A Distributed Algorithm for Large-Scale Generalized Matching

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informatik

Recommender systems

Given:

- A user-item feedback matrix

Goal:

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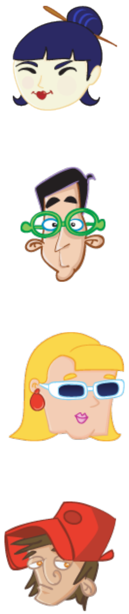

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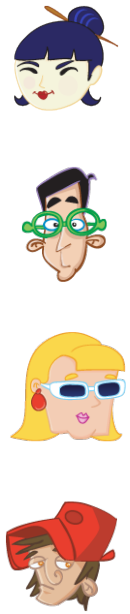

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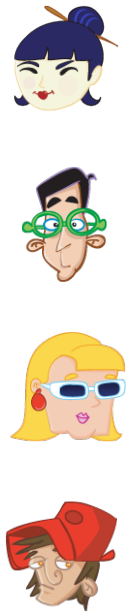

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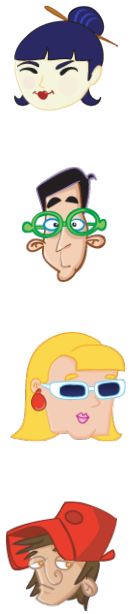
How to recommend items under constraints?



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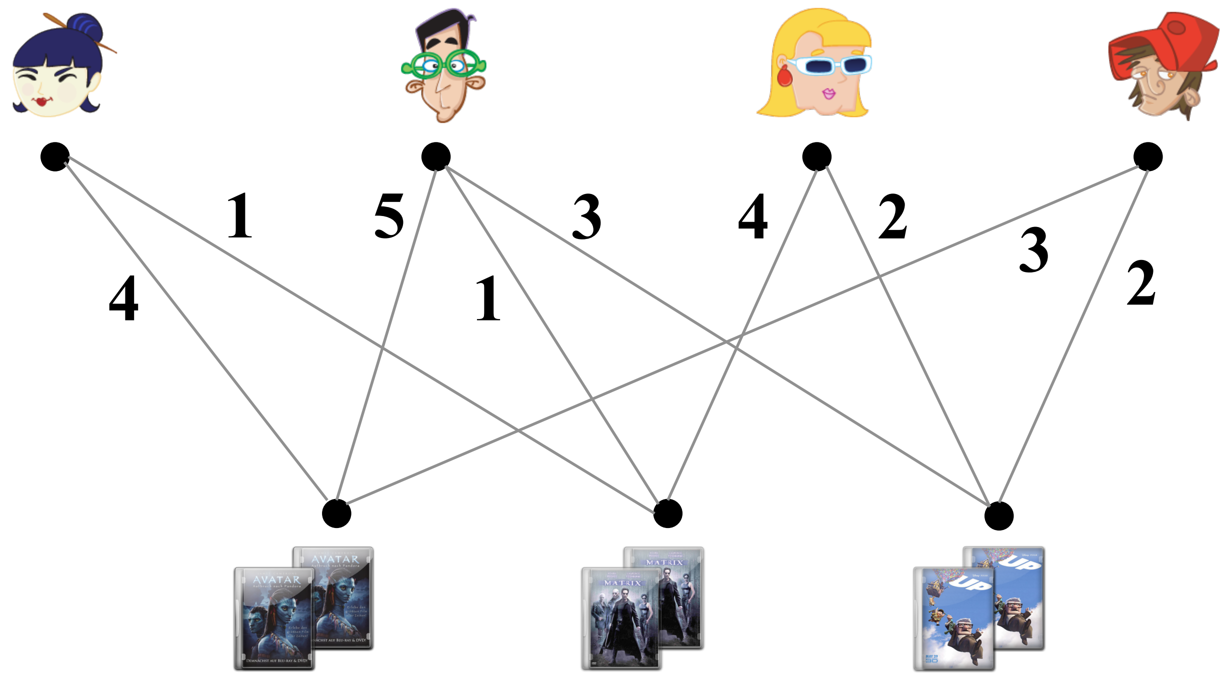
Generalized Bipartite Matching (GBM)

\# Recommendation = 1

Users

Predicted ratings

DVDs



Available DVD per movie = 2

Constraints:

- Neither too few nor too many recommendations
- Number of DVDs limited
- …

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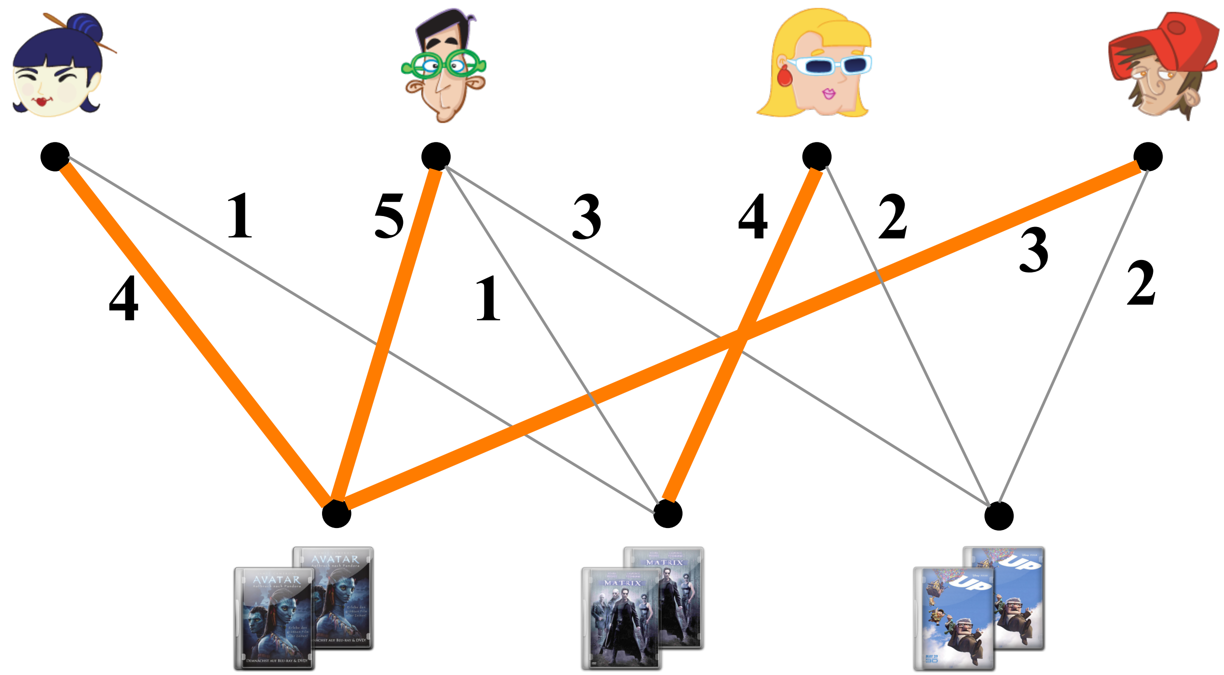

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Generalized Bipartite Matching (GBM)

