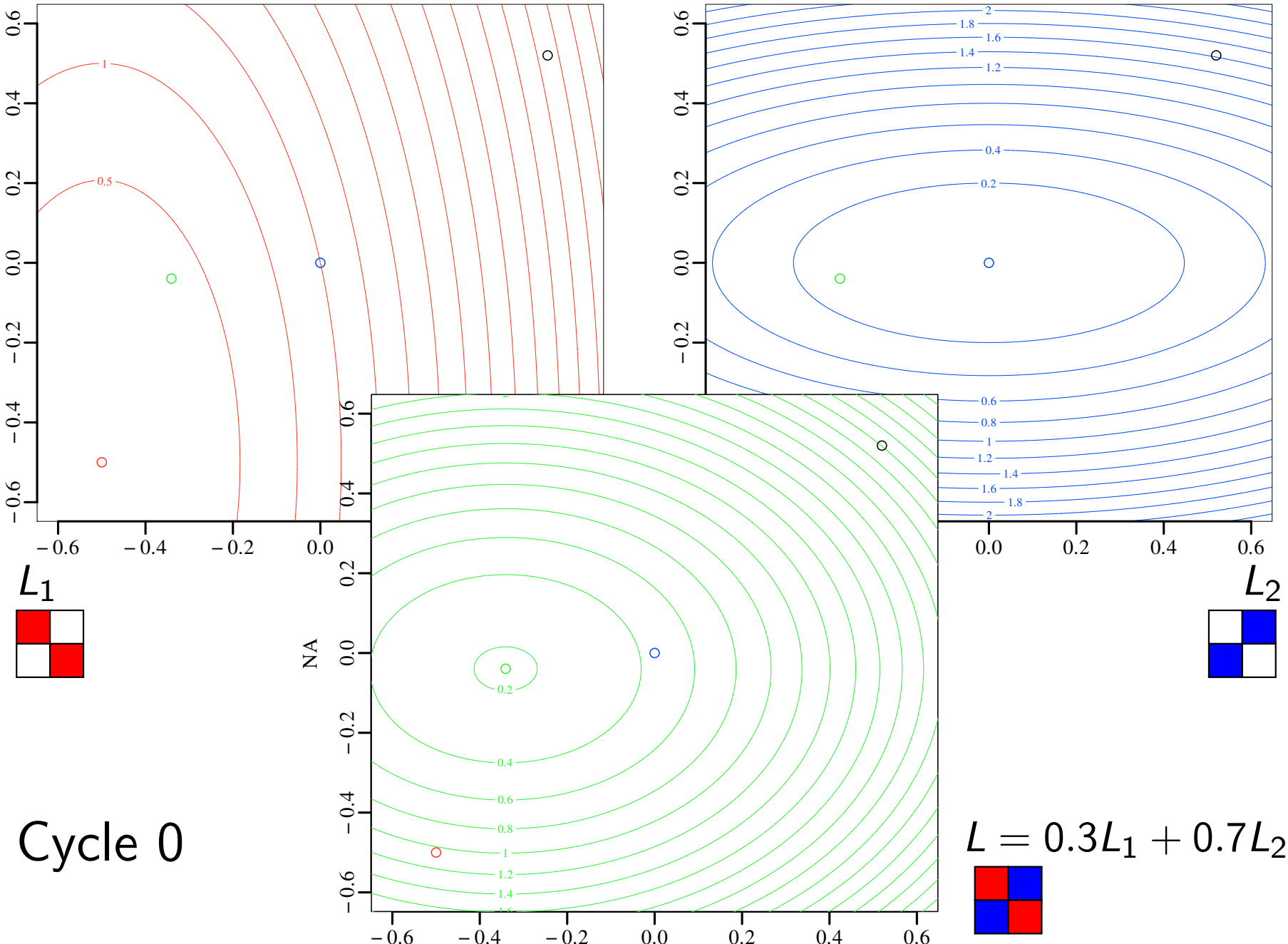
- Instance of "stratified SGD"
- Provably correct



How does it work?



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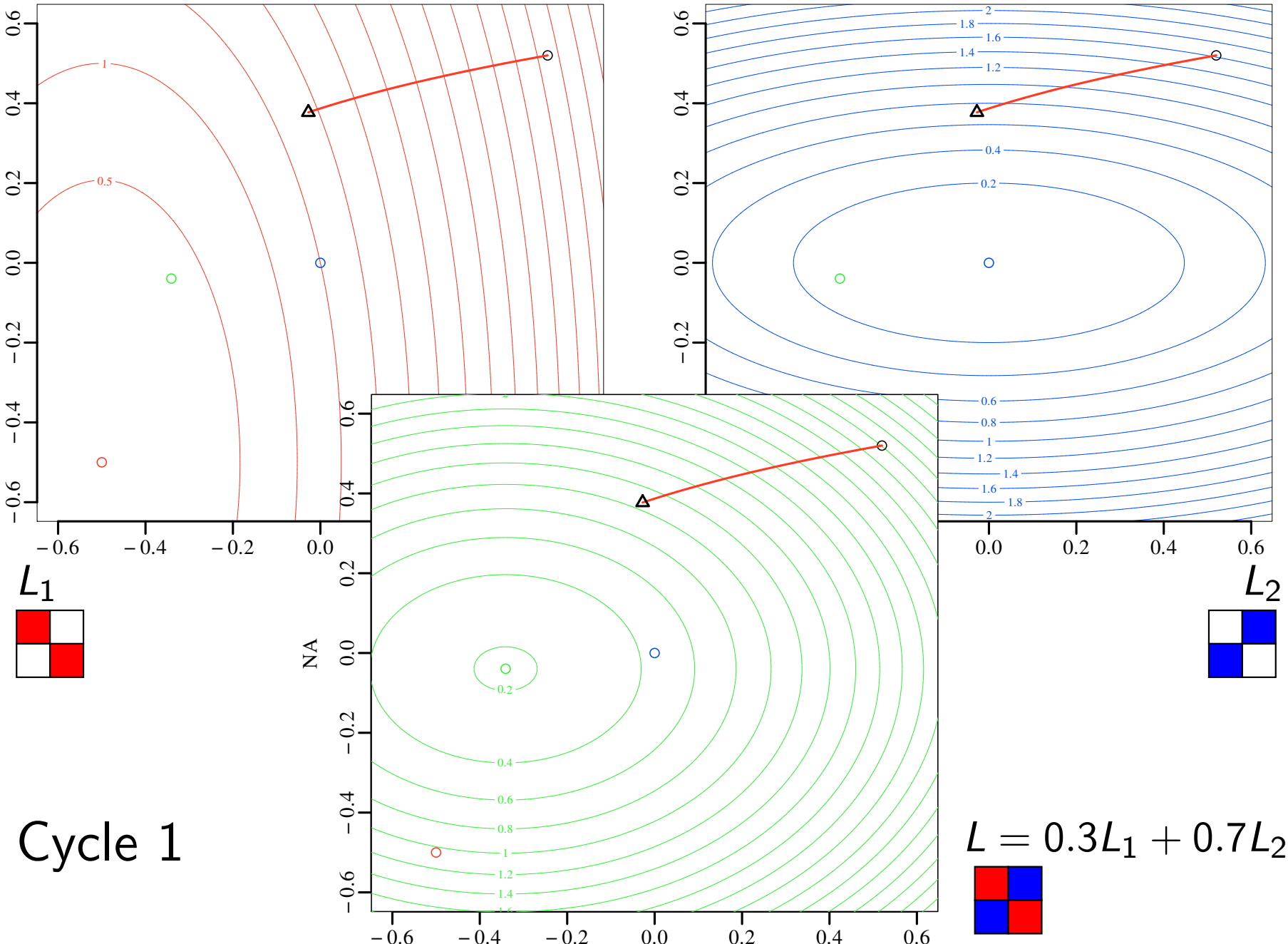

