# Large-Scale Matrix Factorization with Distributed Stochastic Gradient Descent 

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## Outline

Matrix Factorization

Stochastic Gradient Descent

Distributed SGD with MapReduce

Experiments

Summary

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- Set of users
- Set of items (movies, books, jokes, products, stories, ...)
- Feedback (ratings, purchase, click-through, tags, ...)


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Avatar
Alice

| The Matrix |
| :--- |


| Bob |
| :--- |
| Charlie |

$\left(\begin{array}{cc}3 & 4\end{array}\right.$
3
5

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$\left.\begin{array}{l}\text { Avatar } \\ \text { Alice } \\ \text { Bob } \\ \text { Charlie Matrix }\end{array} \begin{array}{ccc}? & \text { Up } \\ 3 & 4 & 2 \\ 5 & 2 & ? \\ 3 & ? & 3\end{array}\right)$
- Netflix competition: 500k users, 20k movies, 100M movie ratings, 3 M question marks


## Semantic Factors (Koren et al., 2009)



## Latent Factor Models

- Discover latent factors ( $r=1$ )

|  | Avatar | The Matrix | Up |
| :---: | :---: | :---: | :---: |
| Alice |  | 4 | 2 |
| Bob | 3 | 2 |  |
| Charlie | 5 |  | 3 |

## Latent Factor Models

- Discover latent factors ( $r=1$ )

|  | Avatar <br> $(2.24)$ | The Matrix <br> $(1.92)$ | Up <br> $(1.18)$ |
| :---: | :---: | :---: | :---: |
| Alice <br> $(1.98)$ |  | $\mathbf{4}$ | $\mathbf{2}$ |
| Bob <br> $(1.21)$ | $\mathbf{3}$ | $\mathbf{2}$ |  |
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- Minimum loss

$$
\min _{\mathbf{W}, \mathbf{H}} \sum_{(i, j) \in Z}\left(\mathbf{V}_{i j}-[\mathbf{W H}]_{i j}\right)^{2}
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- Bias


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+\lambda(\|\mathbf{W}\|+\|\mathbf{H}\|+\|\mathbf{u}\|+\|\mathbf{m}\|)
\end{array}
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- Bias, regularization


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- Minimum loss

$$
\begin{array}{r}
\min _{\mathbf{w}, \mathbf{H}, \mathbf{u}, \mathbf{m}} \sum_{(i, j, j) \in Z_{t}}\left(\mathbf{V}_{i j}-\mu-\mathbf{u}_{i}(t)-\mathbf{m}_{j}(t)-[\mathbf{W}(t) \mathbf{H}]_{i j}\right)^{2} \\
+\lambda(\|\mathbf{W}(t)\|+\|\mathbf{H}\|+\|\mathbf{u}(t)\|+\|\mathbf{m}(t)\|)
\end{array}
$$

- Bias, regularization, time


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- A general machine learning problem
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- Model
- $L_{i j}\left(\mathbf{W}_{i *}, \mathbf{H}_{* j}\right)$ : loss at element $(i, j)$
- Includes prediction error, regularization, auxiliary information, ...
- Constraints (e.g., non-negativity)



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- Includes prediction error, regularization, auxiliary information, ...
- Constraints (e.g., non-negativity)
- Find best model

$$
\underset{\mathbf{W}, \mathbf{H}}{\operatorname{argmin}} \sum_{(i, j) \in Z} L_{i j}\left(\mathbf{W}_{i *}, \mathbf{H}_{* j}\right)
$$



## Successful Applications

- Movie recommendation (Netflix, competition papers)
- $>12 \mathrm{M}$ users, $>20 \mathrm{k}$ movies, 2.4 B ratings (projected)
- 36 GB data, 9.2 GB model (projected)
- Latent factor model
- Website recommendation (Microsoft, WWW10)
- 51 M users, 15 M URLs, 1.2 B clicks
- 17.8 GB data, 161 GB metadata, 49 GB model
- Gaussian non-negative matrix factorization
- News personalization (Google, WWW07)
- Millions of users, millions of stories, ? clicks
- Probabilistic latent semantic indexing


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## Distributed processing is necessary!

