Large-Scale Matrix Factorization with Distributed Stochastic Gradient Descent

Rainer Gemulla

August 23, 2011

Peter J. Haas Yannis Sismanis Erik Nijkamp







Outline

Matrix Factorization

Stochastic Gradient Descent

Distributed SGD with MapReduce

 ${\sf Experiments}$

Summary

Outline

Matrix Factorization

Stochastic Gradient Descent

Distributed SGD with MapReduce

Experiments

Summary

- Problem
 - Set of users
 - ▶ Set of items (movies, books, jokes, products, stories, ...)
 - ► Feedback (ratings, purchase, click-through, tags, ...)

- Problem
 - Set of users
 - ▶ Set of items (movies, books, jokes, products, stories, ...)
 - ► Feedback (ratings, purchase, click-through, tags, ...)
- Predict additional items a user may like
 - lacktriangle Assumption: Similar feedback \Longrightarrow Similar taste

- Problem
 - Set of users
 - ▶ Set of items (movies, books, jokes, products, stories, ...)
 - ► Feedback (ratings, purchase, click-through, tags, ...)
- Predict additional items a user may like
 - ► Assumption: Similar feedback ⇒ Similar taste
- Example

	Avatar	The Matrix	Up
Alice	/	4	2 \
Bob	3	2	
Charlie	5		3 <i>]</i>

- Problem
 - Set of users
 - ▶ Set of items (movies, books, jokes, products, stories, ...)
 - ► Feedback (ratings, purchase, click-through, tags, ...)
- Predict additional items a user may like
 - ► Assumption: Similar feedback ⇒ Similar taste
- Example

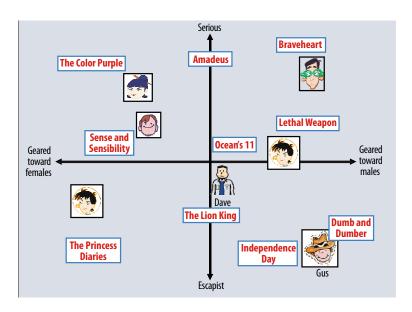
	Avatar	The Matrix	Up
Alice	?	4	2 \
Bob	3	2	?
Charlie	5	?	3 <i>]</i>

- Problem
 - Set of users
 - ▶ Set of items (movies, books, jokes, products, stories, ...)
 - ► Feedback (ratings, purchase, click-through, tags, ...)
- Predict additional items a user may like
 - ► Assumption: Similar feedback ⇒ Similar taste
- Example

	Α	vatar	The Matrix	Up
Alice	/	?	4	2 \
Bob	1	3	2	?
$\it Charlie$		5	?	3 <i>]</i>

▶ Netflix competition: 500k users, 20k movies, 100M movie ratings, 3M question marks

Semantic Factors (Koren et al., 2009)



▶ Discover latent factors (r = 1)

	Avatar	The Matrix	Up
Alice		4	2
Bob	3	2	
Charlie	5		3

▶ Discover latent factors (r = 1)

	Avatar (2.24)	The Matrix (1.92)	Up (1.18)
Alice (1.98)		4	2
Bob (1.21)	3	2	
Charlie (2.30)	5		3

▶ Discover latent factors (r = 1)

	Avatar (2.24)	The Matrix (1.92)	Up (1.18)
Alice (1.98)		4 (3.8)	2 (2.3)
Bob (1.21)	3 (2.7)	2 (2.3)	
Charlie (2.30)	5 (5.2)		3 (2.7)

Minimum loss

$$\min_{\mathbf{W},\mathbf{H}} \sum_{(i,j) \in Z} (\mathbf{V}_{ij} - [\mathbf{W}\mathbf{H}]_{ij})^2$$

▶ Discover latent factors (r = 1)

	Avatar (2.24)	The Matrix (1.92)	Up (1.18)
Alice (1.98)	? (4.4)	4 (3.8)	2 (2.3)
Bob (1.21)	3 (2.7)	2 (2.3)	? (1.4)
Charlie (2.30)	5 (5.2)	? (4.4)	3 (2.7)

Minimum loss

$$\min_{\mathbf{W},\mathbf{H}} \sum_{(i,j) \in Z} (\mathbf{V}_{ij} - [\mathbf{W}\mathbf{H}]_{ij})^2$$

▶ Discover latent factors (r = 1)

