UNIVERSITY OF MANNHEIM

Data Mining I

Association Analysis

Example Applications in which Co-Occurrence Matters

- We are often interested in co-occurrence relationships.
- Marketing
  1. identify items that are bought together by sufficiently many customers.
  2. use this information for marketing or supermarket shelf management purposes.

- Inventory Management
  1. identify parts that are often needed together for repairs.
  2. use this information to equip your repair vehicles with the right parts.

- Usage Mining
  1. identify words that frequently appear together in search queries.
  2. use this information to offer auto-completion features to the user.

Outline

1. Correlation Analysis
2. Association Analysis
    1. Frequent Itemset Generation
    2. Rule Generation
    3. Handling Continuous and Categorical Attributes
    4. Interestingness Measures

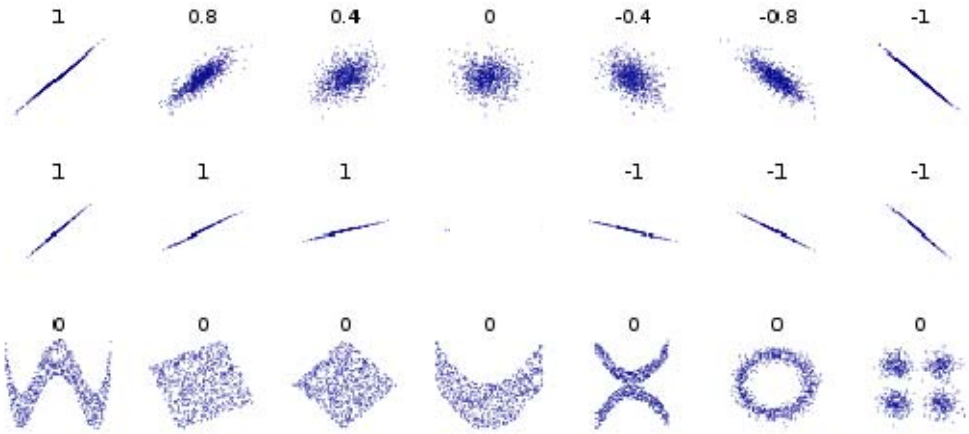
1. Correlation Analysis

– Correlation analysis measures the degree of dependency between two variables.
  - Continuous variables: Pearson's correlation coefficient (PCC)
  - Binary variables: Phi coefficient

$$PCC(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

$$Phi(x,y) = \frac{f_{11} f_{00} - f_{01} f_{10}}{\sqrt{f_{1+} f_{+1} f_{0+} f_{+0}}}$$

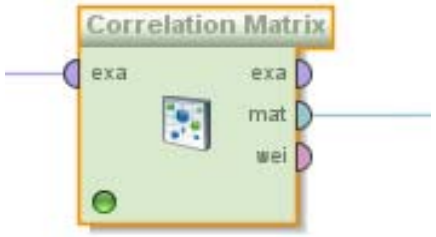
– Value range [-1,1]
  - 1 : positive correlation
  - 0 : variables independent
  - -1 : negative correlation



Correlations between Products in Shopping Baskets

	P1	P2	P3	P4	P5
Basket 1	1	1	0	1	1
Basket 2	1	0	0	1	1
Basket 3	1	0	0	0	1

1 : always bought together
0 : sometimes bought together
-1 : never bought together



Correlation Matrix (Correlation Matrix)

Table View | Pairwise Table | Plot View | Annotations

Attributes	ThinkPad X...	Asus EeePC	HP Laserjet...	2 GB DDR3...	8 GB DDR3...	Lenovo Tab...	Netbook-Sc...	HP CE50 T...	LT Laser M...	LT Minimaus
ThinkPad X2	1	-1	0.356	-0.816	0.612	0.583	-0.667	0.356	0.167	-0.408
Asus EeePC	-1	1	-0.356	0.816	-0.612	-0.583	0.667	-0.356	-0.167	0.408
HP Laserjet	0.356	-0.356	1	-0.218	-0.327	0.356	-0.535	1	-0.089	-0.655
2 GB DDR3	-0.816	0.816	-0.218	1	-0.500	-0.816	0.816	-0.218	0	0.200
8 GB DDR3	0.612	-0.612	-0.327	-0.500	1	0.102	-0.408	-0.327	0.102	0
Lenovo Tabl	0.583	-0.583	0.356	-0.816	0.102	1	-0.667	0.356	-0.250	0
Netbook-Sch	-0.667	0.667	-0.535	0.816	-0.408	-0.667	1	-0.535	0.167	0.408
HP CE50 To	0.356	-0.356	1	-0.218	-0.327	0.356	-0.535	1	-0.089	-0.655
LT Laser Ma	0.167	-0.167	-0.089	0	0.102	-0.250	0.167	-0.089	1	-0.408
LT Minimaus	-0.408	0.408	-0.655	0.200	0	0	0.408	-0.655	-0.408	1

Shortcoming: Measures correlation only between two items but not between multiple items, e.g. {ThinkPad, Cover} → {Minimaus}

2. Association Analysis

- Association analysis can find multiple item co-occurrence relationships (descriptive method)
- focuses on occurring items, not absent items
- first algorithms developed in the early 90s at IBM by Agrawal & Srikant
- initially used for shopping basket analysis to find how items purchased by customers are related
- later extended to more complex data structures
  - sequential patterns
  - subgraph patterns
- and other application domains
  - web usage mining, social science, life science

Association Analysis

Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction.

Shopping Transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Examples of Association Rules

{Diaper} → {Beer}
{Beer, Bread} → {Milk}
{Milk, Bread} → {Eggs, Coke}

Implication means co-occurrence, not causality!

Definition: Support and Frequent Itemset

- Itemset
  - A collection of one or more items
  - Example: {Milk, Bread, Diaper}
  - k-itemset: An itemset that contains k items
- Support count (σ)
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread, Diaper}) = 2
- Support (s)
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5 = 0.4
- Frequent Itemset
  - An itemset whose support is greater than or equal to a minimal support (*minsup)* threshold specified by the user

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

- Association Rule
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - An association rule states that when X occurs, Y occurs with certain probability.
  - Example:
    {Milk, Diaper} → {Beer}
    Condition Consequent

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- Rule Evaluation Metrics
  - Support (s)
    Fraction of transactions that contain both X and Y
  - Confidence (c)
    Measures how often items in Y appear in transactions that contain X

$$s(X \rightarrow Y) = \frac{|X \cup Y|}{|T|} \qquad s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)} \qquad c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Main Challenges concerning Association Analysis

1. Mining associations from large amounts of data can be computationally expensive
   - Algorithms need to apply smart pruning strategies
2. Algorithms often discover a large number of associations
   - many of them are uninteresting or redundant
   - the user needs to select the subset of the associations that is relevant given her task at hand

The Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  1. support ≥ *minsup* threshold
  2. confidence ≥ *minconf* threshold
- *Minsup* and *minconf* are provided by the user.
- Brute Force Approach:
  1. List all possible association rules
  2. Compute the support and confidence for each rule
  3. Remove rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive due to large number of candidates!