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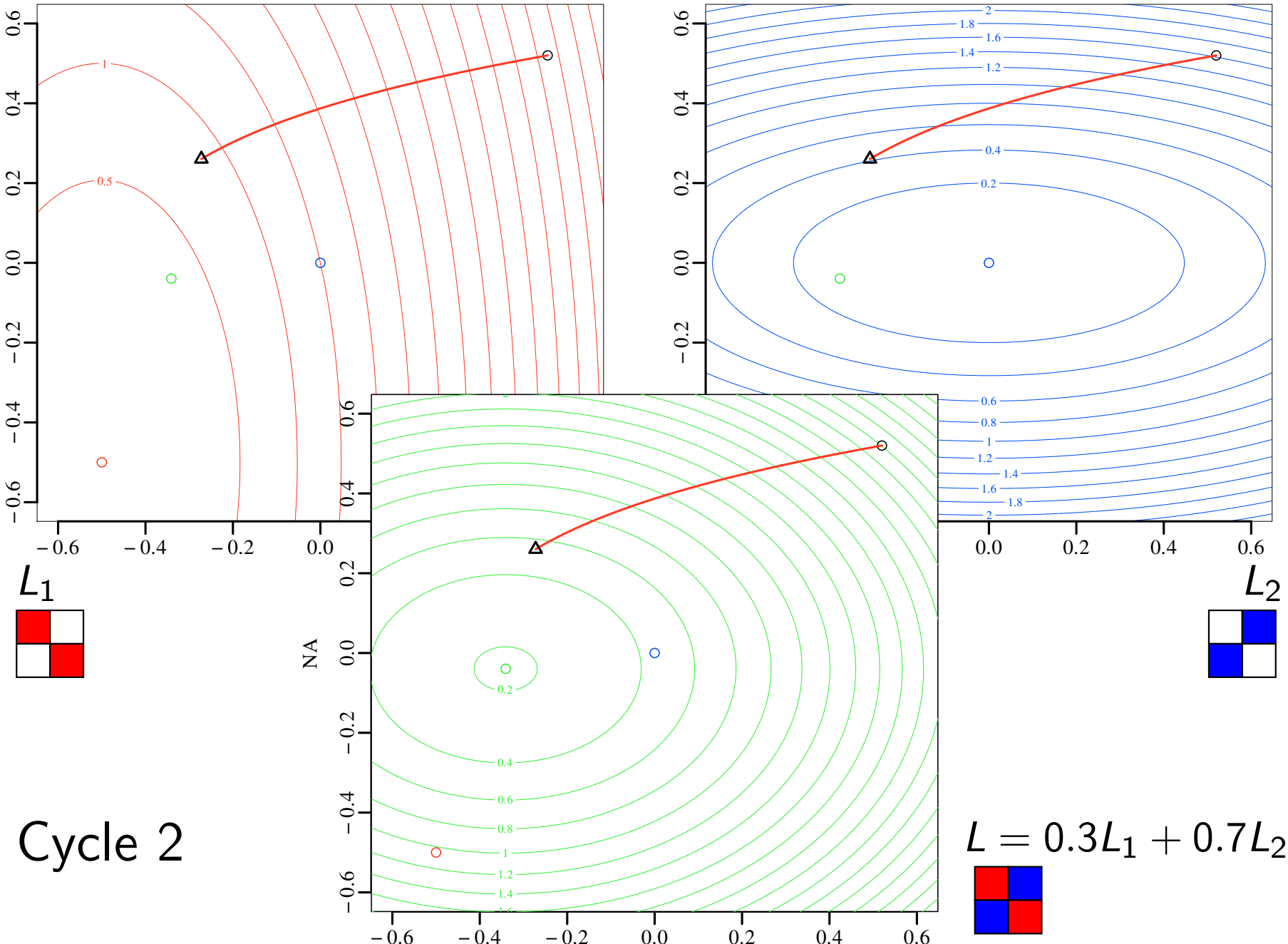
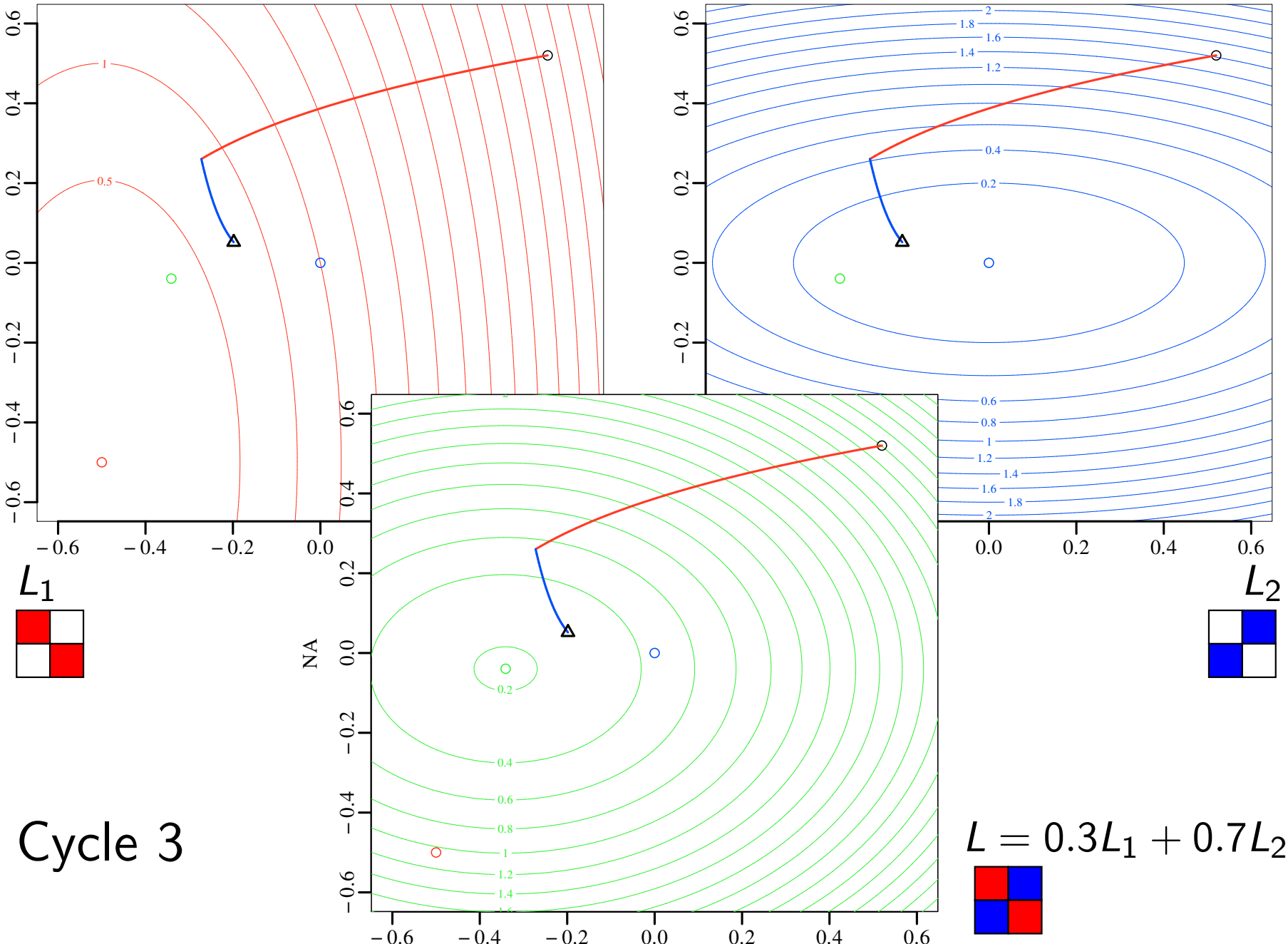


Cycle 2

$L = 0.3L_1 + 0.7L_2$

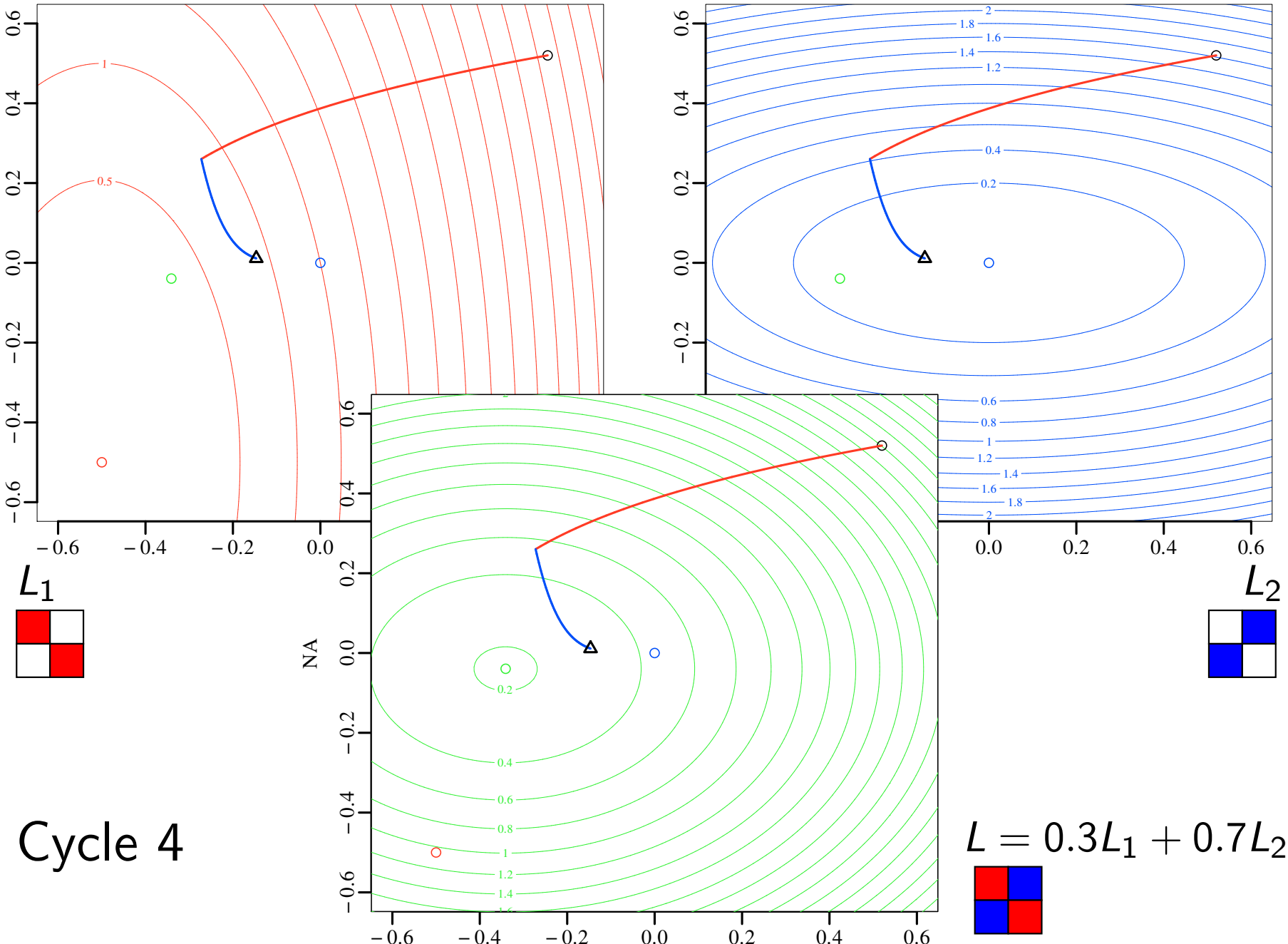
How does it work?