	Avatar (2.24)	The Matrix (1.92)	Up (1.18)
Alice (1.98)	? (4.4)	4 (3.8)	2 (2.3)
Bob (1.21)	3 (2.7)	2 (2.3)	? (1.4)
Charlie (2.30)	5 (5.2)	? (4.4)	3 (2.7)

Minimum loss

$$\min_{\mathbf{W},\mathbf{H},\mathbf{u},\mathbf{m}} \sum_{(i,j) \in \mathcal{Z}} (\mathbf{V}_{ij} - \mu - \mathbf{u}_i - \mathbf{m}_j - [\mathbf{W}\mathbf{H}]_{ij})^2$$

Bias

▶ Discover latent factors (r = 1)

	Avatar (2.24)	The Matrix (1.92)	Up (1.18)
Alice (1.98)	? (4.4)	4 (3.8)	2 (2.3)
Bob (1.21)	3 (2.7)	2 (2.3)	? (1.4)
Charlie (2.30)	5 (5.2)	? (4.4)	3 (2.7)

Minimum loss

$$\begin{aligned} \min_{\mathbf{W}, \mathbf{H}, \mathbf{u}, \mathbf{m}} & \sum_{(i, j) \in \mathcal{Z}} (\mathbf{V}_{ij} - \mu - \mathbf{u}_i - \mathbf{m}_j - [\mathbf{W}\mathbf{H}]_{ij})^2 \\ & + \lambda \left(\|\mathbf{W}\| + \|\mathbf{H}\| + \|\mathbf{u}\| + \|\mathbf{m}\| \right) \end{aligned}$$

Bias, regularization

▶ Discover latent factors (r = 1)

	Avatar (2.24)	The Matrix (1.92)	Up (1.18)
Alice (1.98)	? (4.4)	4 (3.8)	2 (2.3)
Bob (1.21)	3 (2.7)	2 (2.3)	? (1.4)
Charlie (2.30)	5 (5.2)	? (4.4)	3 (2.7)

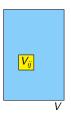
Minimum loss

$$\begin{aligned} \min_{\mathbf{W}, \mathbf{H}, \mathbf{u}, \mathbf{m}} \sum_{(i, j, t) \in \mathcal{Z}_t} (\mathbf{V}_{ij} - \mu - \mathbf{u}_i(t) - \mathbf{m}_j(t) - [\mathbf{W}(t)\mathbf{H}]_{ij})^2 \\ + \lambda \left(\|\mathbf{W}(t)\| + \|\mathbf{H}\| + \|\mathbf{u}(t)\| + \|\mathbf{m}(t)\| \right) \end{aligned}$$

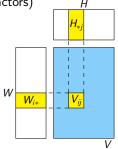
Bias, regularization, time

- ► A general machine learning problem
 - ▶ Recommender systems, text indexing, face recognition, ...

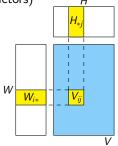
- ► A general machine learning problem
 - ▶ Recommender systems, text indexing, face recognition, ...
- Training data
 - **V**: $m \times n$ input matrix (e.g., rating matrix)
 - ► *Z*: training set of indexes in **V** (e.g., subset of known ratings)



- ► A general machine learning problem
 - ▶ Recommender systems, text indexing, face recognition, . . .
- ▶ Training data
 - **V**: $m \times n$ input matrix (e.g., rating matrix)
 - ► *Z*: training set of indexes in **V** (e.g., subset of known ratings)
- Parameter space
 - ▶ **W**: row factors (e.g., $m \times r$ latent customer factors)
 - ▶ **H**: column factors (e.g., $r \times n$ latent movie factors)

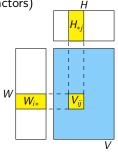


- ► A general machine learning problem
 - ▶ Recommender systems, text indexing, face recognition, . . .
- ▶ Training data
 - **V**: $m \times n$ input matrix (e.g., rating matrix)
 - ► *Z*: training set of indexes in **V** (e.g., subset of known ratings)
- ► Parameter space
 - **W**: row factors (e.g., $m \times r$ latent customer factors)
 - ▶ **H**: column factors (e.g., $r \times n$ latent movie factors)
- Model
 - ▶ $L_{ii}(\mathbf{W}_{i*}, \mathbf{H}_{*i})$: loss at element (i, j)
 - ► Includes prediction error, regularization, auxiliary information, ...
 - ► Constraints (e.g., non-negativity)