$1 \leq$ # Recommendations ≤ 2

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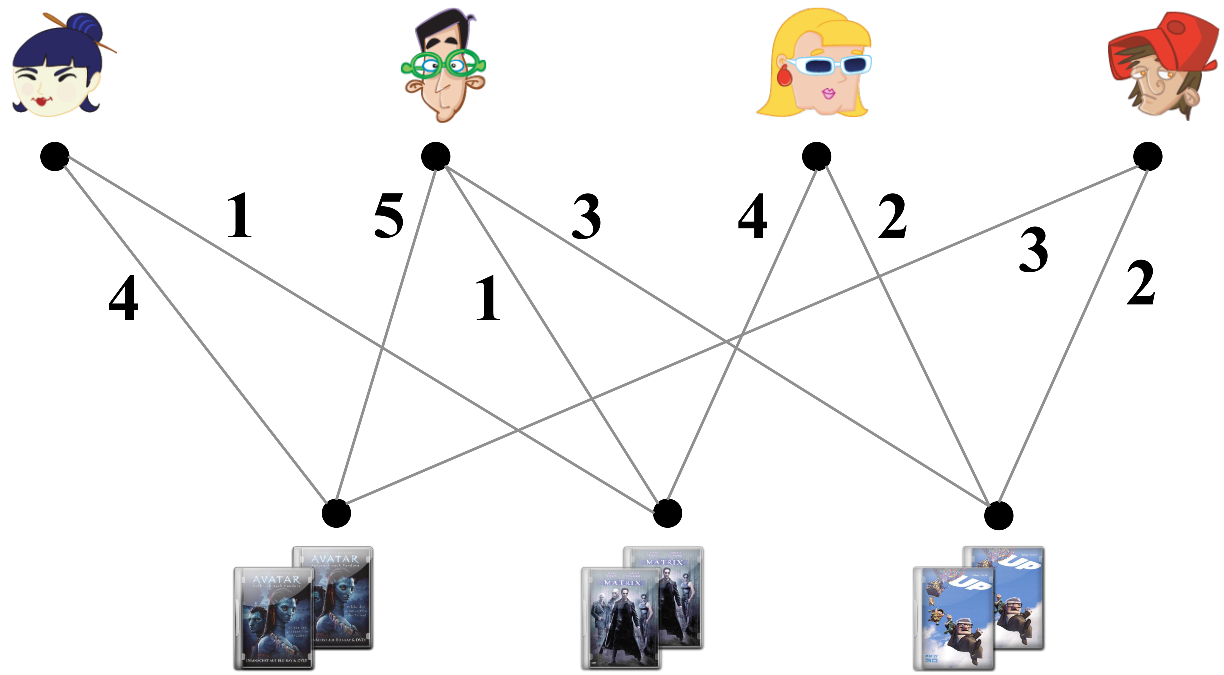

Available DVD per movie = 2

Goal

- Recommend items to users s.t.
  - I. Interesting for users
  - II. Neither too few nor too many
  - III. Availability of items satisfied

s.t. Maximum weight matching (I); LB and UB constraints (II, III)

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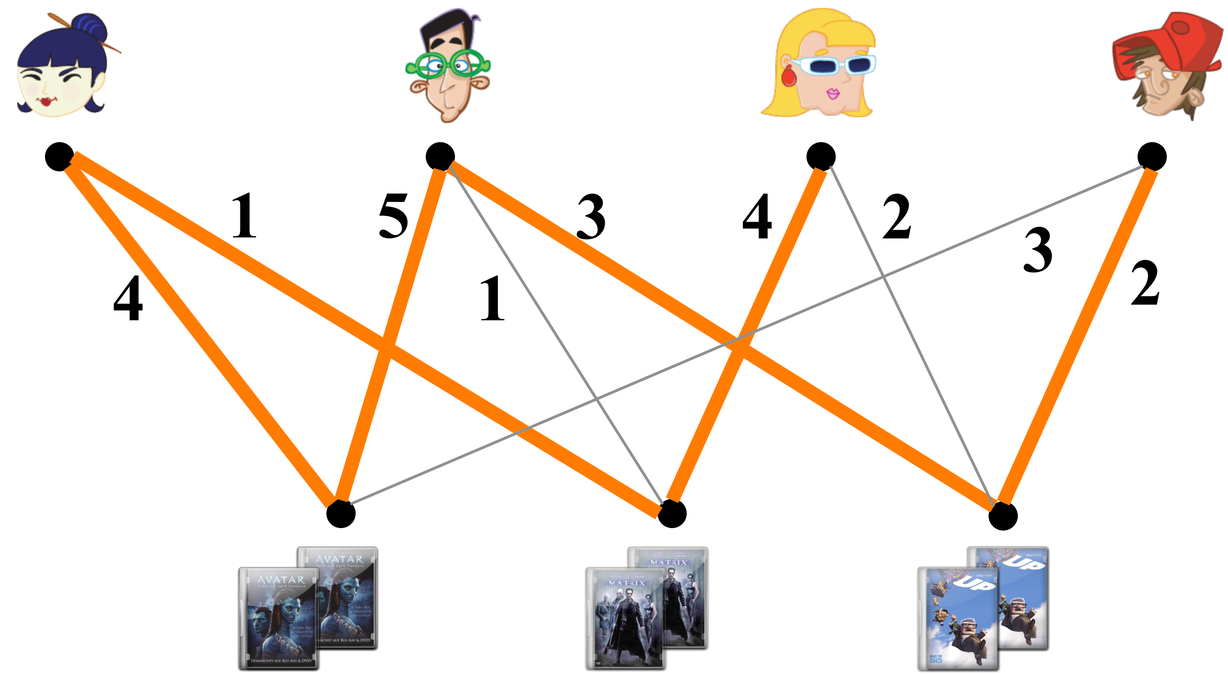

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Maximum weight matching s.t. LB and UB constraints

Challenge

GBM optimally solvable in polynomial time

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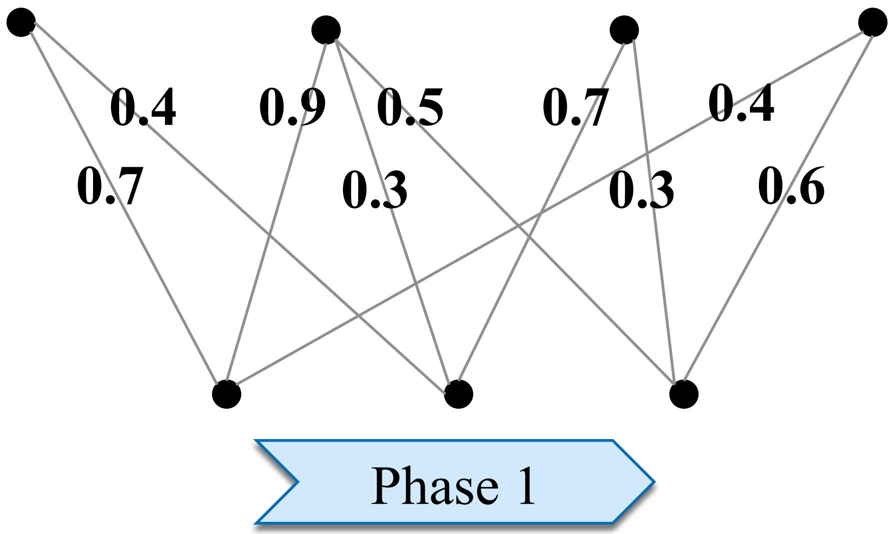
Goal:

- **Efficient** and **scalable** algorithm for large-scale GBM instances

Framework

Phase 1: Approximate LP

- Compute "edge probabilities" using linear programming



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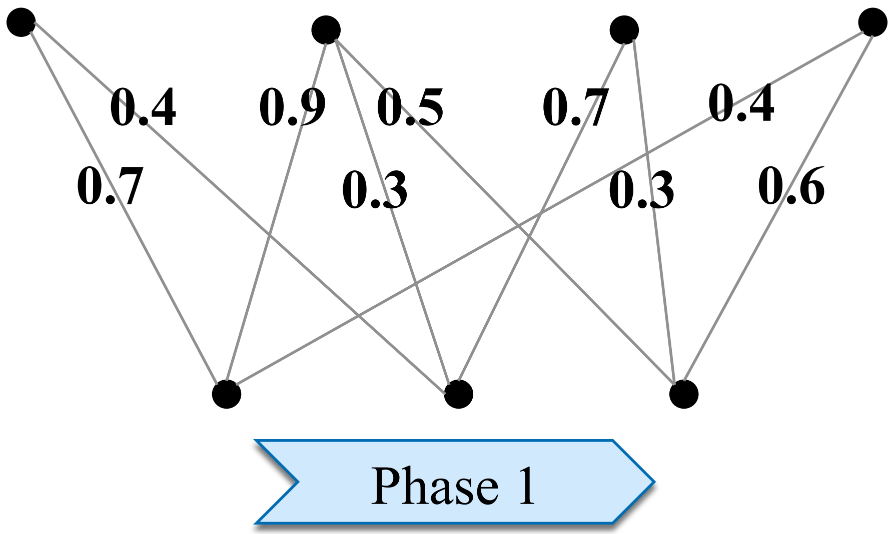
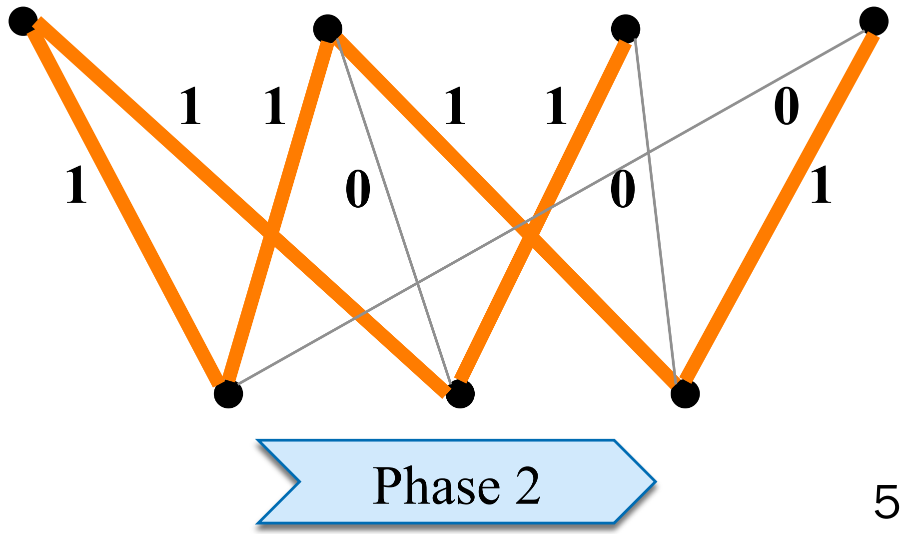


Phase 2: Round

- Select edges based on probabilities from phase 1



Phase 1: Computing edge probabilities

Contrib. 1

Algorithm for Mixed Packing Covering (MPC) LPs (like GBM LP)

- Gradient-based multiplicative weights update algorithm
- Approximately solves MPC LPs (ε: approx. parameter)
  - LB and UB constraints satisfied up to $(1\pm\varepsilon)$ ⟵ Almost feasible
  - Objective value $(1-\varepsilon)$ of the optimum ⟵ Near-optimal
- Poly-log rounds
- Easy to implement: Each round involves matrix-vector multiplications

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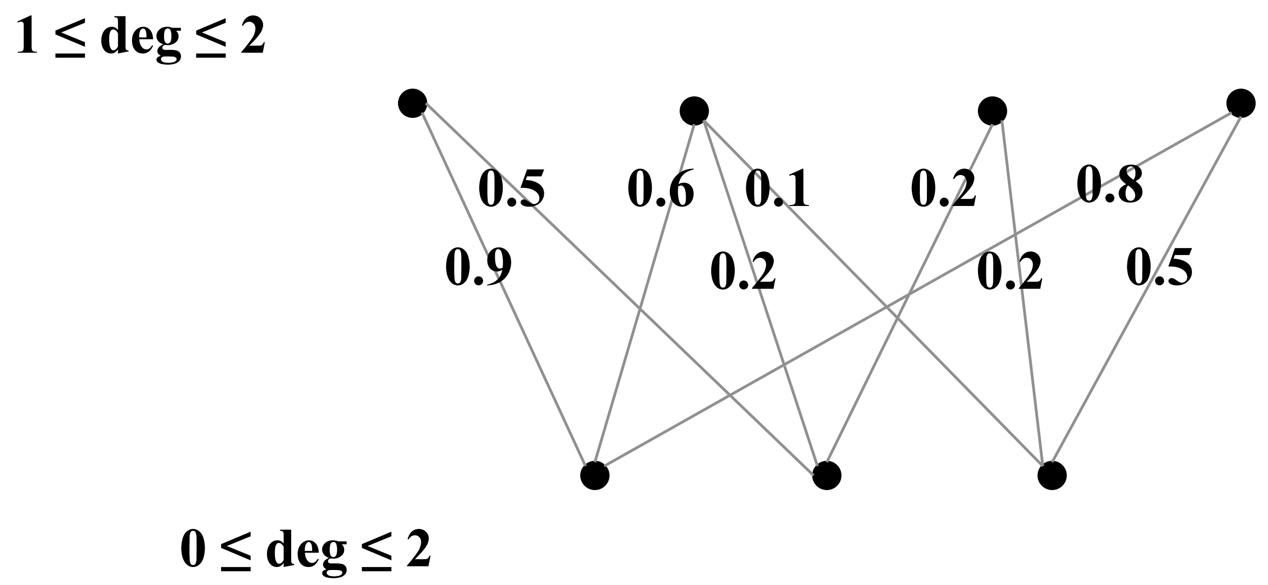

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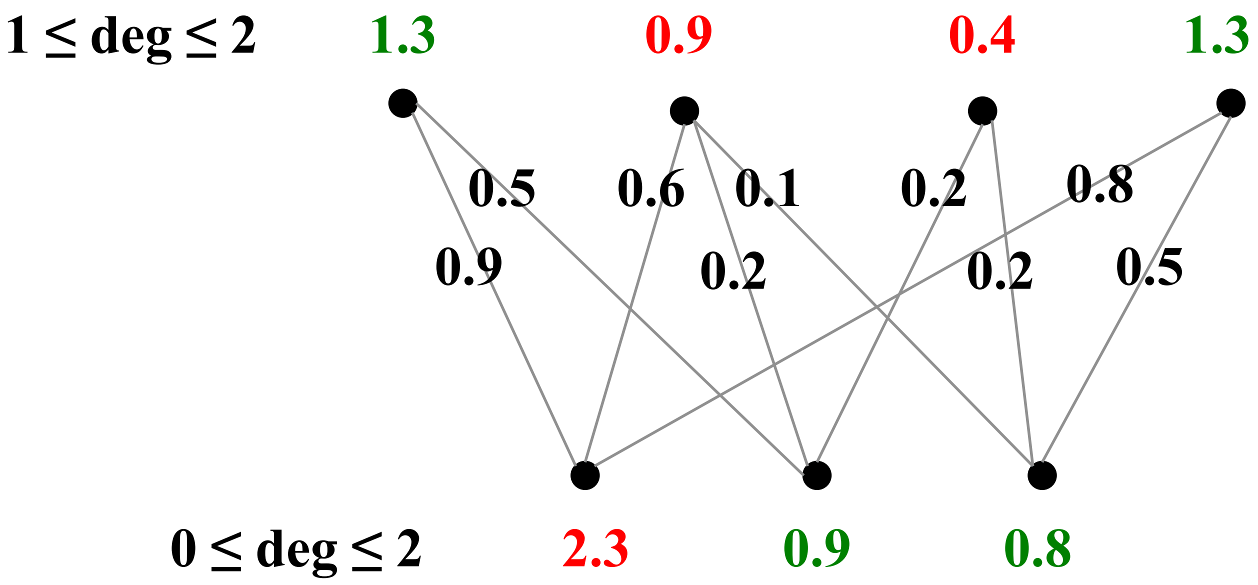