Cycle 3

$L = 0.3L_1 + 0.7L_2$

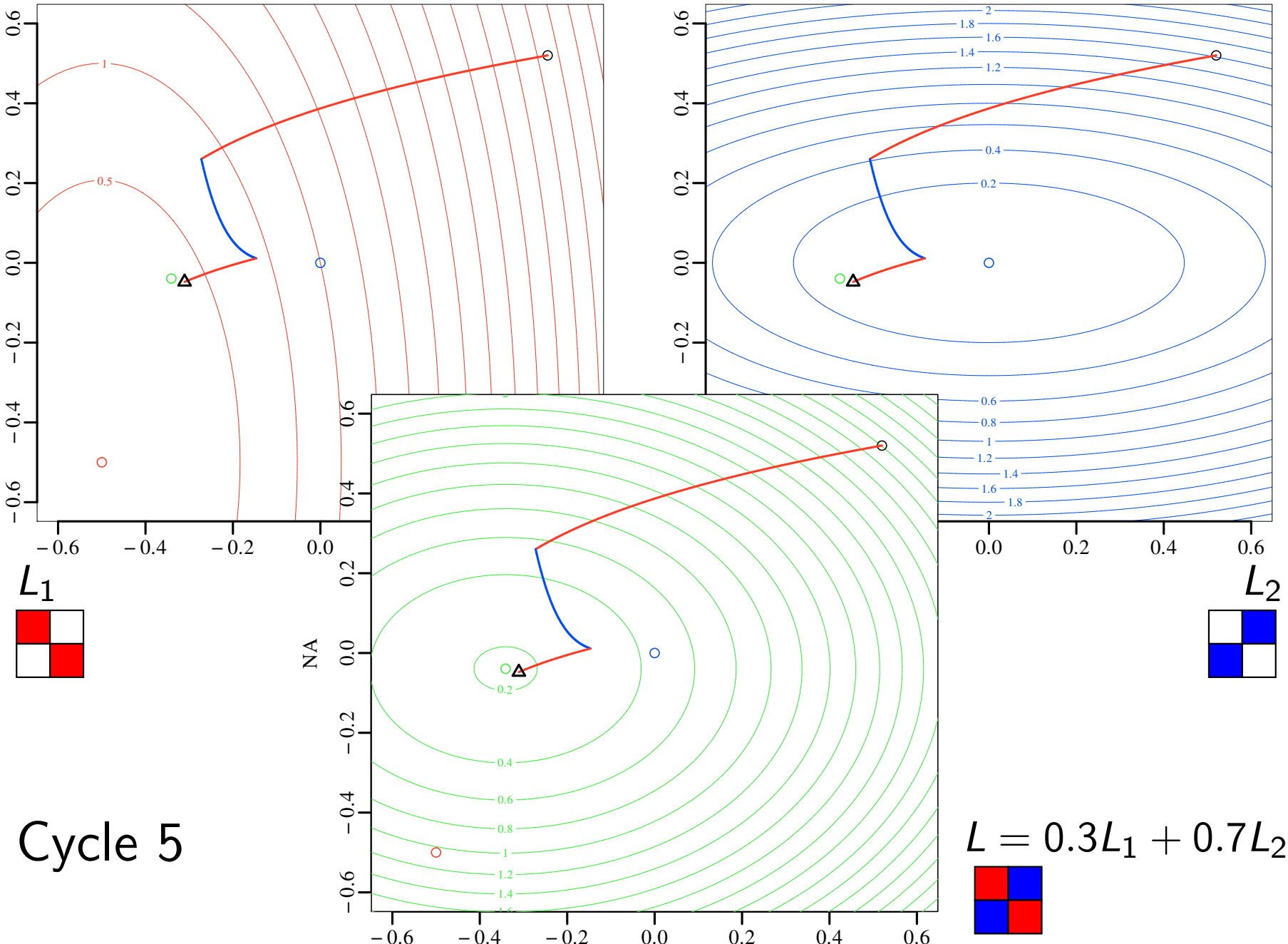
How does it work?



Cycle 4

$L = 0.3L_1 + 0.7L_2$

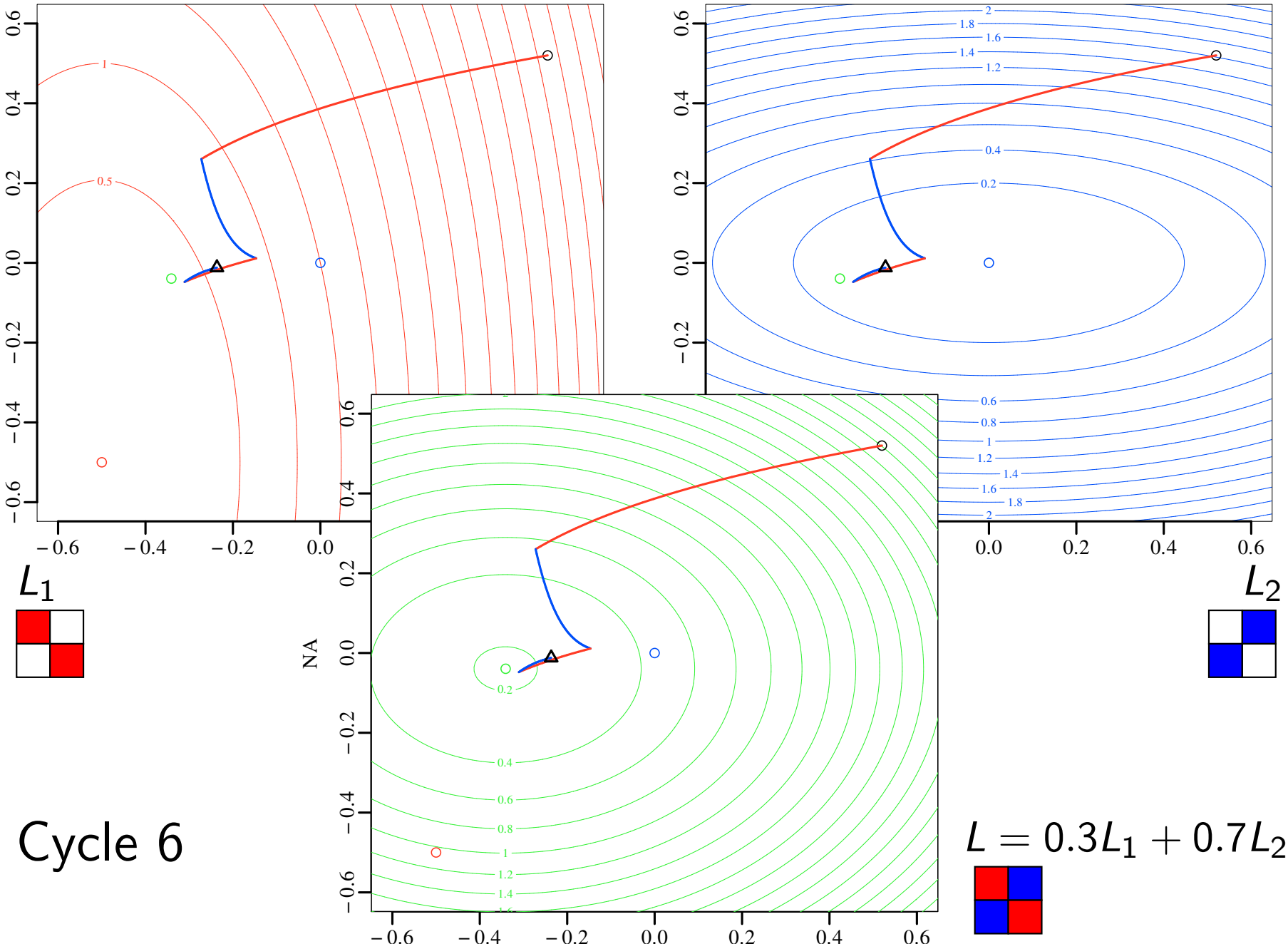
How does it work?