- ► A general machine learning problem
 - ▶ Recommender systems, text indexing, face recognition, . . .
- ▶ Training data
 - **V**: $m \times n$ input matrix (e.g., rating matrix)
 - ► *Z*: training set of indexes in **V** (e.g., subset of known ratings)
- ▶ Parameter space
 - **W**: row factors (e.g., $m \times r$ latent customer factors)
 - ▶ **H**: column factors (e.g., $r \times n$ latent movie factors)
- Model
 - ▶ $L_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$: loss at element (i, j)
 - ► Includes prediction error, regularization, auxiliary information, . . .
 - Constraints (e.g., non-negativity)
- ► Find best model

$$\operatorname*{argmin}_{\mathbf{W},\mathbf{H}} \sum_{(i,j) \in \mathcal{Z}} L_{ij}(\mathbf{W}_{i*},\mathbf{H}_{*j})$$



Successful Applications

- Movie recommendation (Netflix, competition papers)
 - >12M users, >20k movies, 2.4B ratings (projected)
 - 36GB data, 9.2GB model (projected)
 - ► Latent factor model
- Website recommendation (Microsoft, WWW10)
 - ▶ 51M users, 15M URLs, 1.2B clicks
 - ▶ 17.8GB data, 161GB metadata, 49GB model
 - Gaussian non-negative matrix factorization
- News personalization (Google, WWW07)
 - Millions of users, millions of stories, ? clicks
 - Probabilistic latent semantic indexing

Successful Applications

- Movie recommendation (Netflix, competition papers)
 - >12M users, >20k movies, 2.4B ratings (projected)
 - 36GB data, 9.2GB model (projected)
 - ▶ Latent factor model
- Website recommendation (Microsoft, WWW10)
 - ▶ 51M users, 15M URLs, 1.2B clicks
 - ▶ 17.8GB data, 161GB metadata, 49GB model
 - Gaussian non-negative matrix factorization
- News personalization (Google, WWW07)
 - Millions of users, millions of stories, ? clicks
 - Probabilistic latent semantic indexing

Distributed processing is necessary!

- Big data
- Large models
- Expensive computations

Outline

Matrix Factorization

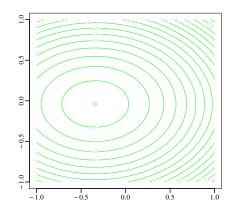
Stochastic Gradient Descent

Distributed SGD with MapReduce

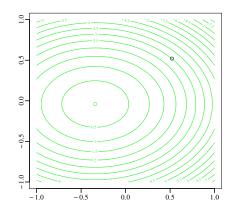
Experiments

Summary

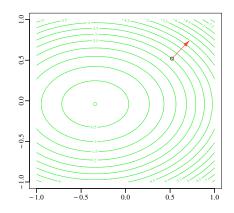
Find minimum θ^* of function L



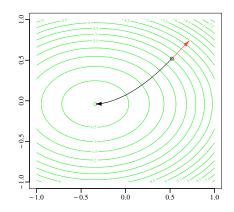
- Find minimum θ^* of function L
- ▶ Pick a starting point θ_0



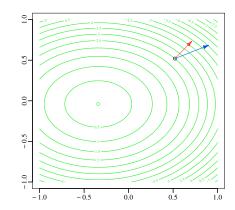
- Find minimum θ^* of function L
- ▶ Pick a starting point θ_0



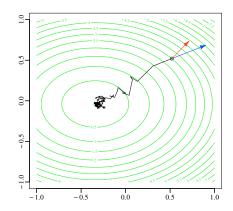
- Find minimum θ^* of function L
- ▶ Pick a starting point θ_0



- Find minimum θ^* of function L
- ▶ Pick a starting point θ_0
- ▶ Approximate gradient $\hat{L}'(\theta_0)$

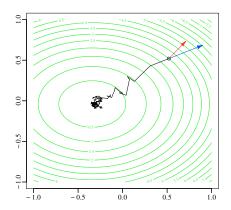


- Find minimum θ^* of function L
- ▶ Pick a starting point θ_0
- ▶ Approximate gradient $\hat{L}'(\theta_0)$
- ▶ Jump "approximately" downhill



- Find minimum θ^* of function L
- ▶ Pick a starting point θ_0
- Approximate gradient $\hat{L}'(\theta_0)$
- ▶ Jump "approximately" downhill
- ► Stochastic difference equation