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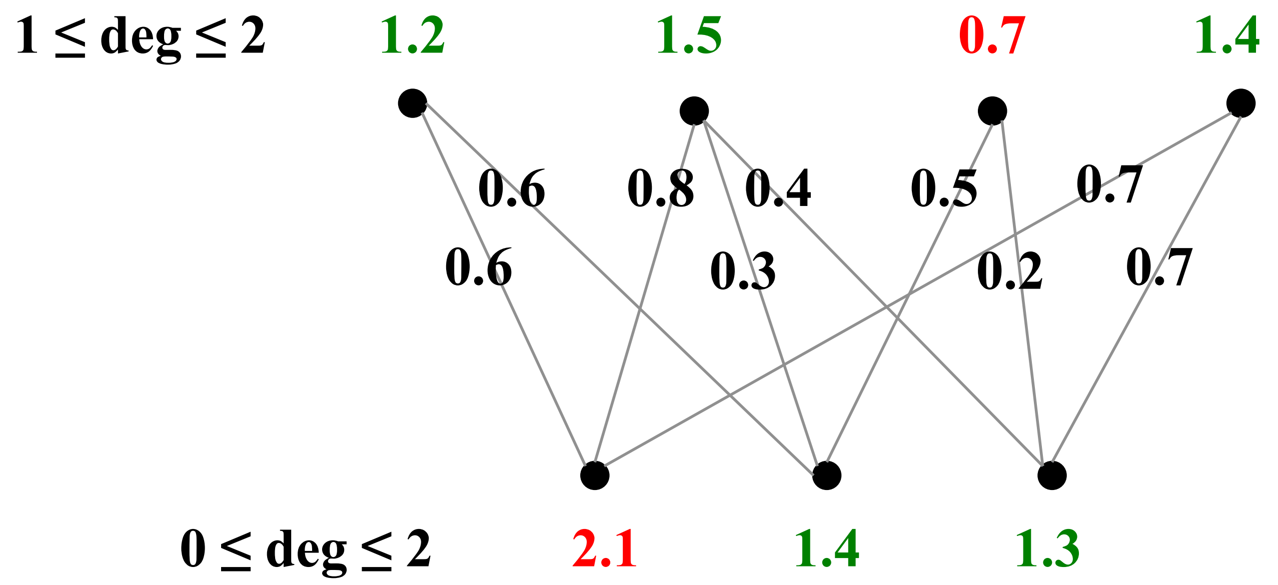

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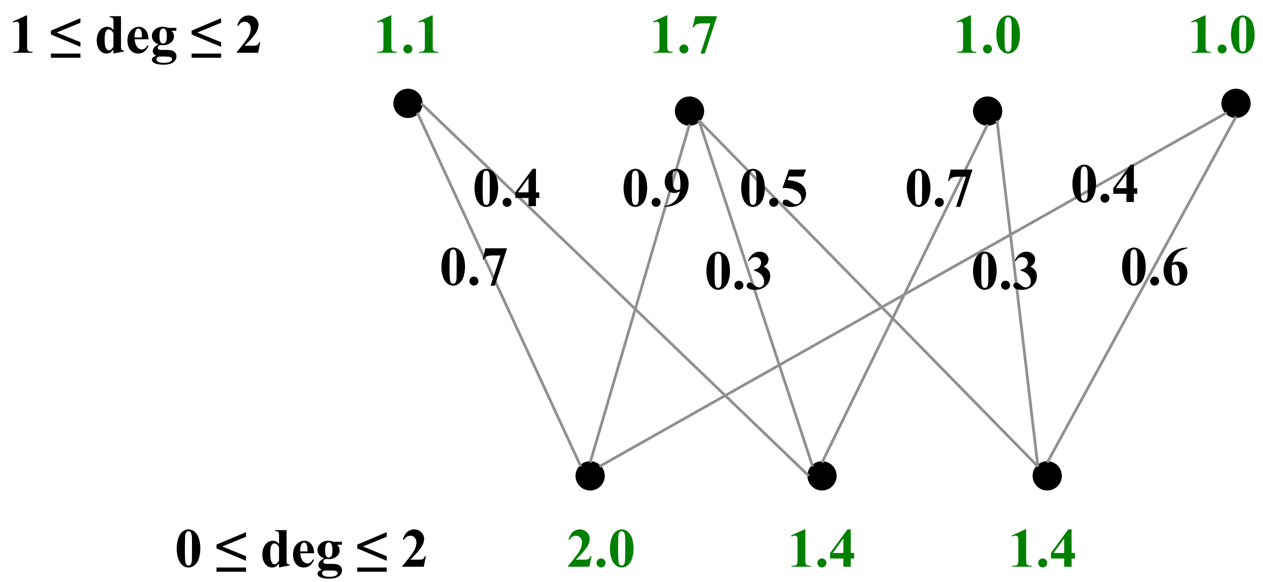

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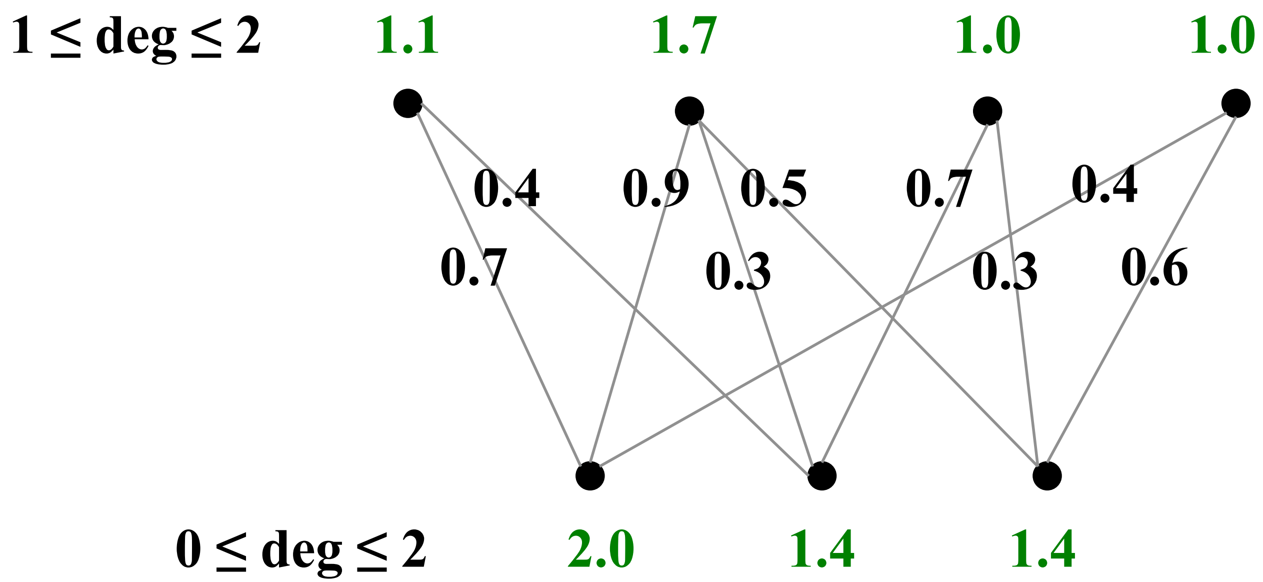