Cycle 5

$L = 0.3L_1 + 0.7L_2$

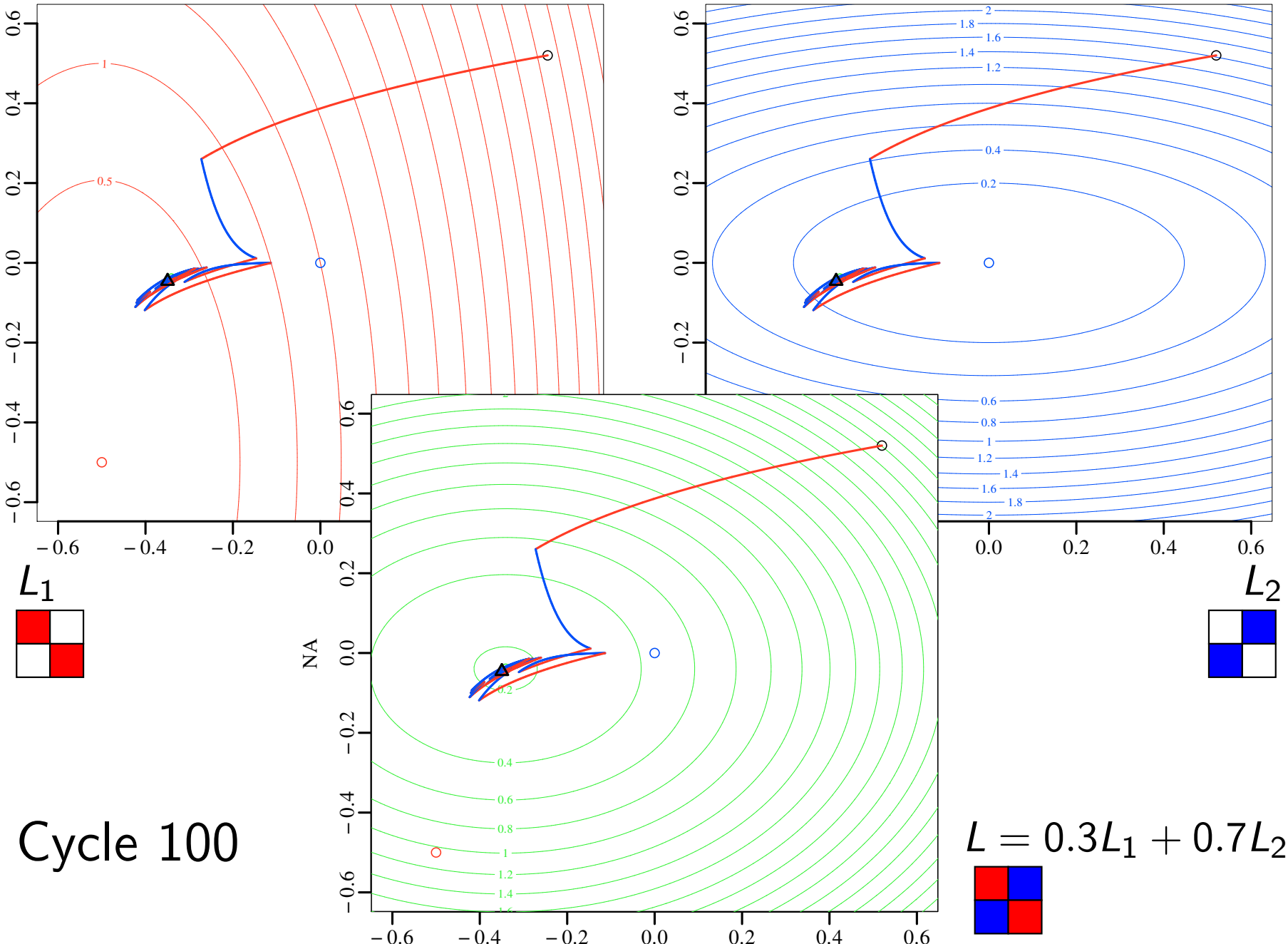
How does it work?



Cycle 6

$L = 0.3L_1 + 0.7L_2$

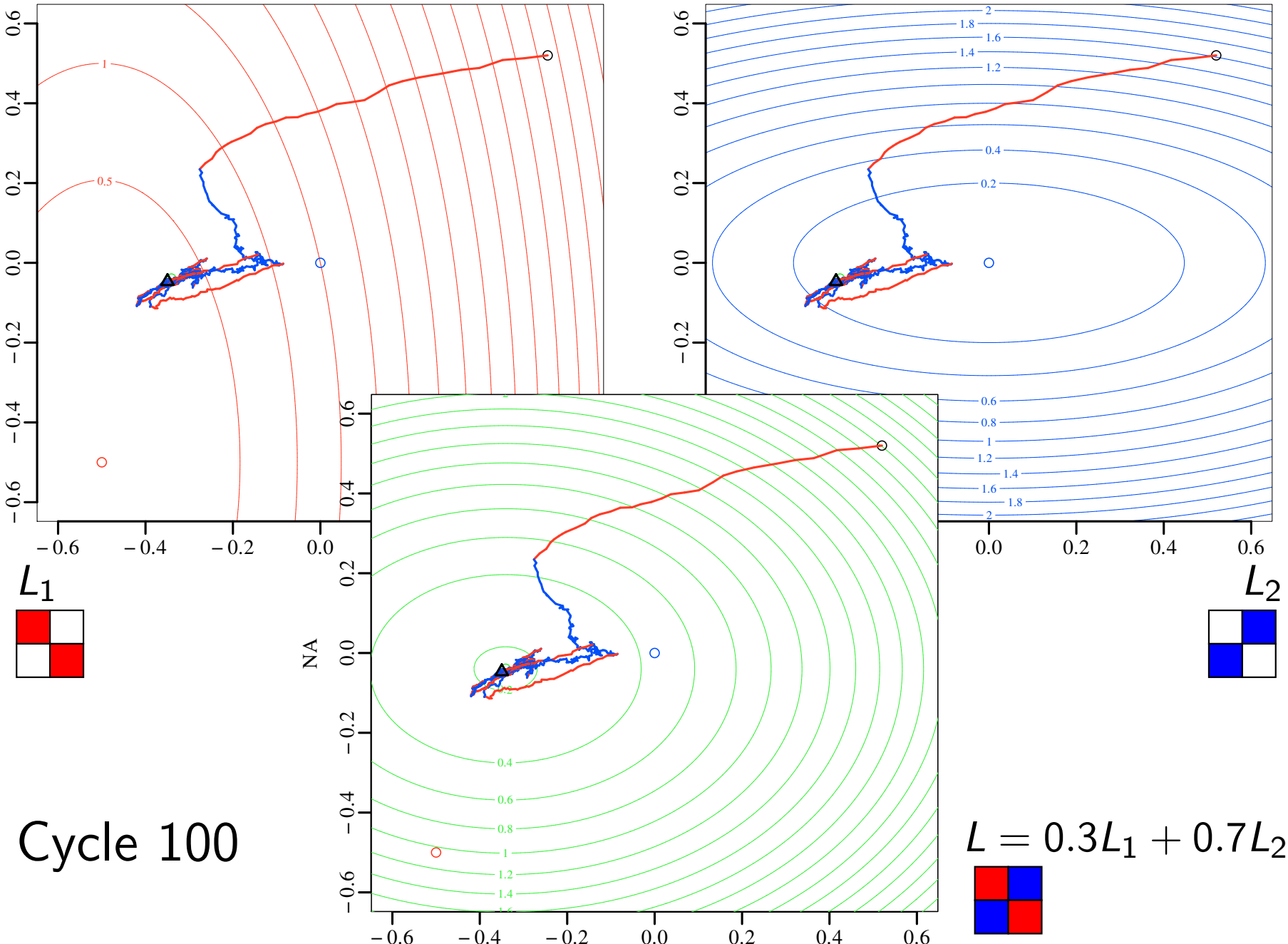
How does it work?



Cycle 100

$L = 0.3L_1 + 0.7L_2$

How does it work?