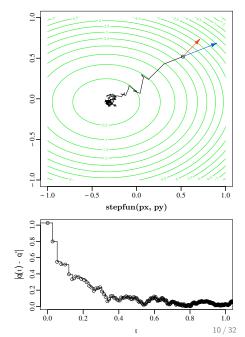
$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$



- ▶ Find minimum θ^* of function L
- ▶ Pick a starting point θ_0
- Approximate gradient $\hat{L}'(\theta_0)$
- Jump "approximately" downhill
- ► Stochastic difference equation

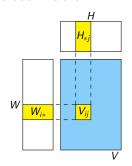
$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

 Under certain conditions, asymptotically approximates (continuous) gradient descent



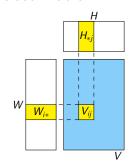
▶ Set $\theta = (\mathbf{W}, \mathbf{H})$ and use

$$L(\theta) = \sum_{(i,j) \in Z} L_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$



▶ Set $\theta = (\mathbf{W}, \mathbf{H})$ and use

$$L(\theta) = \sum_{(i,j)\in\mathcal{Z}} L_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$
$$L'(\theta) = \sum_{(i,j)\in\mathcal{Z}} L'_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$



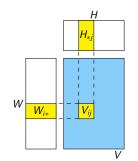
▶ Set $\theta = (\mathbf{W}, \mathbf{H})$ and use

$$L(\theta) = \sum_{(i,j)\in\mathcal{Z}} L_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$

$$L'(\theta) = \sum_{(i,j)\in\mathcal{Z}} L'_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$

$$\hat{L}'(\theta, z) = NL'_{i_zj_z}(\mathbf{W}_{i_z*}, \mathbf{H}_{*j_z}),$$

where
$$N = |Z|$$



▶ Set $\theta = (\mathbf{W}, \mathbf{H})$ and use

$$L(\theta) = \sum_{(i,j)\in\mathcal{Z}} L_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$

$$L'(\theta) = \sum_{(i,j)\in\mathcal{Z}} L'_{ij}(\mathbf{W}_{i*}, \mathbf{H}_{*j})$$

$$\hat{L}'(\theta, z) = NL'_{i_z j_z}(\mathbf{W}_{i_z *}, \mathbf{H}_{*j_z}),$$

$$W = \begin{bmatrix} H \\ H_{*j} \\ \vdots \\ V_{ij} \end{bmatrix}$$

► SGD epoch

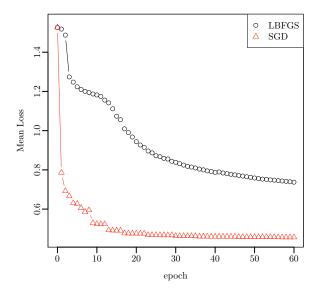
where N = |Z|

- 1. Pick a random entry $z \in Z$
- 2. Compute approximate gradient $\hat{L}'(\theta, z)$
- 3. Update parameters

$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n, z)$$

Repeat N times

Stochastic Gradient Descent on Netflix Data



Comparison

 \triangleright Per epoch, assuming O(r) gradient computation per element

	GD	SGD
Algorithm	Deterministic	Randomized
Gradient computations	1	Ν
Gradient types	Exact	Approximate
Parameter updates	1	Ν
Time	O(rN)	O(rN)
Space	O((m+n)r)	O((m+n)r)

- ► Why stochastic?
 - ▶ Fast convergence to vicinity of optimum
 - Randomization may help escape local minima
 - Exploitation of "repeated structure"

Outline

Matrix Factorization

Stochastic Gradient Descent

Distributed SGD with MapReduce

Experiments

Summary

▶ SGD steps depend on each other

$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

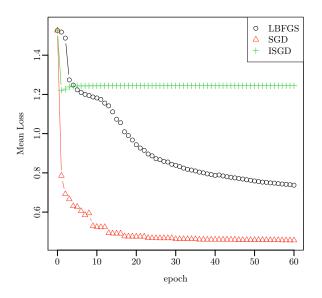
How to distribute?

SGD steps depend on each other

$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

How to distribute?

- ► Parameter mixing (ISGD)
 - ► *Map*: Run independent instances of SGD on subsets of the data (until convergence)
 - Reduce: Average results



SGD steps depend on each other

$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

How to distribute?