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example rules:

{Milk, Diaper} → {Beer} (s=0.4, c=0.67)
{Milk, Beer} → {Diaper} (s=0.4, c=1.0)
{Diaper, Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk, Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk, Beer} (s=0.4, c=0.5)
{Milk} → {Diaper, Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence.
- Thus, we may decouple the support and confidence requirements.

Mining Association Rules

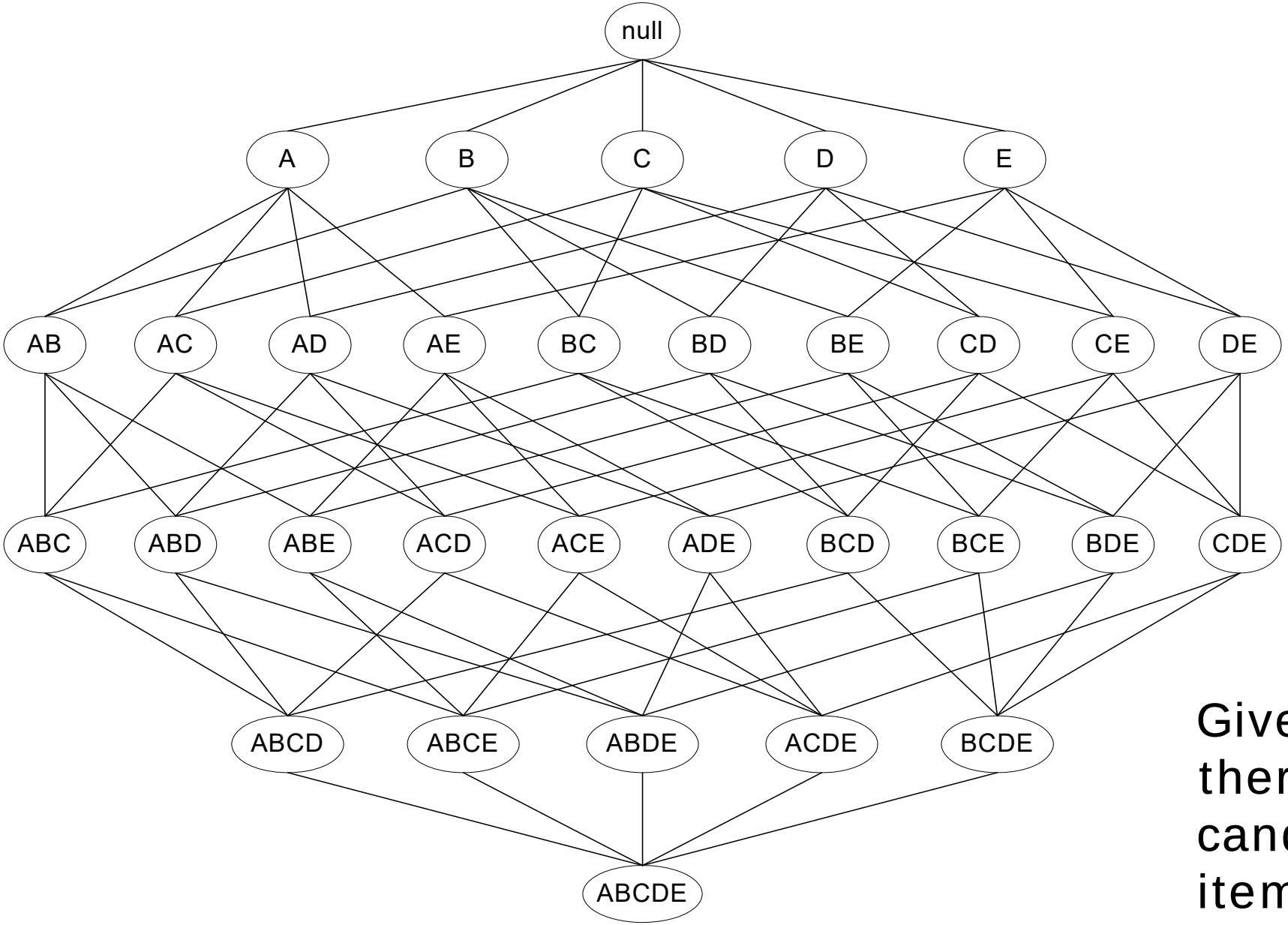
- Two-step approach:

1. Frequent Itemset Generation
   - Generate all itemsets whose support $\geq$ minsup

2. Rule Generation
   - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive.

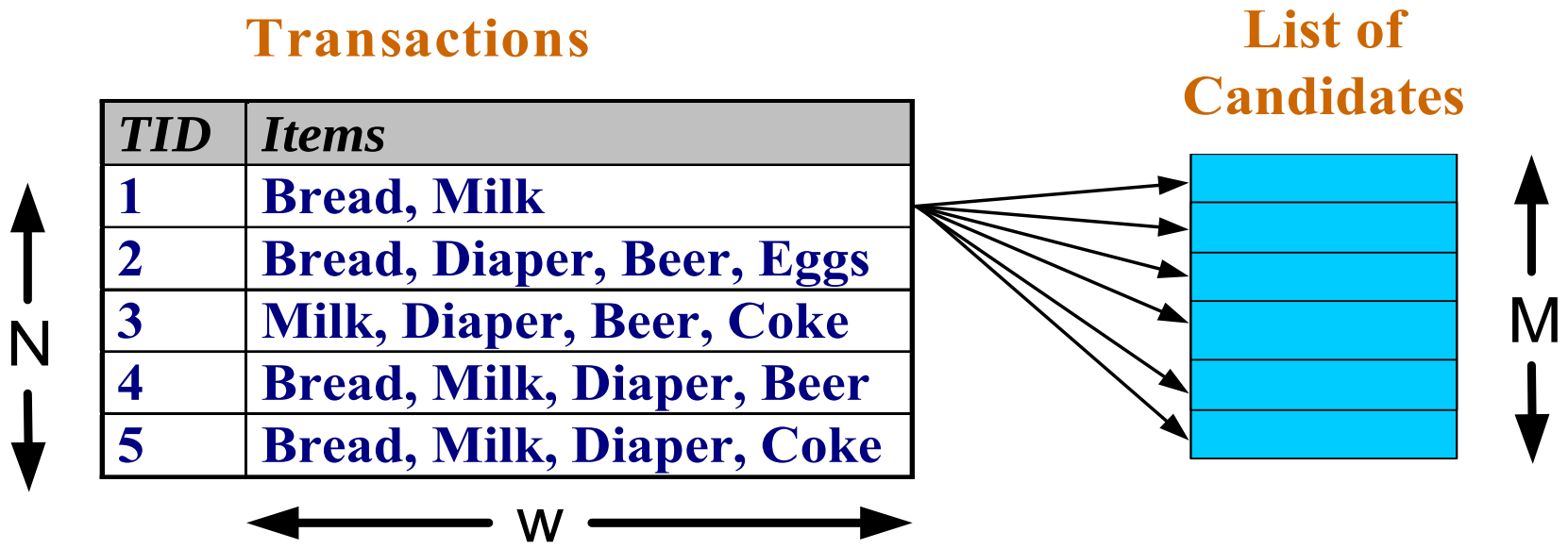
2.1 Frequent Itemset Generation



Given d items, there are 2^d candidate itemsets!

Brute Force Approach

- Each itemset in the lattice is a candidate frequent itemset
- Count the support of each candidate by scanning the database
- Match each transaction against every candidate



- Complexity ~ O(NMw) ➔ Expensive since $M = 2^d$!!!
- A smarter algorithm is required

Example: Brute Force Approach

- Example:
  - Amazon has 10 million books (i.e., Amazon Germany, as of 2011)
- That is $2^{10.000.000}$ possible itemsets
- As a number:
  - $9.04981... \times 10^{3.010.299}$
  - That is: a number with 3 million digits!

- However:
  - most itemsets will not be important at all, e.g., books on Chinese calligraphy, Inuit cooking, and data mining bought together
  - thus, smarter algorithms should be possible.
  - intuition for the algorithm: All itemsets containing Inuit cooking are likely infrequent.

Reducing the Number of Candidates

– Apriori Principle

If an itemset is frequent, then all of its subsets must also be frequent.

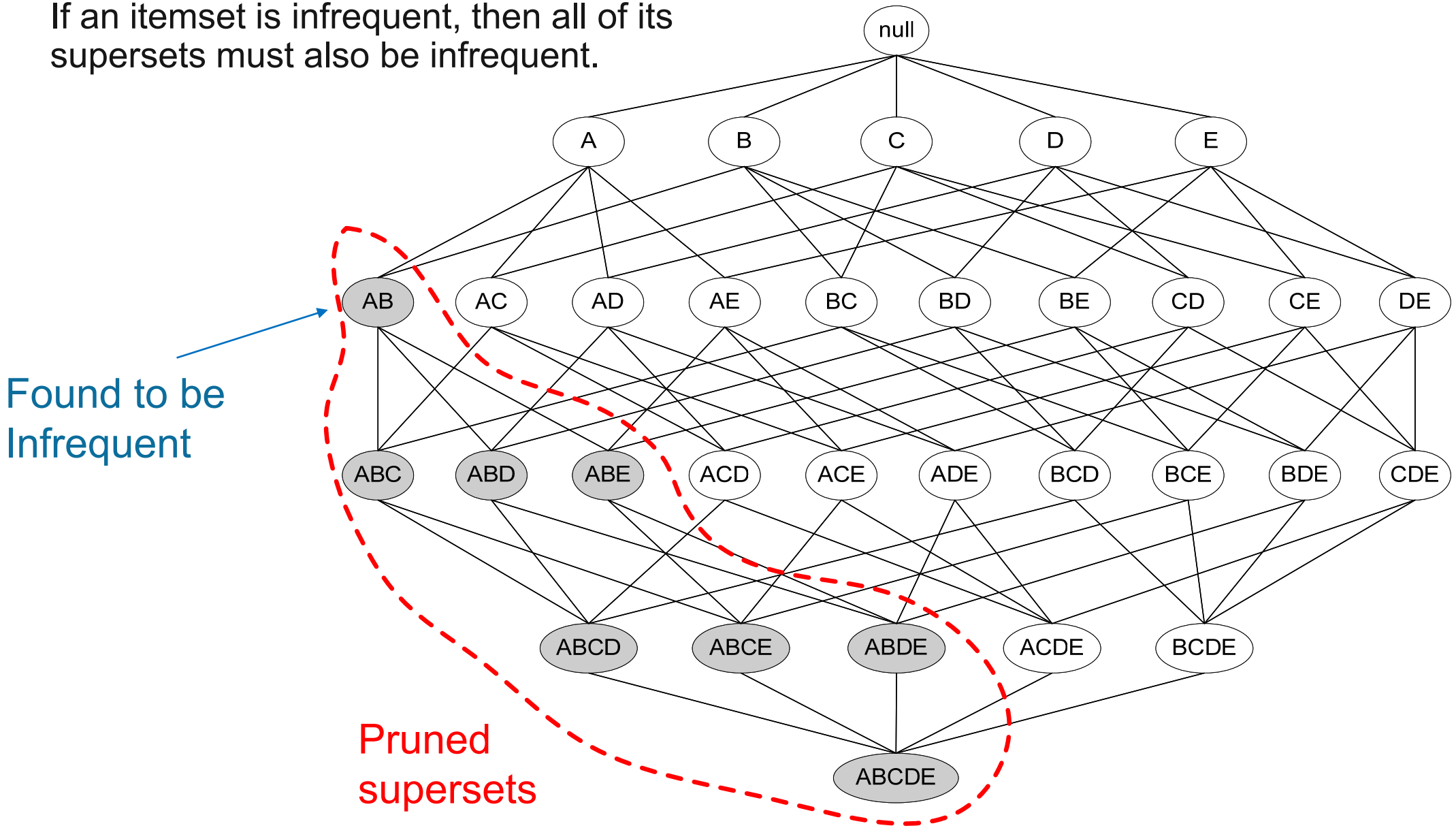
– Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the anti-monotone property of support

Using the Apriori Principle for Pruning

If an itemset is infrequent, then all of its supersets must also be infrequent.



Example: Using the Apriori Principle for Pruning

Minimum Support Count = 3

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

No need to generate candidates involving Coke or Eggs.

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3

No need to generate candidate {Milk, Diaper, Beer} as count {Milk, Beer} = 2.