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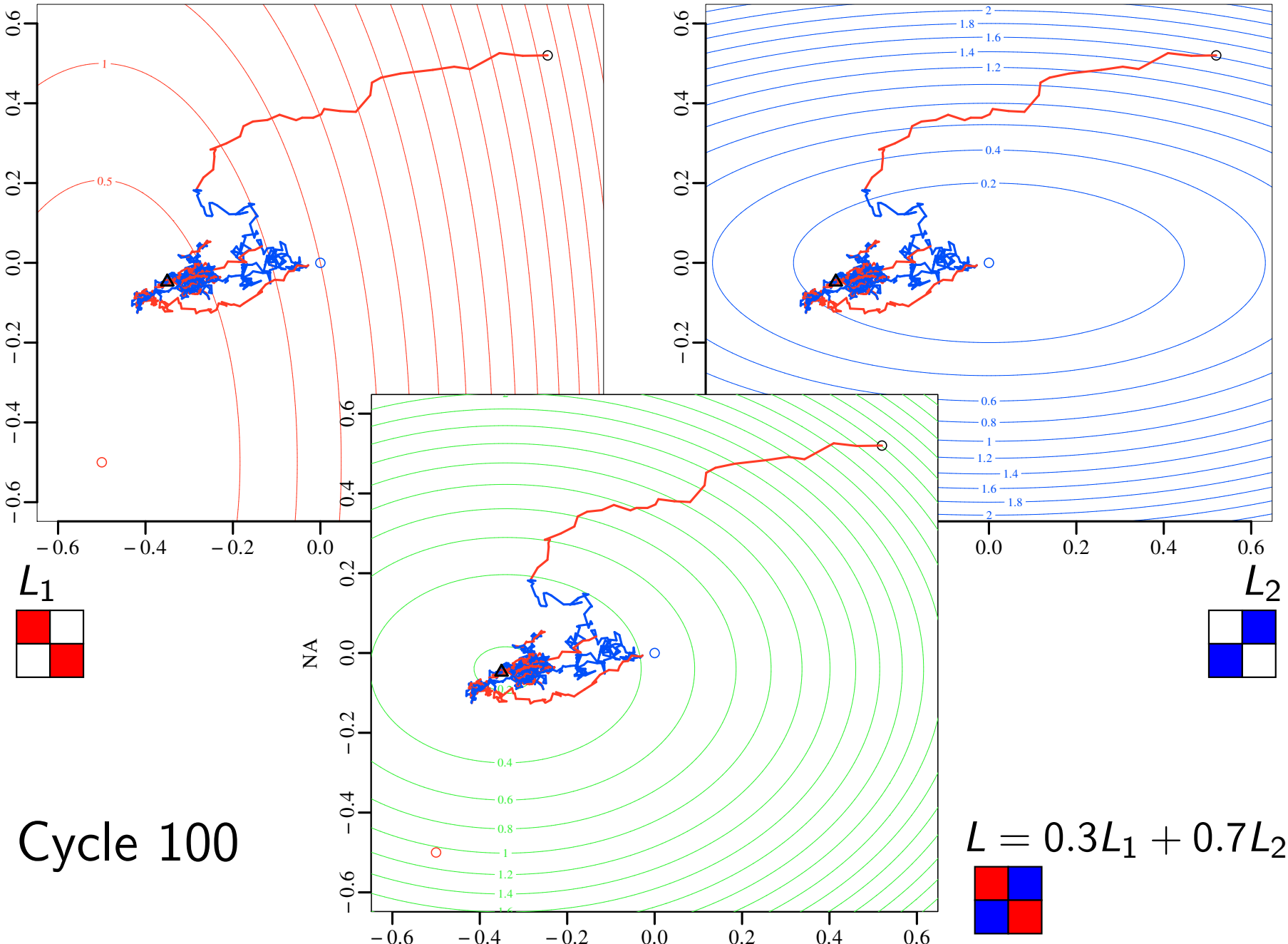
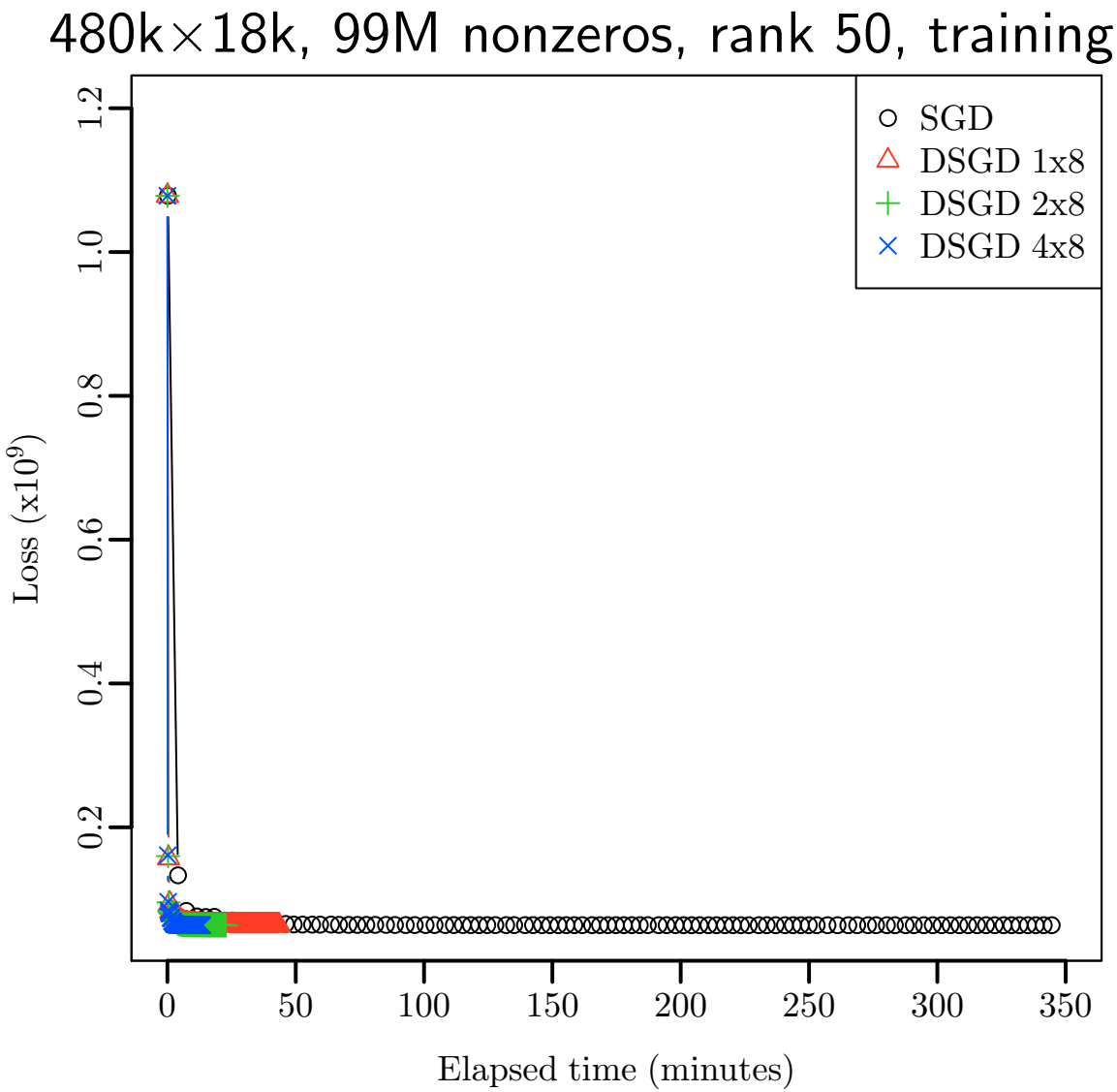
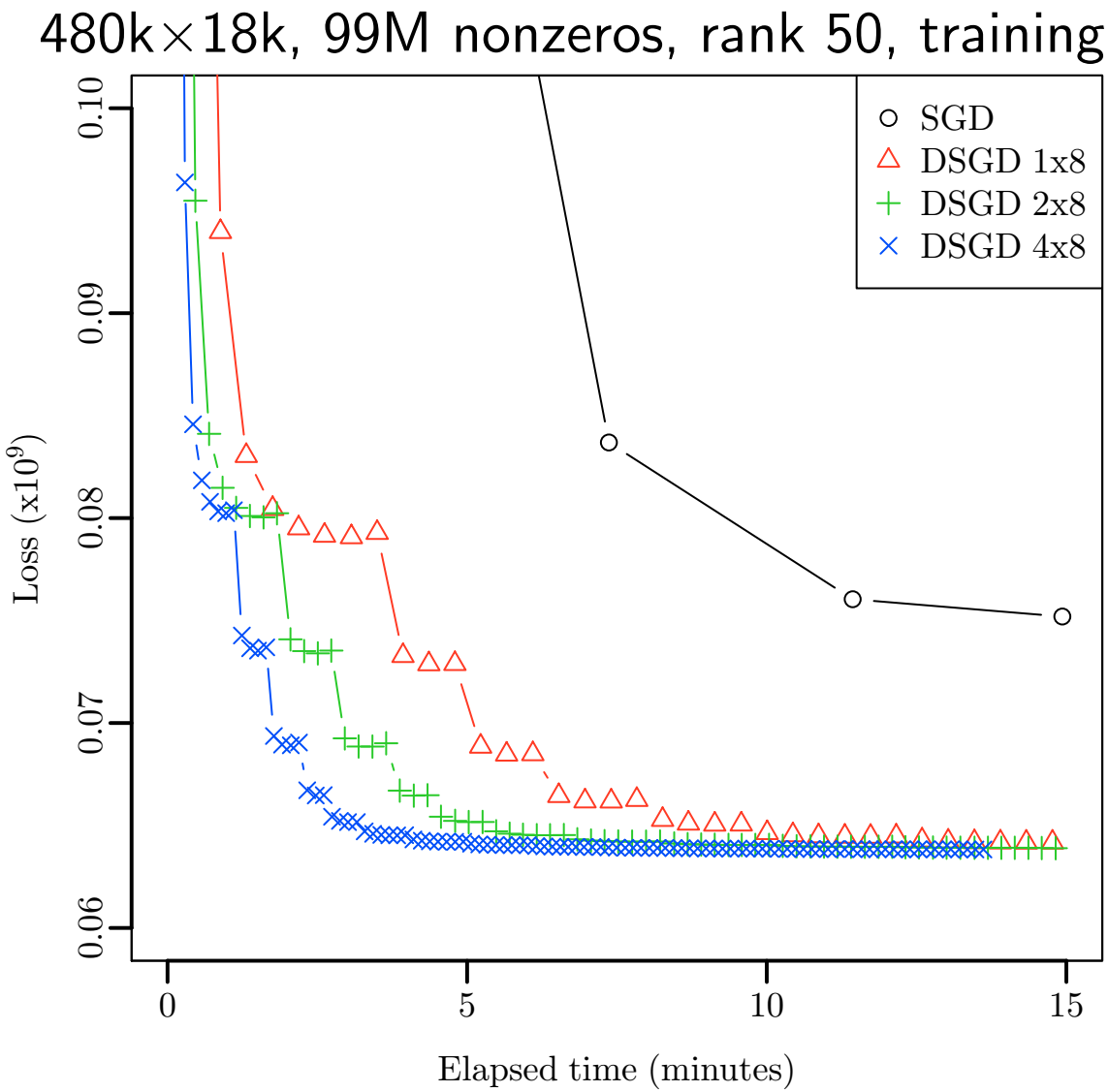