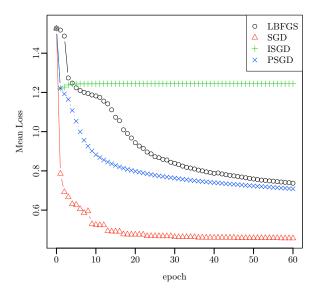
- ► Parameter mixing (ISGD)
 - ► *Map*: Run independent instances of SGD on subsets of the data (until convergence)
 - ► Reduce: Average results
 - Does not converge to correct solution!

SGD steps depend on each other

$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

How to distribute?

- Parameter mixing (ISGD)
 - Map: Run independent instances of SGD on subsets of the data (until convergence)
 - ► Reduce: Average results
 - Does not converge to correct solution!
- Iterative Parameter mixing (PSGD)
 - Map: Run independent instances of SGD on subsets of the data (for some time)
 - ► Reduce: Average results
 - Repeat



SGD steps depend on each other

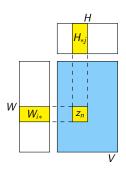
$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

How to distribute?

- Parameter mixing (ISGD)
 - ► *Map*: Run independent instances of SGD on subsets of the data (until convergence)
 - Reduce: Average results
 - Does not converge to correct solution!
- ▶ Iterative Parameter mixing (PSGD)
 - Map: Run independent instances of SGD on subsets of the data (for some time)
 - ► Reduce: Average results
 - Repeat
 - Converges slowly!

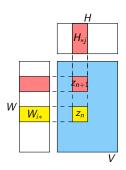
$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

- ▶ An SGD step on example $z \in Z ...$
 - 1. Reads W_{i_z*} and H_{*i_z}
 - 2. Performs gradient computation $L'_{ij}(W_{i_z*}, H_{*j_z})$
 - 3. Updates W_{i_z*} and H_{*j_z}



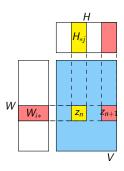
$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

- ▶ An SGD step on example $z \in Z ...$
 - 1. Reads W_{i_z*} and H_{*j_z}
 - 2. Performs gradient computation $L'_{ij}(W_{i_z*}, H_{*j_z})$
 - 3. Updates W_{i_z*} and H_{*j_z}



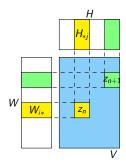
$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

- ▶ An SGD step on example $z \in Z ...$
 - 1. Reads W_{i_z*} and H_{*j_z}
 - 2. Performs gradient computation $L'_{ij}(W_{i_z*}, H_{*j_z})$
 - 3. Updates W_{i_z*} and H_{*j_z}

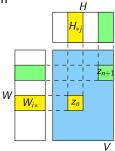


$$\theta_{n+1} = \theta_n - \epsilon_n \hat{L}'(\theta_n)$$

- ▶ An SGD step on example $z \in Z \dots$
 - 1. Reads W_{i_z*} and H_{*j_z}
 - 2. Performs gradient computation $L'_{ij}(W_{i_z*}, H_{*j_z})$
 - 3. Updates W_{i_z*} and H_{*j_z}
- Not all steps are dependent



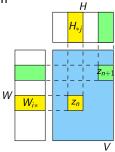
▶ Two elements $z_1, z_2 \in Z$ are *interchangeable* if they share neither row nor column



▶ When z_n and z_{n+1} are interchangeable, the SGD steps

$$\theta_{n+1} = \theta_n - \epsilon \hat{L}'(\theta_n, z_n)$$

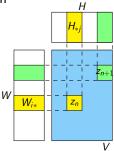
▶ Two elements $z_1, z_2 \in Z$ are *interchangeable* if they share neither row nor column



▶ When z_n and z_{n+1} are interchangeable, the SGD steps

$$\theta_{n+2} = \theta_n - \epsilon \hat{L}'(\theta_n, z_n) - \epsilon \hat{L}'(\theta_{n+1}, z_{n+1})$$

▶ Two elements $z_1, z_2 \in Z$ are *interchangeable* if they share neither row nor column

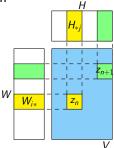


▶ When z_n and z_{n+1} are interchangeable, the SGD steps

$$\theta_{n+2} = \theta_n - \epsilon \hat{L}'(\theta_n, z_n) - \epsilon \hat{L}'(\theta_{n+1}, z_{n+1})$$

= $\theta_n - \epsilon \hat{L}'(\theta_n, z_n) - \epsilon \hat{L}'(\theta_n, z_{n+1}),$