The Apriori Algorithm

1. Let k=1
2. Generate frequent itemsets of length 1
3. Repeat until no new frequent itemsets are identified
    1. Generate length (k+1) candidate itemsets from length k frequent itemsets
    2. Prune candidate itemsets that can not be frequent because they contain subsets of length k that are infrequent (Apriori Principle)
    3. Count the support of each candidate by scanning the DB
    4. Eliminate candidates that are infrequent, leaving only those that are frequent

Example: Apriori Algorithm

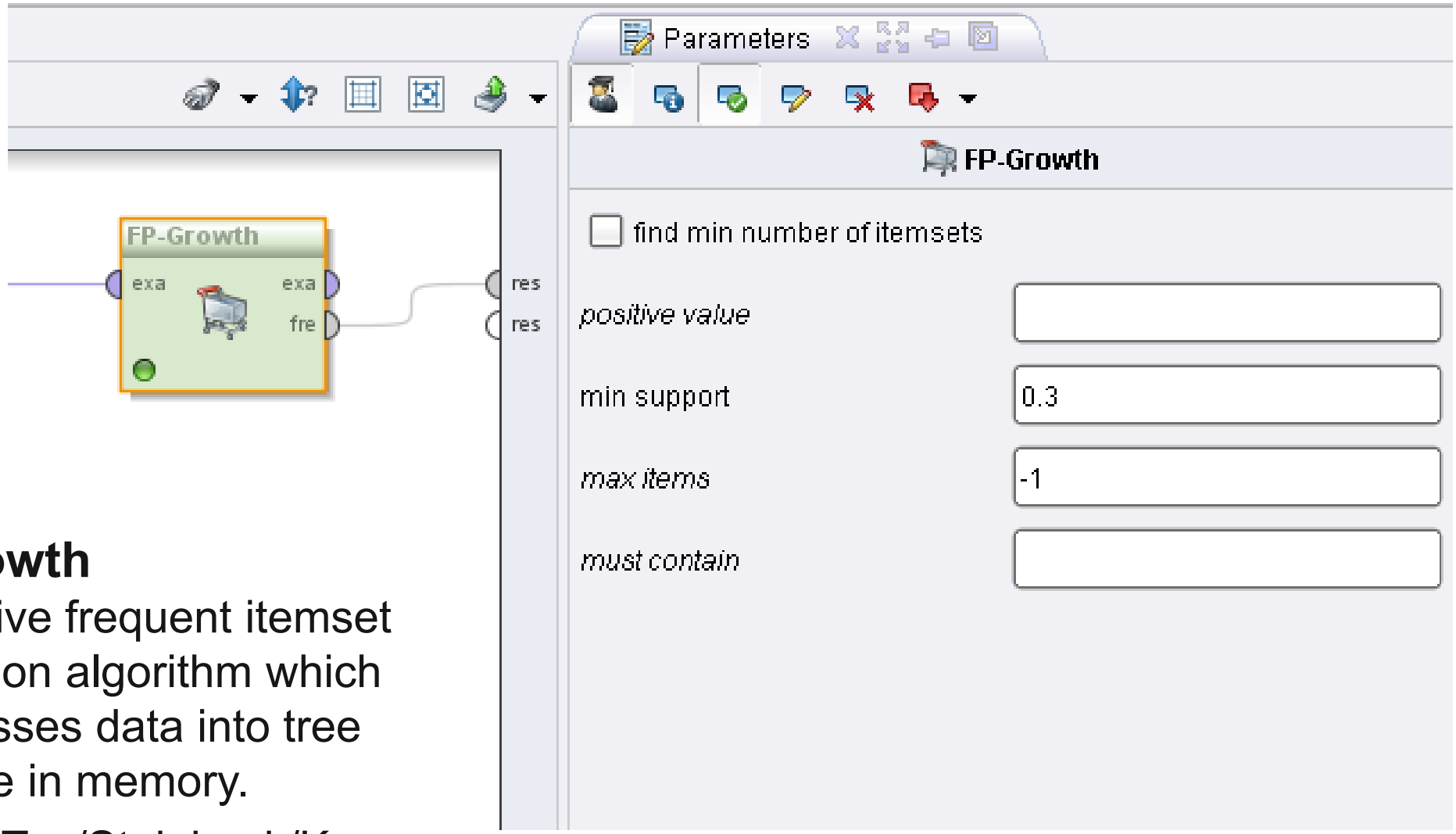
itemset:count minsup=0.5

Dataset T

TID	Items
T100	1, 3, 4
T200	2, 3, 5
T300	1, 2, 3, 5
T400	2, 5

1. scan T
   - ➔ $Cand_1$: {1}:2, {2}:3, {3}:3, {4}:1, {5}:3
   - ➔ $Fequ_1$: {1}:2, {2}:3, {3}:3, {5}:3
   - ➔ $Cand_2$: {1,2}, {1,3}, {1,5}, {2,3}, {2,5}, {3,5}
2. scan T
   - ➔ $Cand_2$: {1,2}:1, {1,3}:2, {1,5}:1, {2,3}:2, {2,5}:3, {3,5}:2
   - ➔ $Fequ_2$: **{1,3}**:2, **{2,3}**:2, **{2,5}**:3, **{3,5}**:2
   - ➔ $Cand_3$: {2, 3, 5}
3. scan T
   - ➔ C_3: **{2, 3, 5}**:2
   - ➔ F_3: **{2, 3, 5}**

Frequent Itemset Generation in Rapidminer



FP-Growth

Alternative frequent itemset generation algorithm which compresses data into tree structure in memory.

Details: Tan/Steinback/Kumar Chapter 6.6

Frequent Itemsets in Rapidminer

No. of Sets: 22

Total Max. Size: 4

Min. Size: 1

Max. Size: 4

Contains Item:

Update View

Size	Support	Item 1	Item 2	Item 3	Item 4
1	0.600	Asus EeePC			
1	0.500	LT Minimaus			
1	0.500	2 GB DDR3			
1	0.400	ThinkPad X2			
1	0.400	Netbook-Sch			
1	0.400	Lenovo Tabl			
1	0.400	LT Laser Ma			
1	0.300	HP Laserjet			
1	0.300	HP CE50 To			
2	0.400	Asus EeePC	LT Minimaus		
2	0.500	Asus EeePC	2 GB DDR3		
2	0.400	Asus EeePC	Netbook-Sch		
2	0.300	LT Minimaus	2 GB DDR3		
2	0.300	LT Minimaus	Netbook-Sch		
2	0.400	2 GB DDR3	Netbook-Sch		
2	0.300	ThinkPad X2	Lenovo Tabl		
2	0.300	HP Laserjet	HP CE50 To		
3	0.300	Asus EeePC	LT Minimaus	2 GB DDR3	
3	0.300	Asus EeePC	LT Minimaus	Netbook-Sch	
3	0.400	Asus EeePC	2 GB DDR3	Netbook-Sch	
3	0.300	LT Minimaus	2 GB DDR3	Netbook-Sch	
4	0.300	Asus EeePC	LT Minimaus	2 GB DDR3	Netbook-Sch

Example Application of Frequent Itemsets

1. Take top-k frequent itemsets of size 2 containing item A
2. Rank second item according to
   - profit made by selling item
   - whether you want to reduce number of items B in stock
   - knowledge about customer preferences
3. Offer special price for combination with top-ranked second item

amazon

Wird oft zusammen gekauft

Preis für beide: EUR 138,00

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Verfügbarkeit und Versanddetails anzeigen

Dieser Artikel: Introduction to Data Mining von Pang-Ning Tan Taschenbuch EUR 85,05

Data Mining: Concepts and Techniques (Morgan Kaufmann Series in Data Management Systems)

2.2 Rule Generation

– Given a frequent itemset L, find all non-empty subsets f $\subset$ L such that f $\rightarrow$ L – f satisfies the minimum confidence requirement.