Outline

Netflix, Nzsl+L2



Netflix, Nzsl+L2



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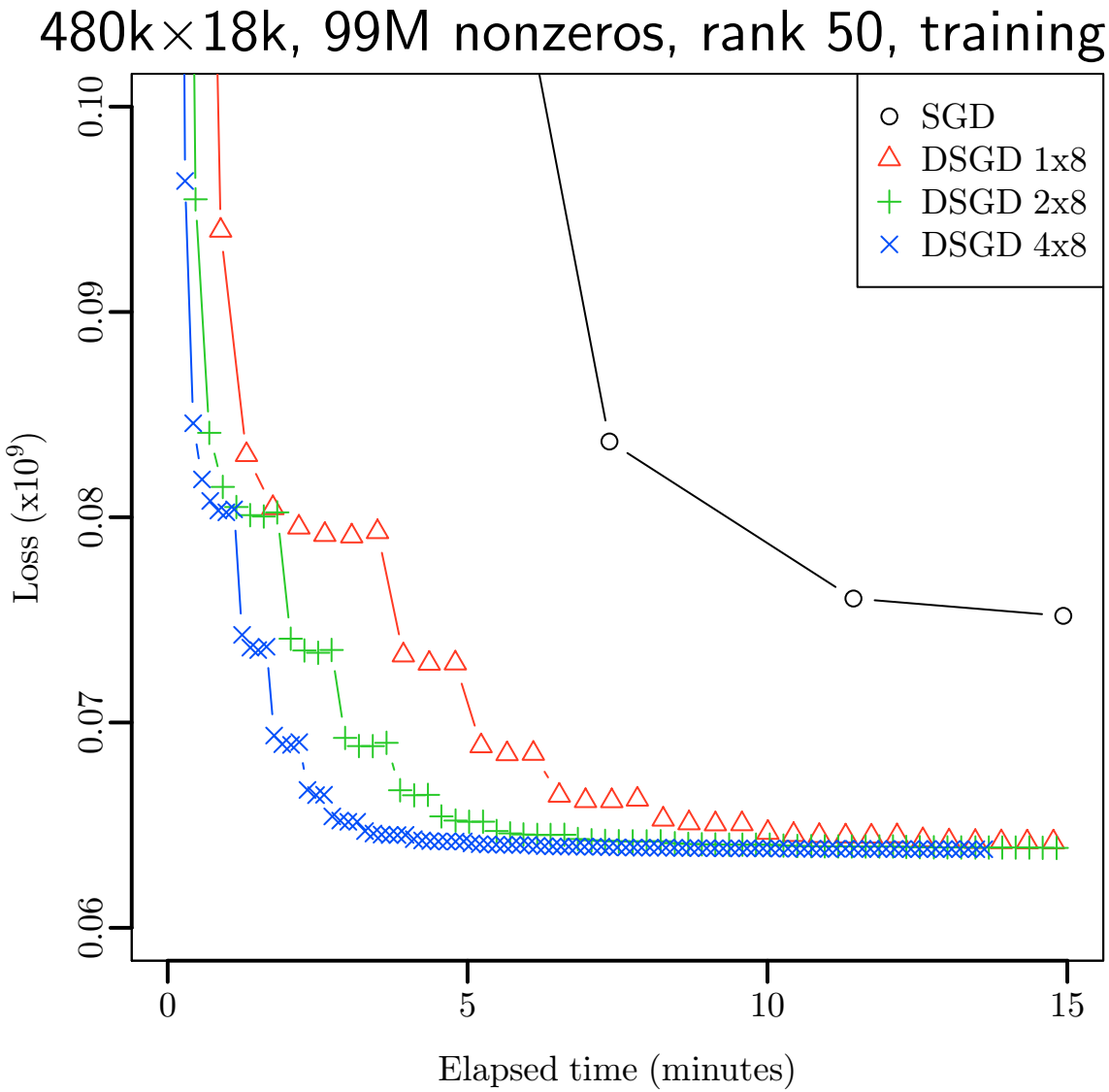
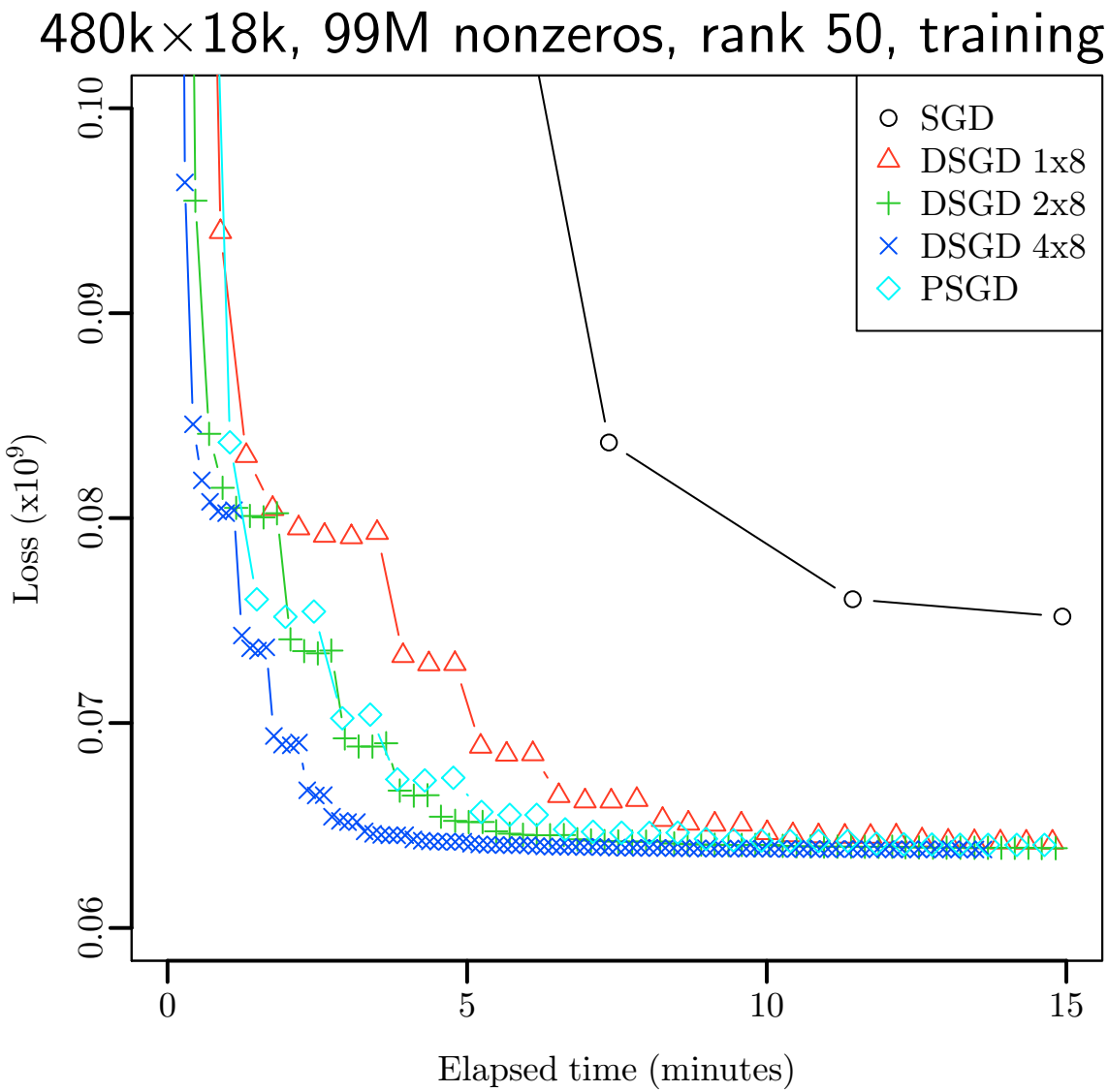