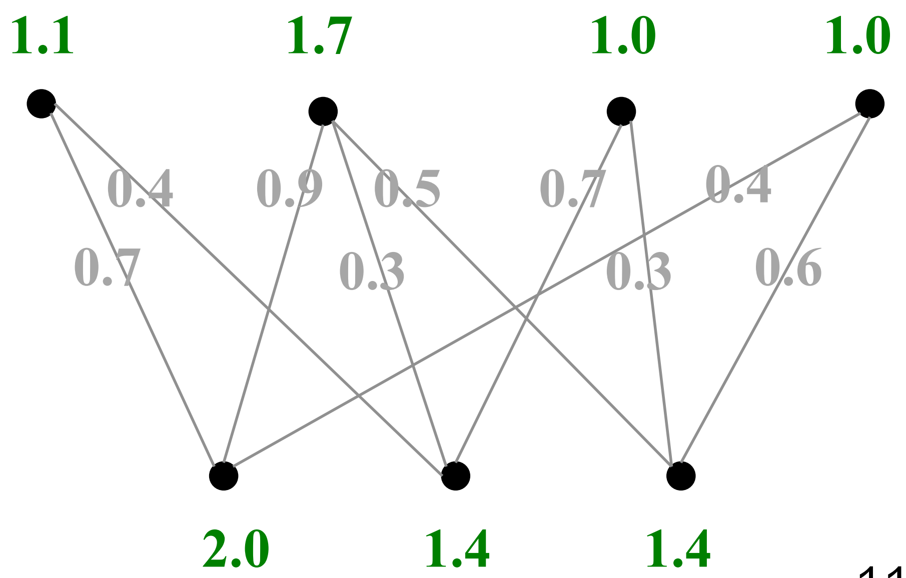
Contrib. 2

Distributed Processing for GBM (details in paper)

- Communication depends on # nodes not # edges
- All computation in parallel

Phase 2: Selecting edges

- **Given:** Edge probabilities from phase 1
- **Goal:** Select edges to be in final solution s. t.
  - I. LB and UB constraints satisfied (up to rounding)
  - II. Approx. guarantee preserved (in expectation)



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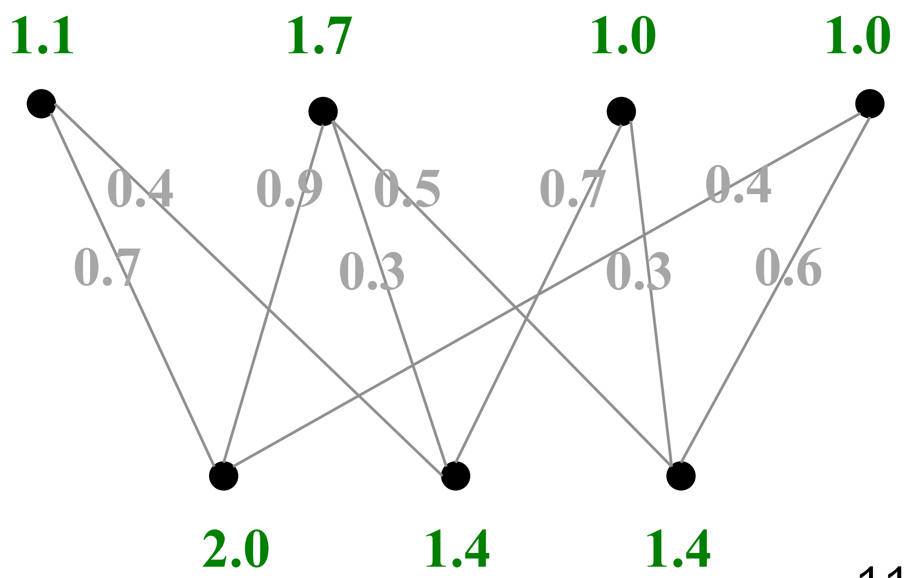

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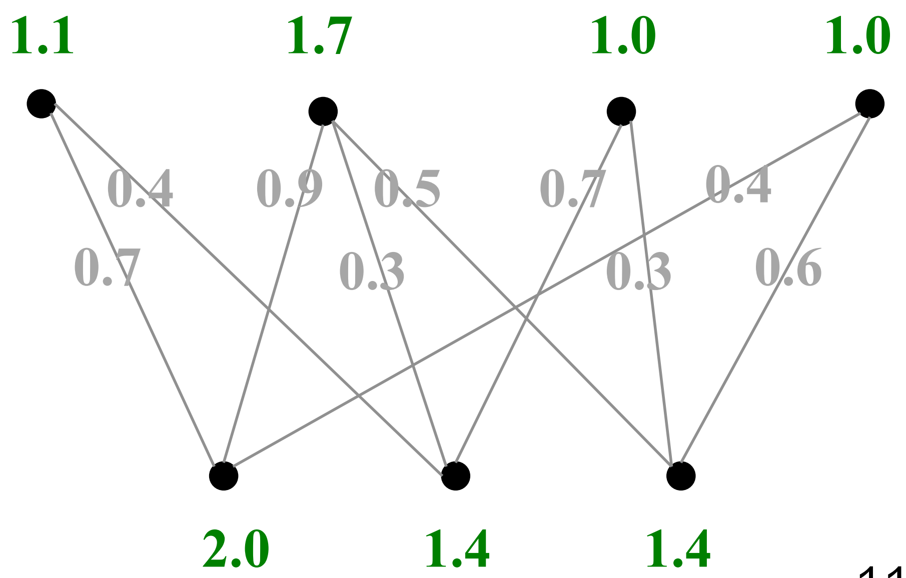

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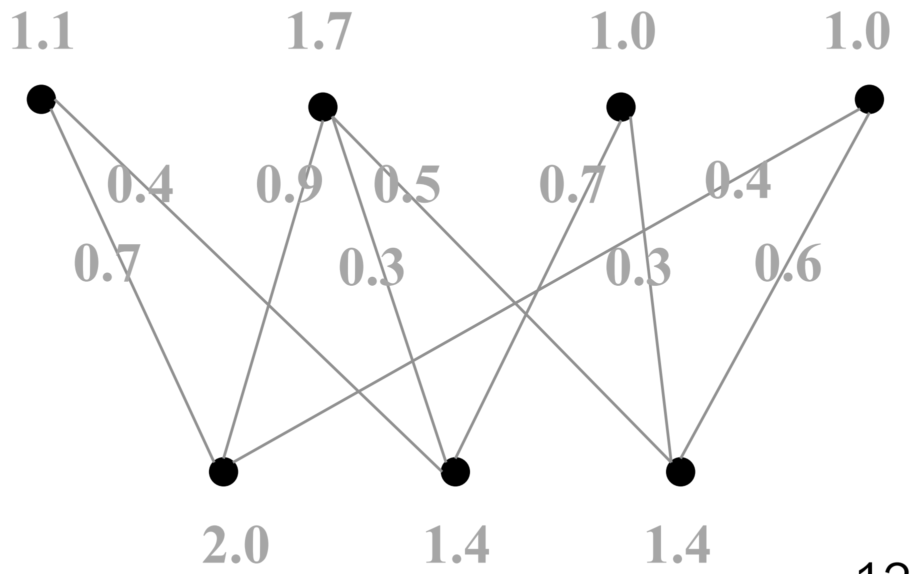