Example Frequent Itemset:

$$\{\text{Milk}, \text{Diaper}, \text{Beer}\}$$

Example Rule:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Challenge: Large Number of Candidate Rules

- If {A,B,C,D} is a frequent itemset, then the candidate rules are:

ABC →D,	ABD →C,	ACD →B,	BCD →A,
A →BCD,	B →ACD,	C →ABD,	D →ABC
AB →CD,	AC → BD,	AD → BC,	BC →AD,
BD →AC,	CD →AB		

- If |L| = k, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \varnothing$ and $\varnothing \rightarrow L$)

Rule Generation

– How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

    c(ABC →D) can be larger or smaller than c(AB →D)

  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., L = {A,B,C,D}:

    $$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - Confidence is anti-monotone with respect to the number of items on the right hand side of the rule

Explanation

Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

- i.e., “moving elements from left to right” cannot increase confidence
- reason:

$$c(AB \rightarrow C) := \frac{s(ABC)}{s(AB)} \qquad c(A \rightarrow BC) := \frac{s(ABC)}{s(A)}$$

- Due to anti-monotone property of support, we know

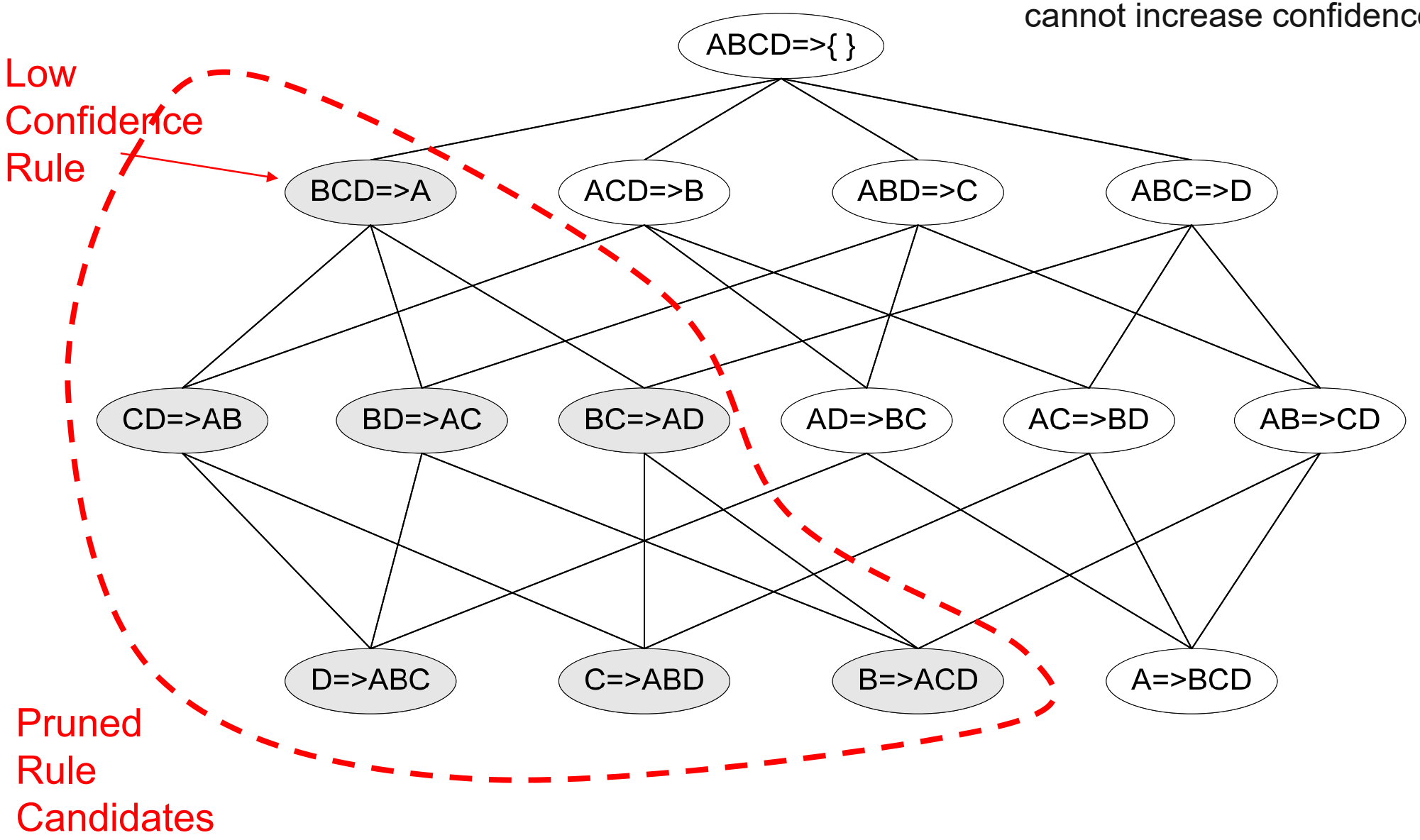
  $s(AB) \leq s(A)$

- Hence

  $c(AB \rightarrow C) \geq C(A \rightarrow BC)$

Candidate Rule Pruning

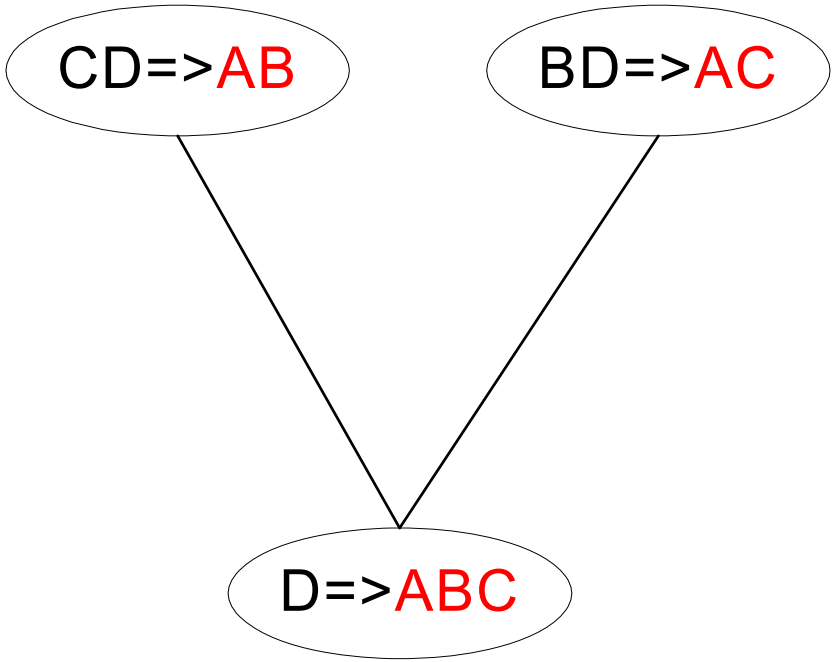
Moving elements from left to right cannot increase confidence.



Candidate Rule Generation within Apriori Algorithm

– Candidate rule is generated by merging two rules that share the same prefix in the rule consequent (right hand side of rule).

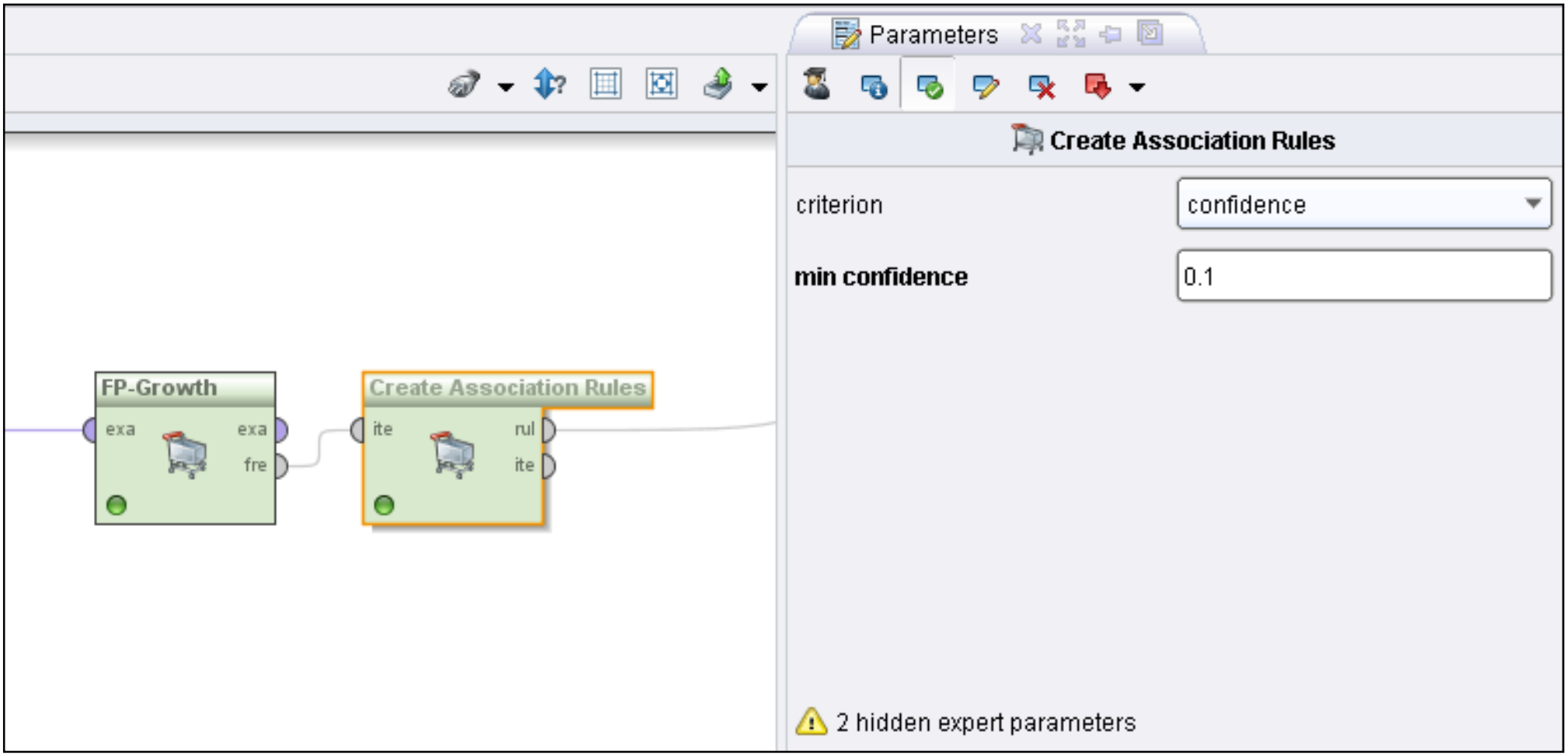
1. join(CD → AB, BD → AC) would produce the candidate rule D → ABC