▶ Two elements $z_1, z_2 \in Z$ are *interchangeable* if they share neither row nor column



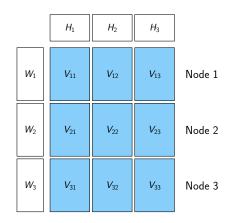
▶ When z_n and z_{n+1} are interchangeable, the SGD steps

$$\theta_{n+2} = \theta_n - \epsilon \hat{L}'(\theta_n, z_n) - \epsilon \hat{L}'(\theta_{n+1}, z_{n+1})$$

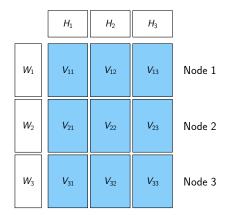
= $\theta_n - \epsilon \hat{L}'(\theta_n, z_n) - \epsilon \hat{L}'(\theta_n, z_{n+1}),$

become parallelizable!

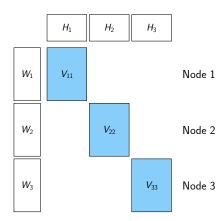
▶ Block and distribute the input matrix **V**



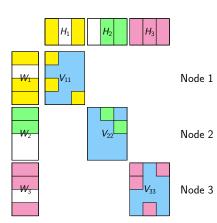
- ▶ Block and distribute the input matrix **V**
- High-level approach (Map only)
 - 1. Pick a "diagonal"
 - 2. Run SGD on the diagonal (in parallel)
 - 3. Merge the results
 - 4. Move on to next "diagonal"
 - ▶ Steps 1–3 form a *cycle*



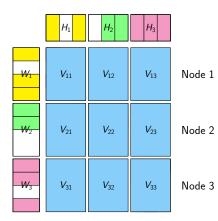
- Block and distribute the input matrix V
- High-level approach (Map only)
 - 1. Pick a "diagonal"
 - 2. Run SGD on the diagonal (in parallel)
 - 3. Merge the results
 - 4. Move on to next "diagonal"
 - ▶ Steps 1–3 form a *cycle*



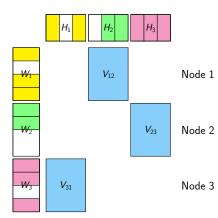
- ▶ Block and distribute the input matrix **V**
- ► High-level approach (Map only)
 - 1. Pick a "diagonal"
 - 2. Run SGD on the diagonal (in parallel)
 - 3. Merge the results
 - 4. Move on to next "diagonal"
 - ► Steps 1–3 form a *cycle*
- Step 2: Simulate sequential SGD
 - Interchangeable blocks
 - Throw dice of how many iterations per block
 - Throw dice of which step sizes per block



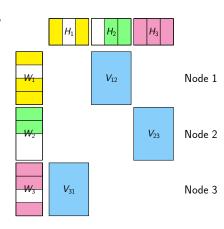
- Block and distribute the input matrix V
- High-level approach (Map only)
 - 1. Pick a "diagonal"
 - 2. Run SGD on the diagonal (in parallel)
 - 3. Merge the results
 - 4. Move on to next "diagonal"
 - ► Steps 1–3 form a *cycle*
- Step 2: Simulate sequential SGD
 - Interchangeable blocks
 - Throw dice of how many iterations per block
 - Throw dice of which step sizes per block

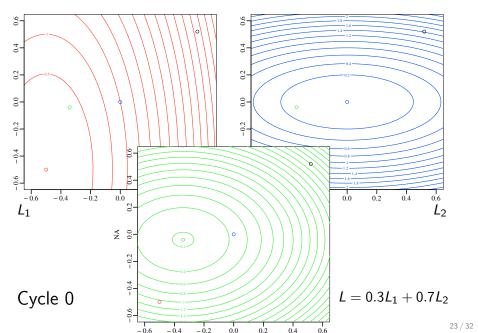


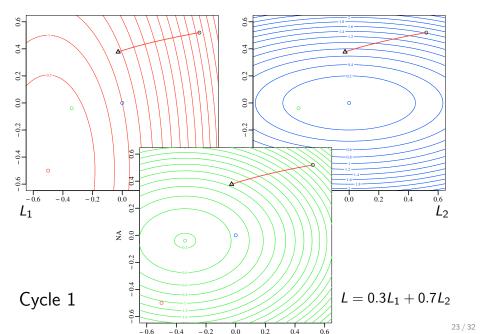
- Block and distribute the input matrix V
- High-level approach (Map only)
 - 1. Pick a "diagonal"
 - 2. Run SGD on the diagonal (in parallel)
 - 3. Merge the results
 - 4. Move on to next "diagonal"
 - ► Steps 1–3 form a *cycle*
- Step 2: Simulate sequential SGD
 - Interchangeable blocks
 - Throw dice of how many iterations per block
 - Throw dice of which step sizes per block