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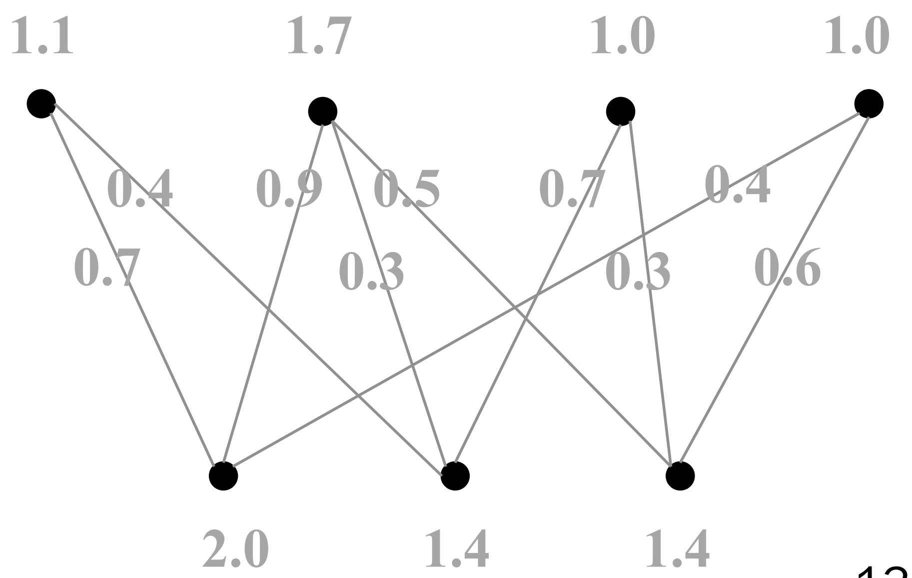

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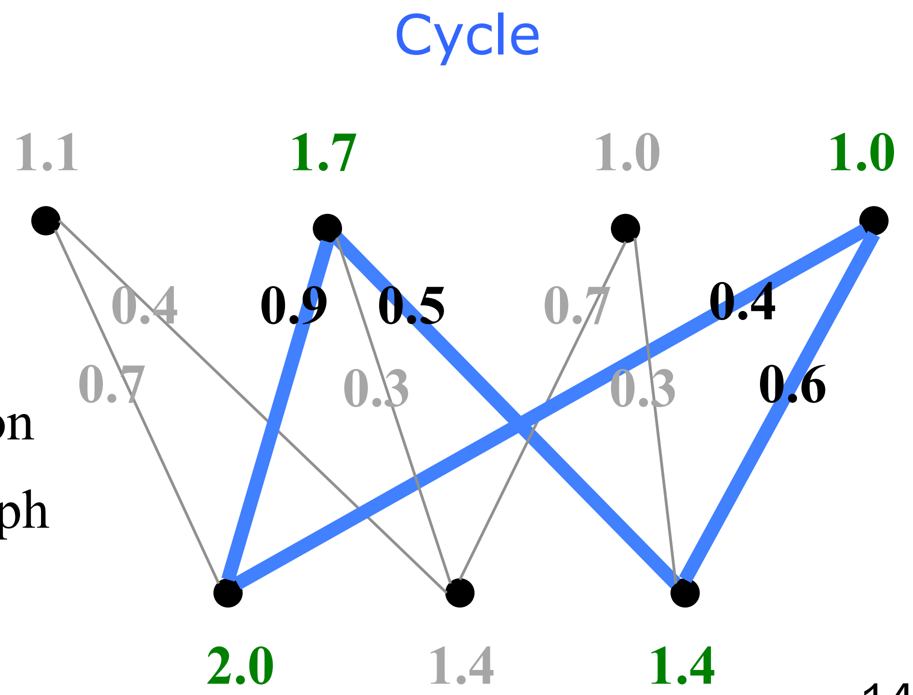

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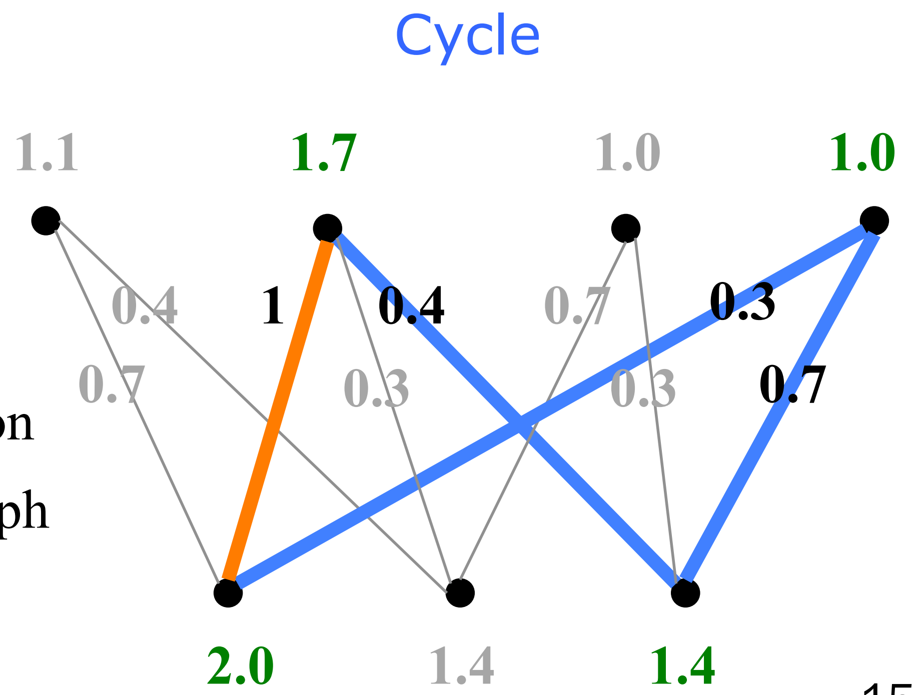

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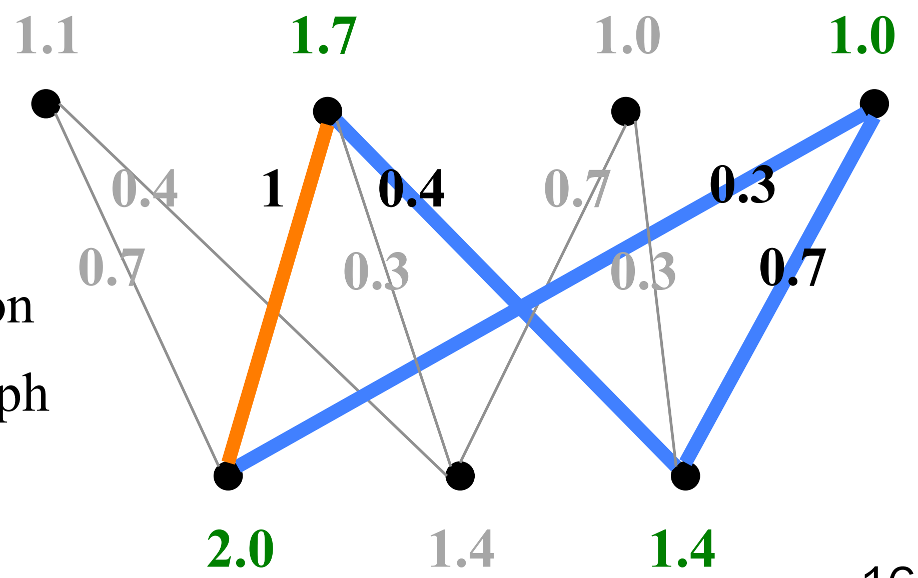
Cycle