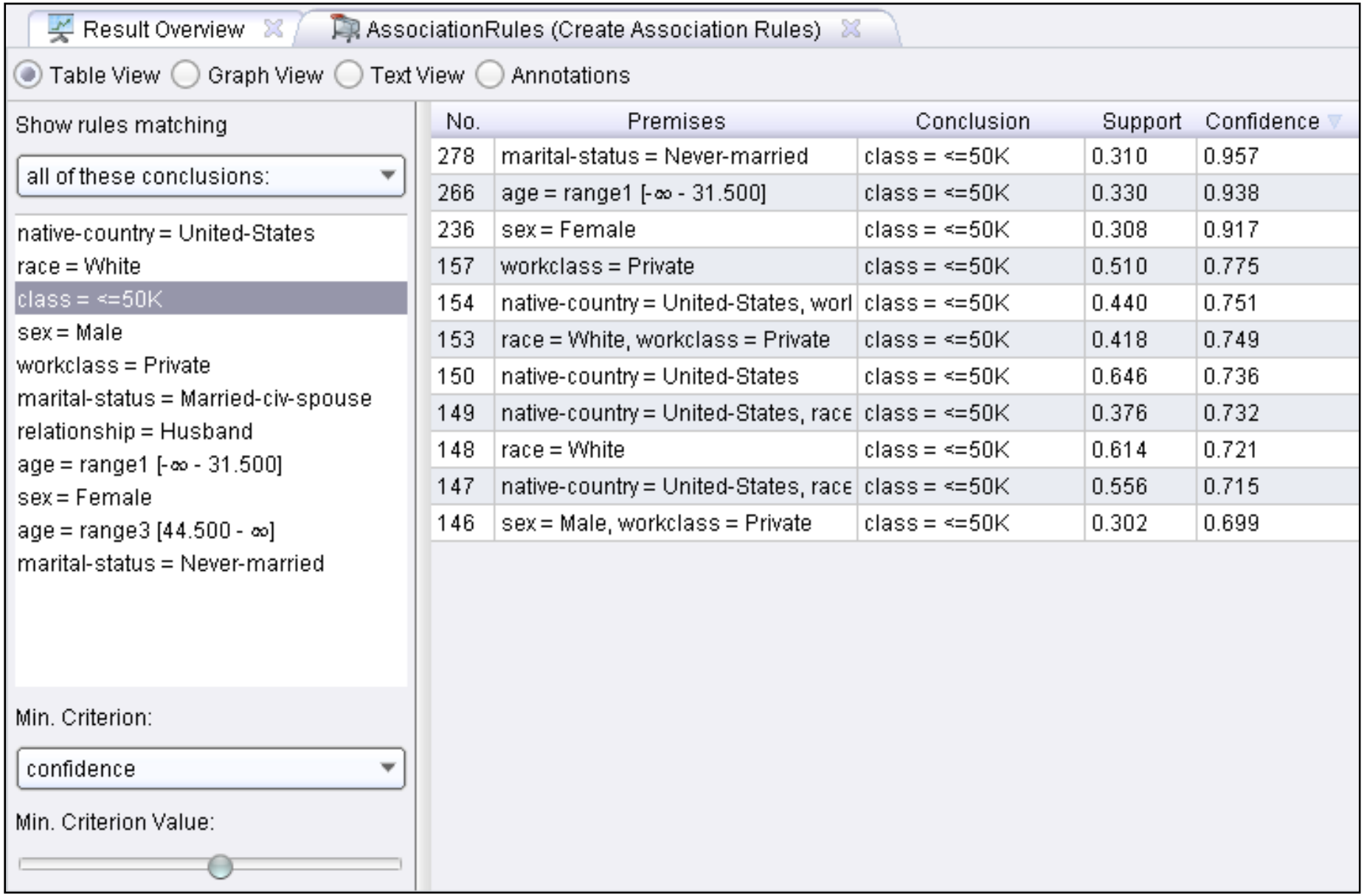
2. Prune rule D → ABC if one of its parent rules does not have high confidence (e.g. AD → BC)

– All the required information for confidence computation has already been recorded in itemset generation.
– Thus, there is no need to scan the transaction data *T* any more.

Creating Association Rules in Rapidminer



Exploring Association Rules in Rapidminer



Filter by conclusion

Filter by confidence

2.3 Handling Continuous and Categorical Attributes

– How to apply association analysis to attributes that are not asymmetric binary variables?

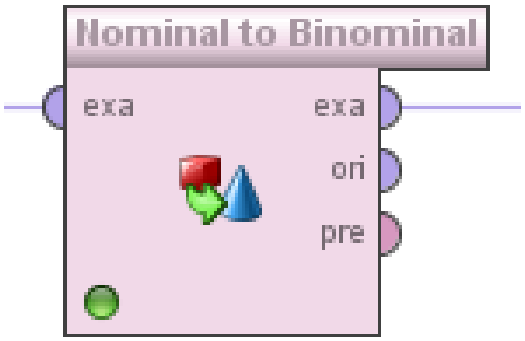
Session Id	Country	Session Length (sec)	Number of Web Pages viewed	Gender	Browser Type	Buy
1	USA	982	8	Male	IE	No
2	China	811	10	Female	Netscape	No
3	USA	2125	45	Female	Mozilla	Yes
4	Germany	596	4	Male	IE	Yes
5	Australia	123	9	Male	Mozilla	No
…	…	…	…	…	…	…

– Example Rule:

$$\{\text{Number of Pages} \in [5,10) \wedge (\text{Browser=Mozilla})\} \rightarrow \{\text{Buy = No}\}$$

Handling Categorical Attributes

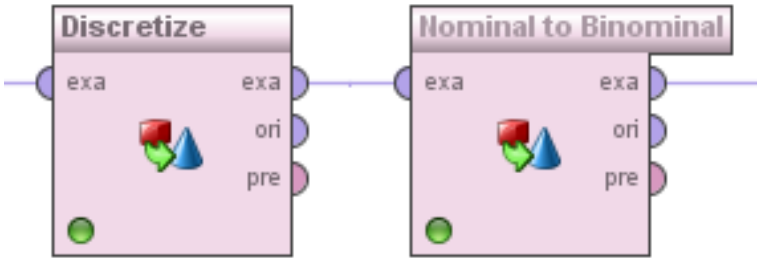
- Transform categorical attribute into asymmetric binary variables
- Introduce a new “item” for each distinct attribute-value pair
  - Example: replace “Browser Type” attribute with
    - attribute: “Browser Type = Internet Explorer”
    - attribute: “Browser Type = Mozilla”
    - …..



- Potential Issues
  - What if attribute has many possible values
    - Many of the attribute values may have very low support
    - Potential solution: Aggregate low-support attribute values
  - What if distribution of attribute values is highly skewed
    - Example: 95% of the visitors have Buy = No
    - Most of the items will be associated with (Buy=No) item
    - Potential solution: drop the highly frequent item

Handling Continuous Attributes

- Transform continuous attribute into binary variables using discretization
  - Equal-width binning
  - Equal-frequency binning



- Issue: Size of the discretization intervals affects support & confidence

$$\{\text{Refund} = \text{No}, (\text{Income} = \$51{,}250)\} \rightarrow \{\text{Cheat} = \text{No}\}$$

$$\{\text{Refund} = \text{No}, (60\text{K} \leq \text{Income} \leq 80\text{K})\} \rightarrow \{\text{Cheat} = \text{No}\}$$

$$\{\text{Refund} = \text{No}, (0\text{K} \leq \text{Income} \leq 1\text{B})\} \rightarrow \{\text{Cheat} = \text{No}\}$$

  - If intervals are too small
    - itemsets may not have enough support
  - If intervals too large
    - rules may not have enough confidence
    - e.g. combination of different age groups compared to a specific age group

2.4 Interestingness Measures

- Association rule algorithms tend to produce too many rules
  - many of them are uninteresting or redundant
  - redundant if {A,B,C} → {D} and {A,B} → {D} have same support & confidence
- Interestingness of patterns depends on application
  - one man's rubbish may be another's treasure
- Interestingness measures can be used to prune or rank the derived rules.
- In the original formulation of association rules, support & confidence were the only interestingness measures used.
- Later, various other measures have been proposed
  - See Tan/Steinback/Kumar, Chapter 6.7
  - We will have a look at one: Lift