- Big data
- Large models
- Expensive computations


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## Stochastic Gradient Descent

- Find minimum $\theta^{*}$ of function $L$



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\theta_{n+1}=\theta_{n}-\epsilon_{n} \hat{L}^{\prime}\left(\theta_{n}\right)
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- Under certain conditions, asymptotically approximates (continuous) gradient descent




## Stochastic Gradient Descent for Matrix Factorization

- Set $\theta=(\mathbf{W}, \mathbf{H})$ and use

$$
L(\theta)=\sum_{(i, j) \in Z} L_{i j}\left(\mathbf{W}_{i *}, \mathbf{H}_{* j}\right)
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\begin{aligned}
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\hat{L}^{\prime}(\theta, z) & =N L_{i_{z} j_{z}}^{\prime}\left(\mathbf{W}_{i_{z} *}, \mathbf{H}_{* j_{z}}\right),
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where $N=|Z|$


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where $N=|Z|$

- SGD epoch

1. Pick a random entry $z \in Z$
2. Compute approximate gradient $\hat{L^{\prime}}(\theta, z)$
3. Update parameters

$$
\theta_{n+1}=\theta_{n}-\epsilon_{n} \hat{L^{\prime}}\left(\theta_{n}, z\right)
$$

4. Repeat $N$ times

## Stochastic Gradient Descent on Netflix Data



## Comparison

- Per epoch, assuming $O(r)$ gradient computation per element

|  | GD | SGD |
| :--- | :---: | :---: |
| Algorithm | Deterministic | Randomized |
| Gradient computations | 1 | $N$ |
| Gradient types | Exact | Approximate |
| Parameter updates | 1 | $N$ |
| Time | $O(r N)$ | $O(r N)$ |
| Space | $O((m+n) r)$ | $O((m+n) r)$ |

- Why stochastic?
- Fast convergence to vicinity of optimum
- Randomization may help escape local minima
- Exploitation of "repeated structure"


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## Averaging Techniques

- SGD steps depend on each other

$$
\theta_{n+1}=\theta_{n}-\epsilon_{n} \hat{L}^{\prime}\left(\theta_{n}\right)
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How to distribute?

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- Map: Run independent instances of SGD on subsets of the data (until convergence)
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- Reduce: Average results
- Does not converge to correct solution!
- Iterative Parameter mixing (PSGD)
- Map: Run independent instances of SGD on subsets of the data (for some time)
- Reduce: Average results
- Repeat
- Converges slowly!


## Problem Structure

- SGD steps depend on each other

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$$

- An SGD step on example $z \in Z \ldots$

1. Reads $W_{i_{z} *}$ and $H_{* j_{z}}$
2. Performs gradient computation $L_{i j}^{\prime}\left(W_{i z *}, H_{* j_{z}}\right)$
3. Updates $W_{i_{2} *}$ and $H_{* j_{2}}$


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- Not all steps are dependent



## Interchangeability

- Two elements $z_{1}, z_{2} \in Z$ are interchangeable if they share neither row nor column

- When $z_{n}$ and $z_{n+1}$ are interchangeable, the SGD steps

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$$

become parallelizable!

## Exploitation

- Block and distribute the input matrix $\mathbf{V}$



## Exploitation

- Block and distribute the input matrix V
- High-level approach (Map only)

1. Pick a "diagonal"
2. Run SGD on the diagonal (in parallel)
3. Merge the results
4. Move on to next "diagonal"

- Steps 1-3 form a cycle



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Simulate sequential SGD

- Interchangeable blocks
- Throw dice of how many iterations per block
- Throw dice of which step sizes per block


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- Interchangeable blocks
- Throw dice of how many iterations per block
- Throw dice of which step sizes per block
- Instance of "stratified SGD"
- Provably correct


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## How does it work?



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## DSGD scales well (Netflix, NZSL+L2)



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## DSGD is fast ( $8 x 8$, Netflix, NZSL)



## DSGD is fast $(8 \times 8$, Netflix, NZSL+L2)



## DSGD is fast ( $8 \times 8$, synth., NZSL+L2)



## DSGD runs on Hadoop


(25.6B entries $>1 / 2 \mathrm{~TB}$ of data)

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## Summary

- Matrix factorization
- Widely applicable via customized loss functions
- Large instances (millions $\times$ millions with billions of entries)
- Distributed Stochastic Gradient Descent
- Simple and versatile
- Avoids averaging via novel "stratified SGD" variant
- Achieves
- Fully distributed data/model
- Fully distributed processing
- Same or better loss
- Faster
- Good scalability
- Future Directions
- More decompositions (e.g., losses at 0)
- Tensors
- Stratified SGD for other models
- ...


## Thank you!