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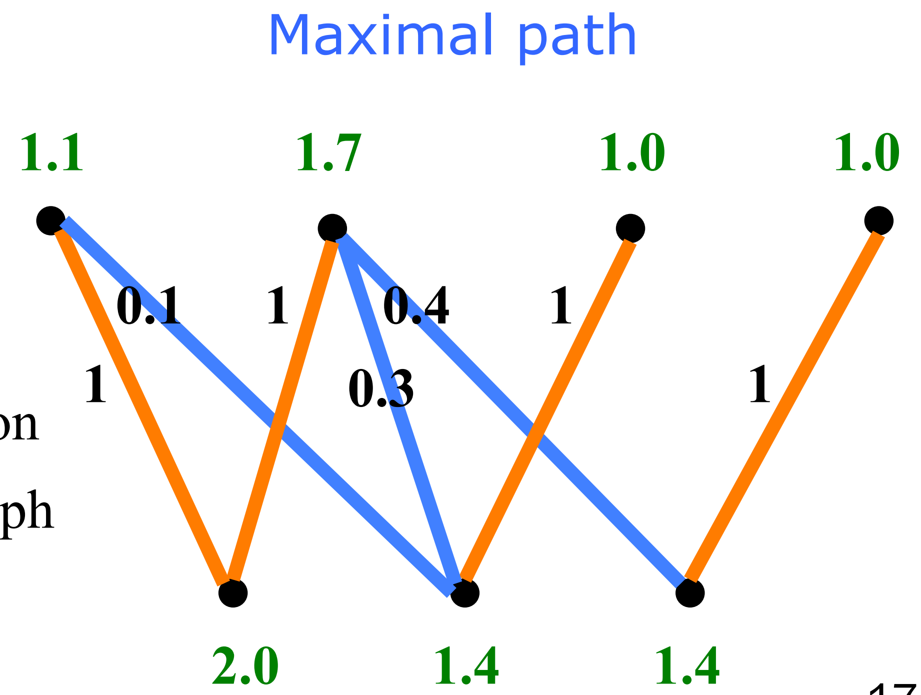

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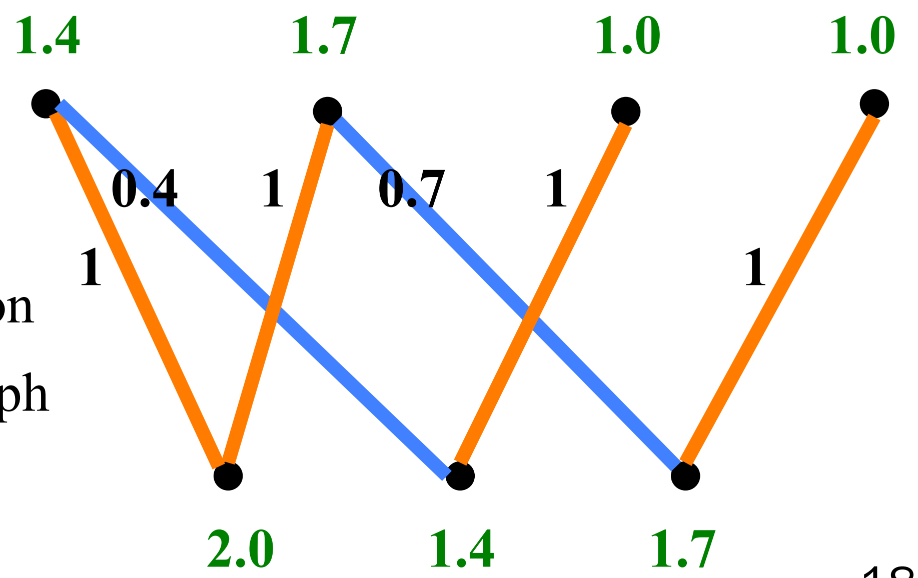
Maximal path



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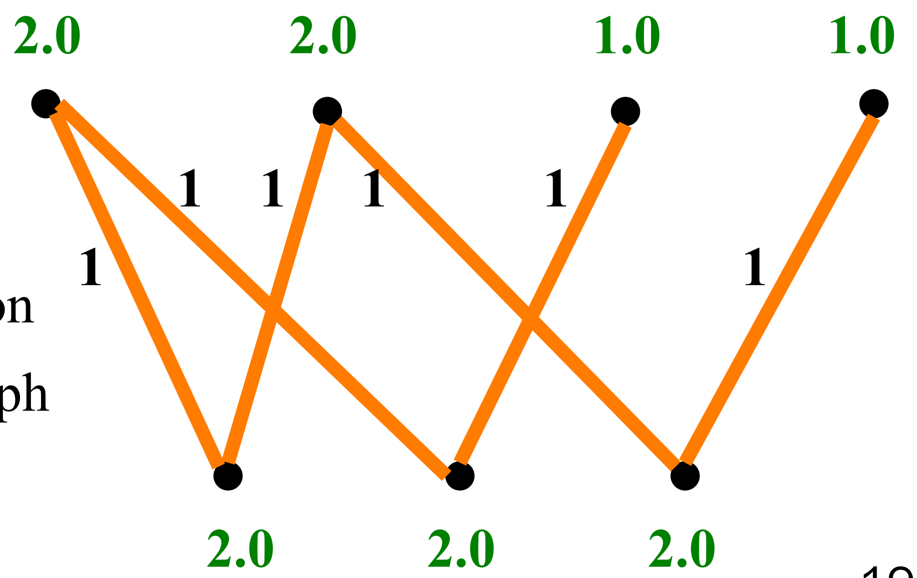

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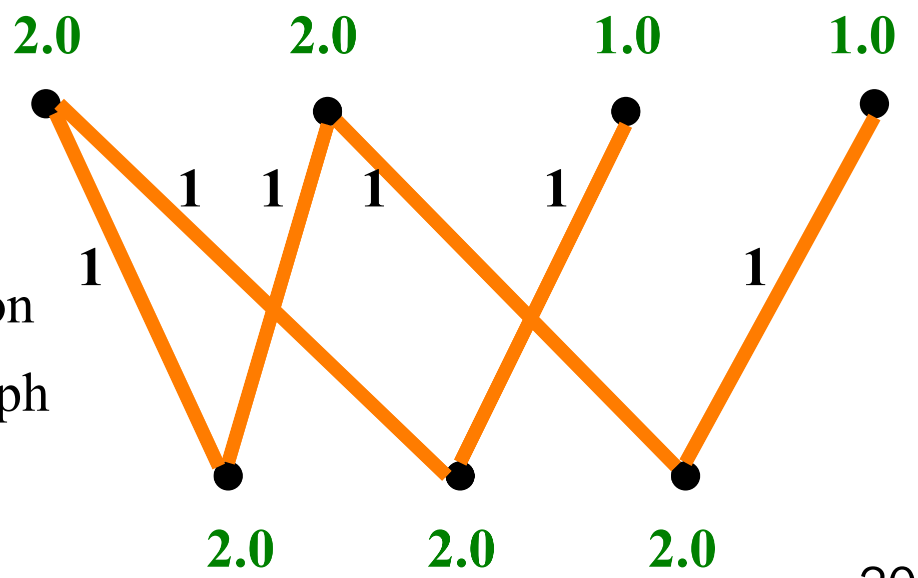

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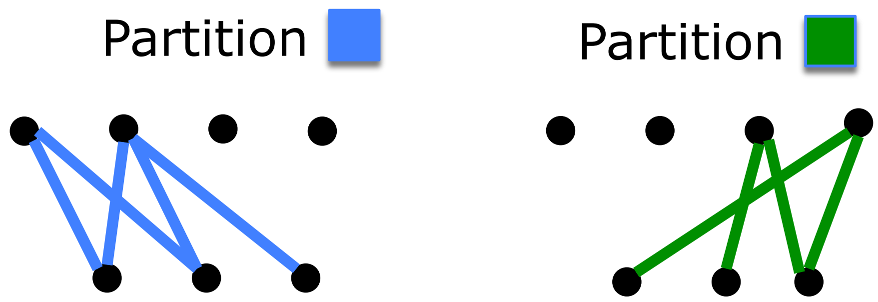
Contrib. 3

How to distribute?