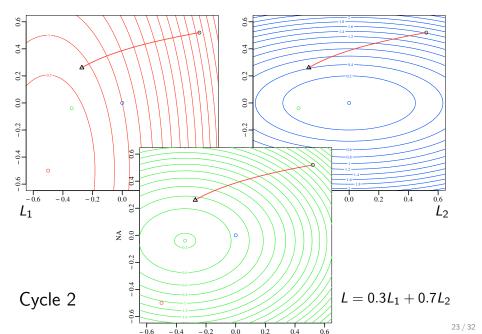


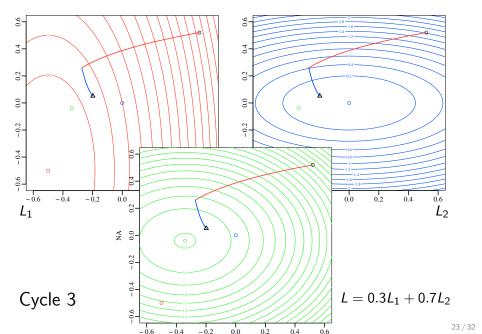
- Block and distribute the input matrix V
- High-level approach (Map only)
 - 1. Pick a "diagonal"
 - 2. Run SGD on the diagonal (in parallel)
 - 3. Merge the results
 - 4. Move on to next "diagonal"
 - ► Steps 1–3 form a *cycle*
- Step 2: Simulate sequential SGD
 - Interchangeable blocks
 - Throw dice of how many iterations per block
 - Throw dice of which step sizes per block
- Instance of "stratified SGD"
- Provably correct