DSGD converges significantly faster than SGD.

Netflix, Nzsl+L2



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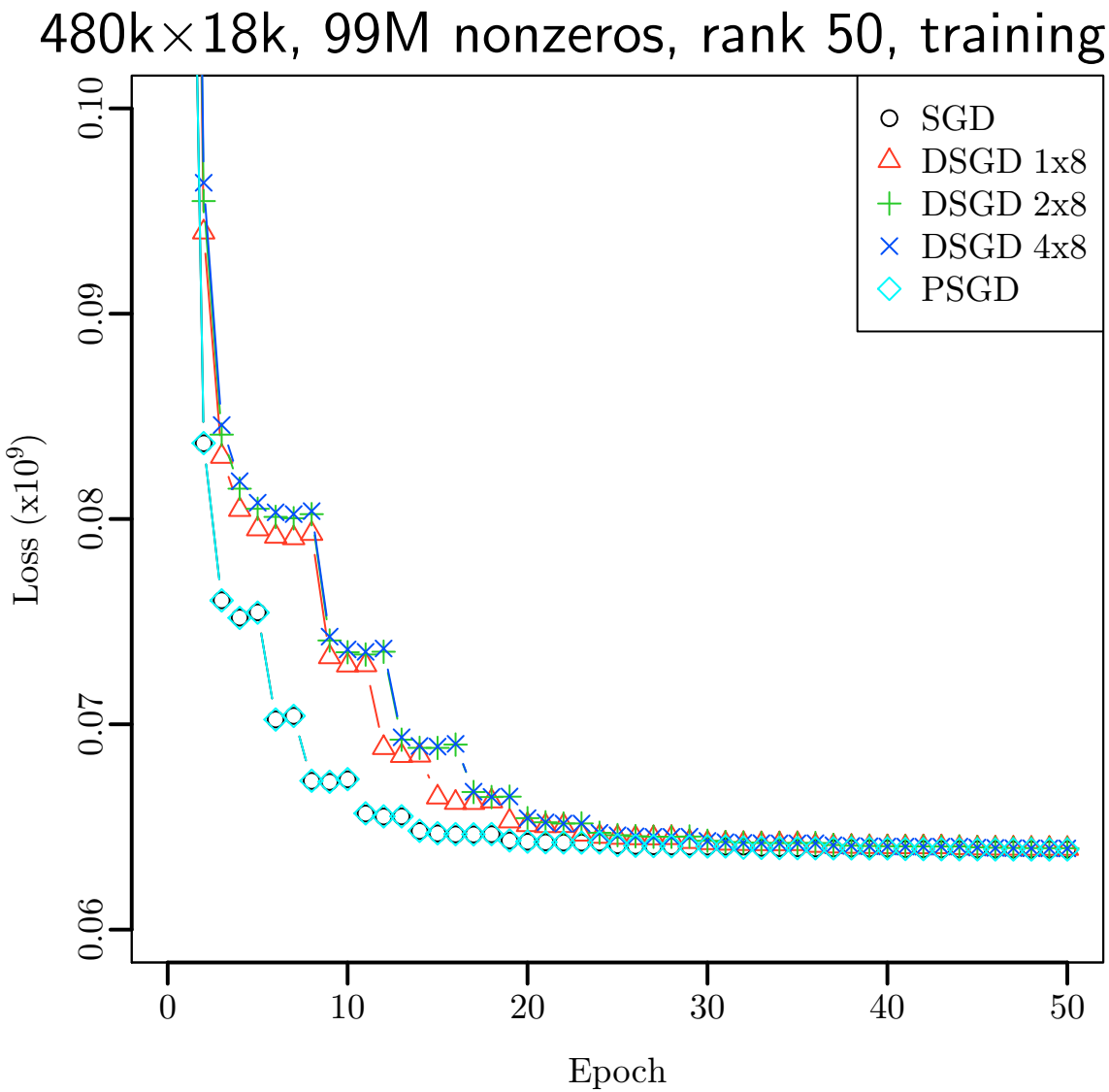