- A local cycle is a global cycle
- A local maximal path is not a global maximal path
- Order of processing cycles has no affect on approx. guarantess



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Contrib. 3

Distributed algorithm:

1. Partition edges uniformly across compute nodes
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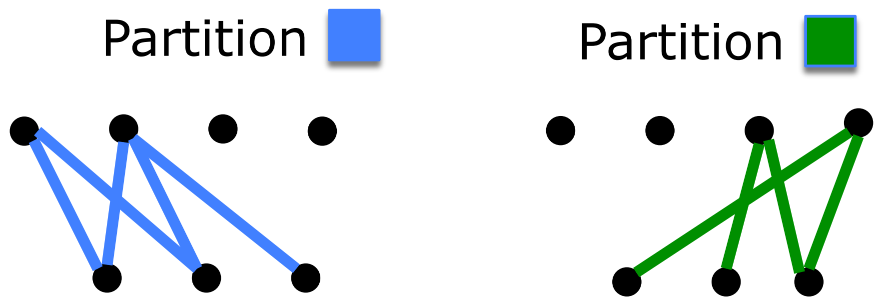

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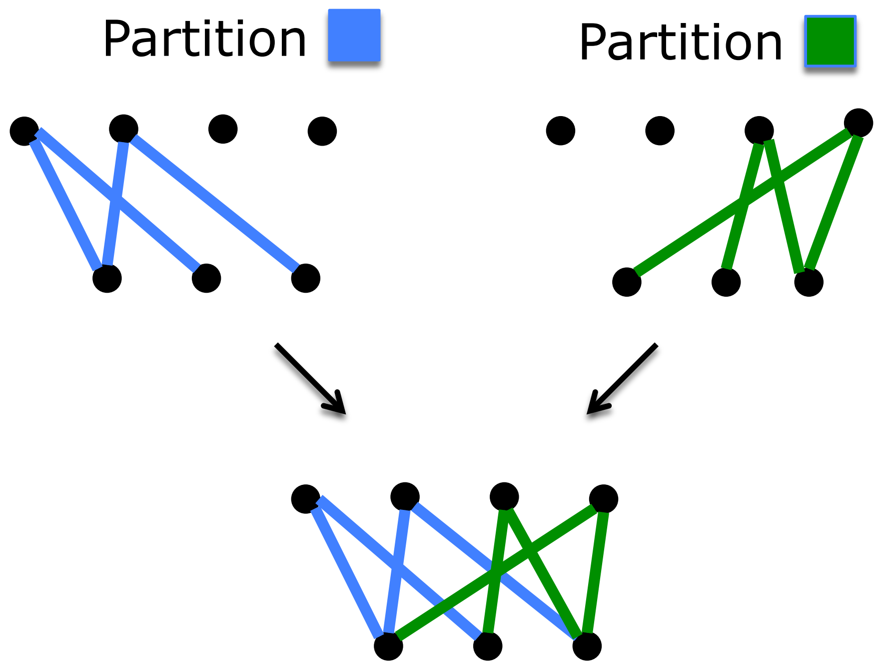

Large-Scale (Semi-) Synthetic Experiments

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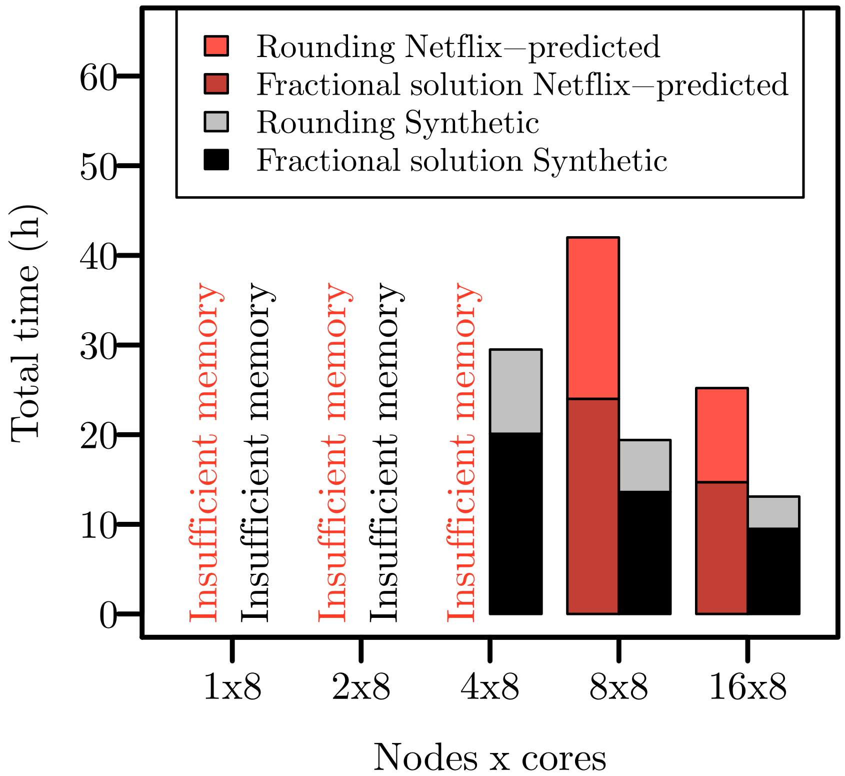
Cluster of commodity nodes



Powerful server

Gurobi ran out of memory on a large-memory server with 512GB RAM

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Scales to very large problem instances

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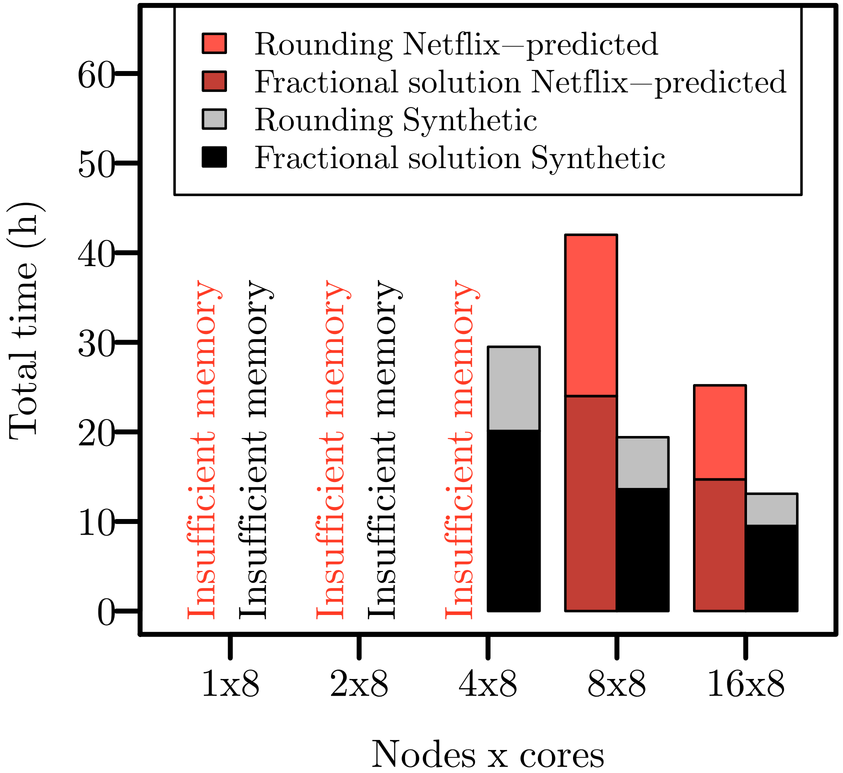

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