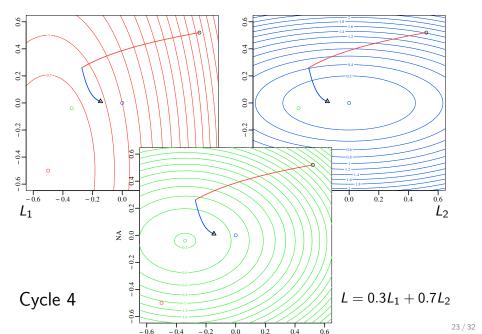


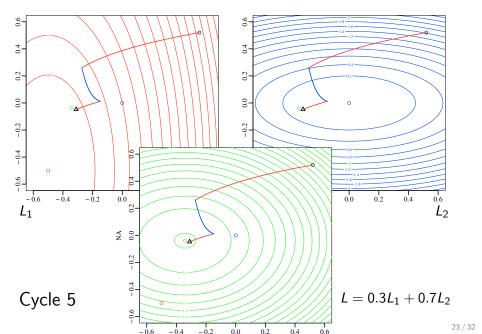


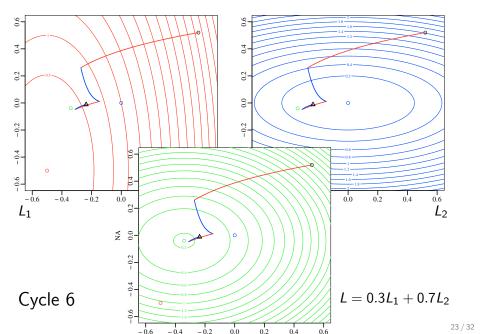


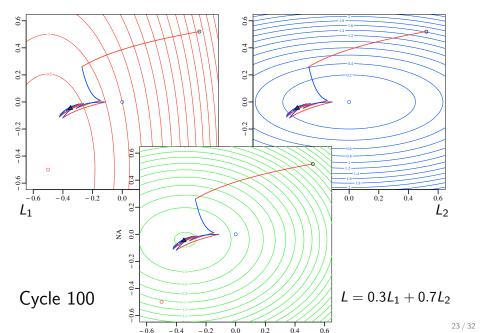


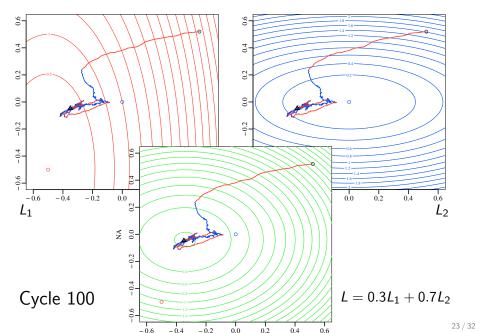


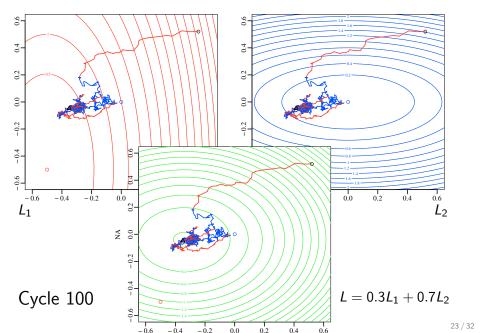












Outline

Matrix Factorization

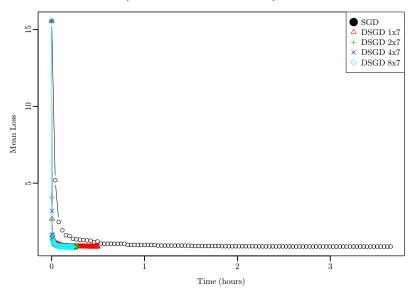
Stochastic Gradient Descent

Distributed SGD with MapReduce

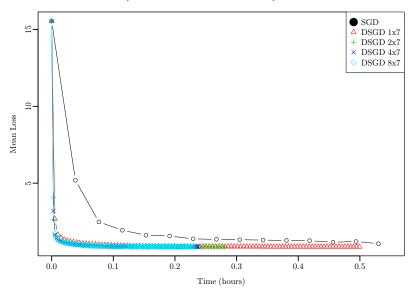
${\sf Experiments}$

Summary

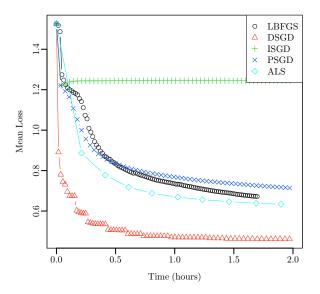
DSGD scales well (Netflix, NZSL+L2)



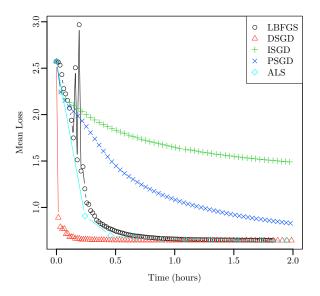
DSGD scales well (Netflix, NZSL+L2)



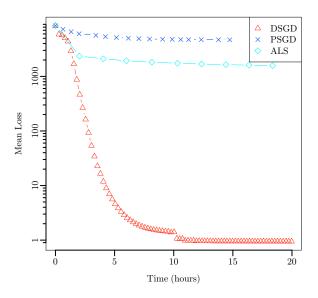
DSGD is fast (8x8, Netflix, NZSL)



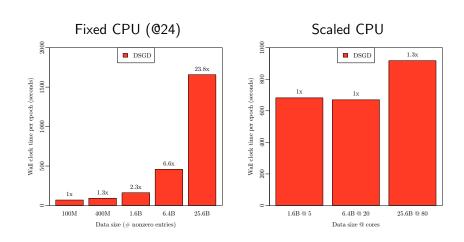
DSGD is fast (8x8, Netflix, NZSL+L2)



DSGD is fast (8x8, synth., NZSL+L2)



DSGD runs on Hadoop



(25.6B entries > 1/2TB of data)

Outline

Matrix Factorization

Stochastic Gradient Descent

Distributed SGD with MapReduce

Experiments

Summary

Summary

- Matrix factorization
 - Widely applicable via customized loss functions
 - ▶ Large instances (millions × millions with billions of entries)
- Distributed Stochastic Gradient Descent
 - Simple and versatile
 - Avoids averaging via novel "stratified SGD" variant
 - Achieves
 - Fully distributed data/model
 - Fully distributed processing
 - Same or better loss
 - Faster
 - Good scalability
- Future Directions
 - More decompositions (e.g., losses at 0)
 - Tensors
 - Stratified SGD for other models
 - · ...

Thank you!