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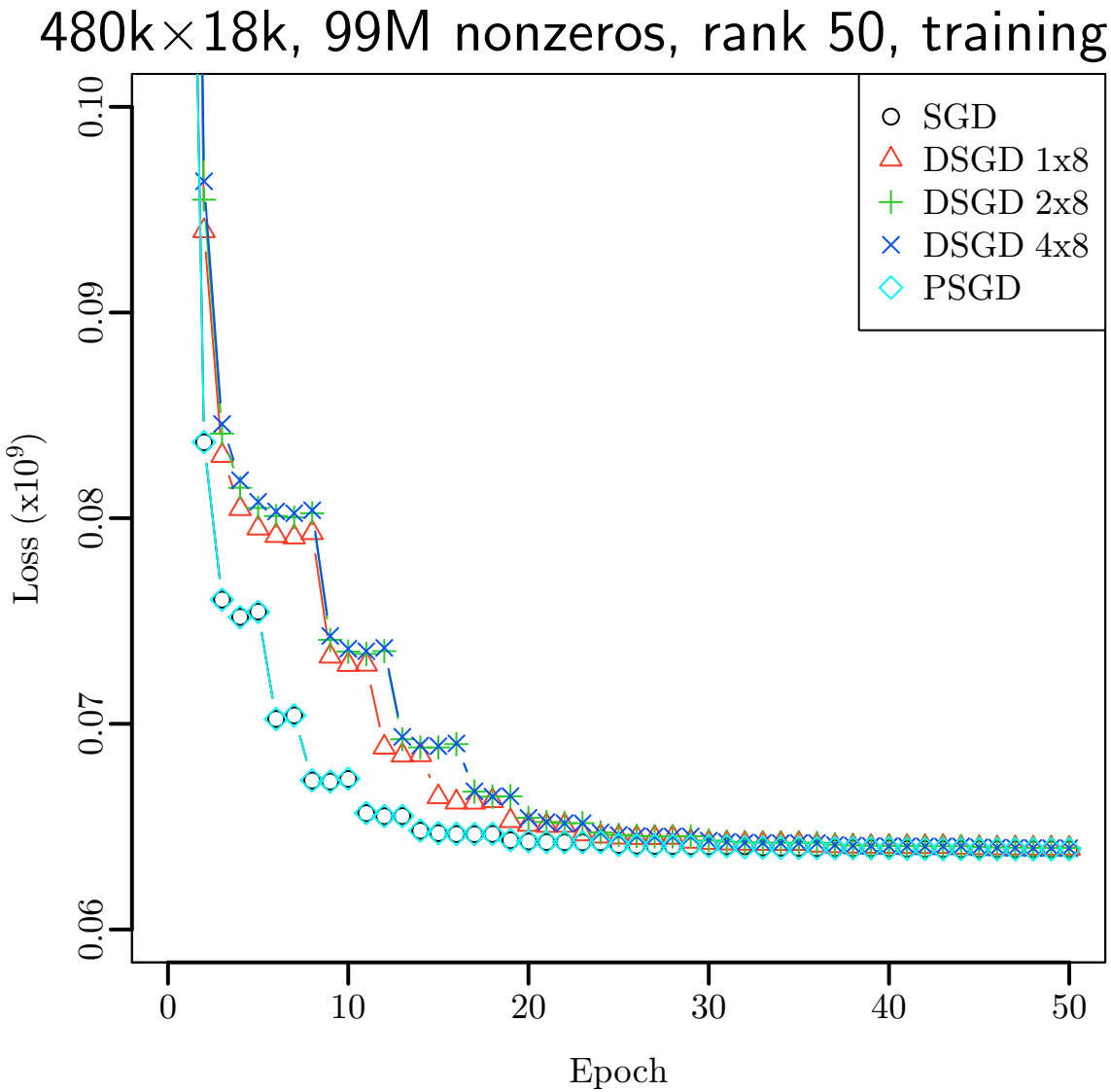
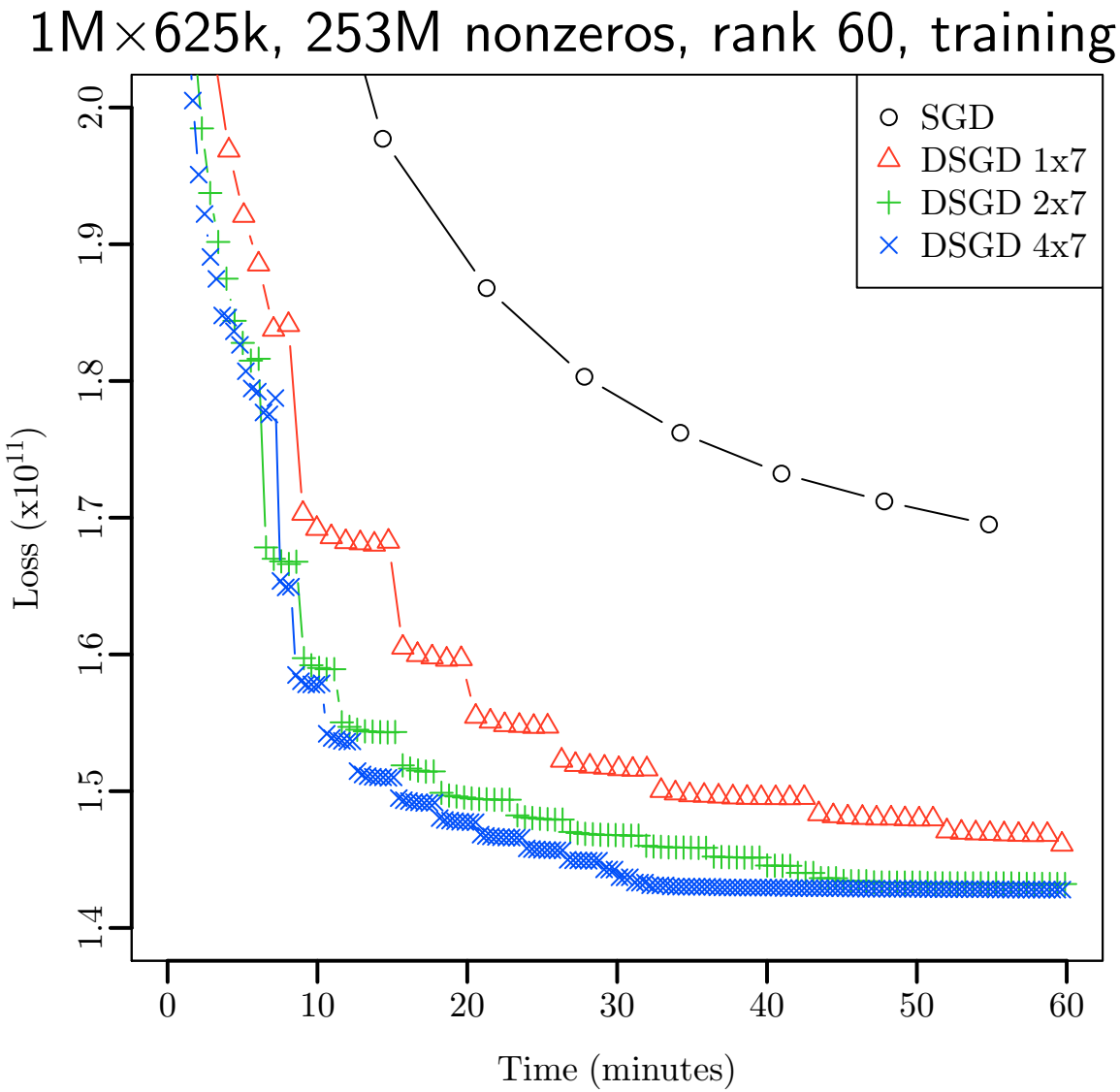
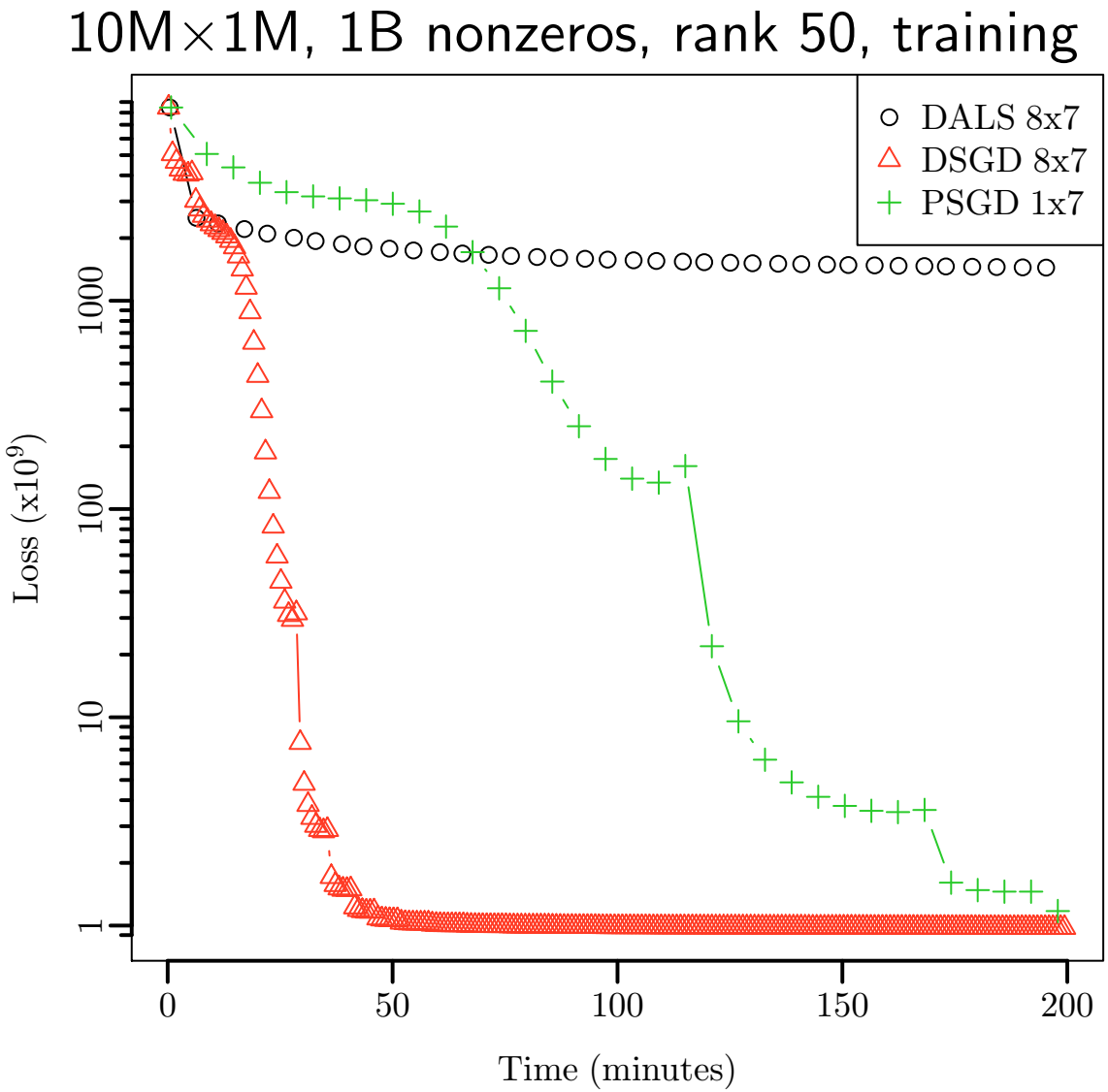


Stratification is (currently) not for free.

KDD Cup, Nzsl+Nzl2+Bias



Synthetic data, Nzsl+L2



Outline

Summary

- Matrix factorization
  - Widely applicable via customized loss functions
  - Large instances (millions $\times$ millions with billions of entries)
- Distributed Stochastic Gradient Descent
  - Simple and versatile
  - Avoids averaging via novel "stratified SGD" variant
  - Achieves
    - Fully distributed data/model
    - Fully distributed processing
    - Competitive to alternative algorithms
    - Fast, scalable
- Future Directions
  - Improved stratification
  - Simultaneous computation & communication
  - Stratification for other models
  - ...

Thank you!