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

confidence(Tea $\rightarrow$ Coffee) = 0.75

but support(Coffee) = 0.9

Although confidence is high, rule is misleading as the fraction of coffee drinkers is higher than the confidence of the rule

Lift

– The *lift* of an association rule $X \rightarrow Y$ is defined as:

$$Lift = \frac{c(X \rightarrow Y)}{s(Y)}$$

– Confidence normalized by support of consequent

– Interpretation
  - if lift > 1, then X and Y are positively correlated
  - if lift = 1, then X and Y are independent
  - if lift < 1, then X and Y are negatively correlated

Example: Lift

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

$$Lift = \frac{c(X \rightarrow Y)}{s(Y)}$$

Association Rule: Tea $\rightarrow$ Coffee

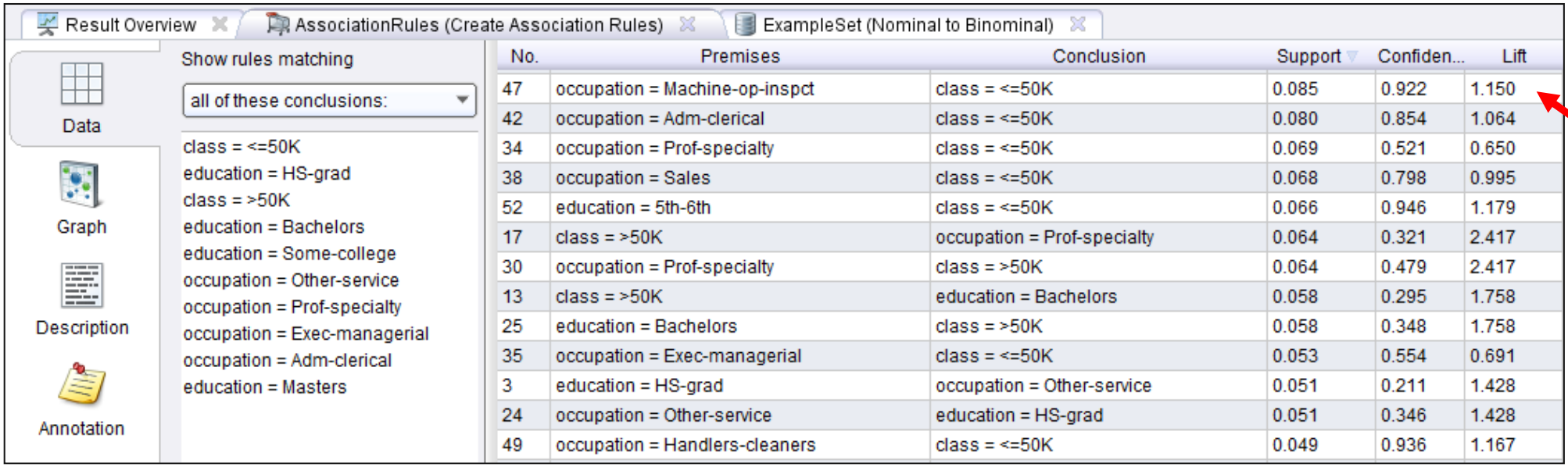
confidence(Tea $\rightarrow$ Coffee) = 0.75

support(Coffee) = 0.9

lift(Tea $\rightarrow$ Coffee) = 0.75/0.9= 0.8333

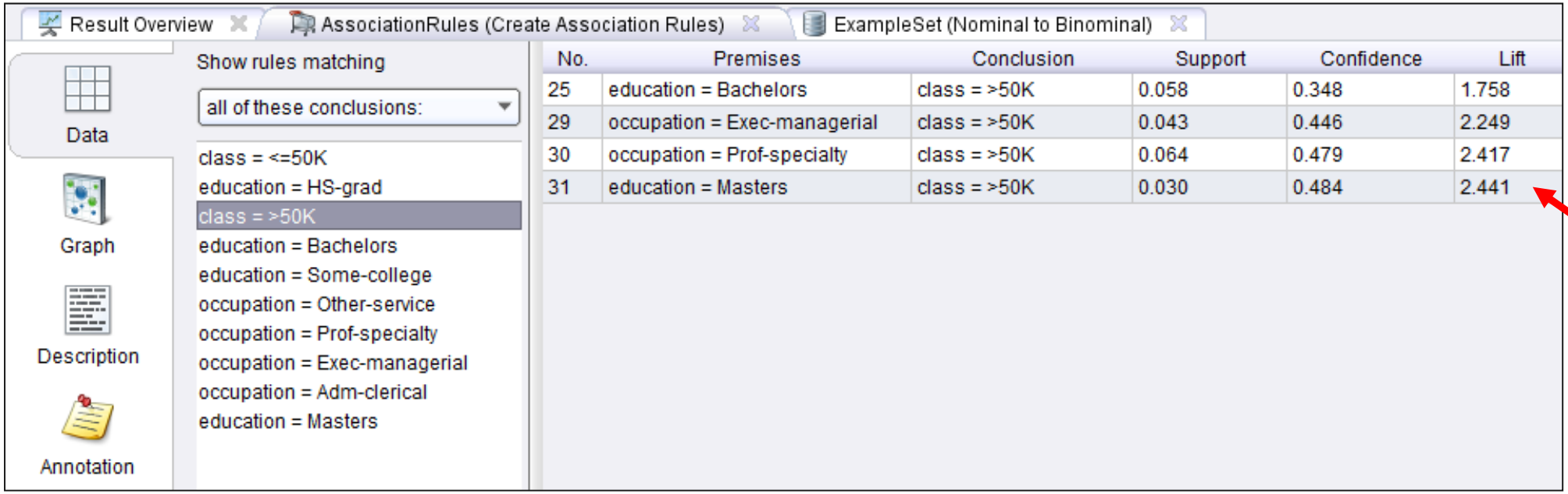
(< 1, therefore is negatively correlated)

Exploring Association Rules in Rapidminer



No.	Premises	Conclusion	Support	Confiden...	Lift
47	occupation = Machine-op-inspct	class = <=50K	0.085	0.922	1.150
42	occupation = Adm-clerical	class = <=50K	0.080	0.854	1.064
34	occupation = Prof-specialty	class = <=50K	0.069	0.521	0.650
38	occupation = Sales	class = <=50K	0.068	0.798	0.995
52	education = 5th-6th	class = <=50K	0.066	0.946	1.179
17	class = >50K	occupation = Prof-specialty	0.064	0.321	2.417
30	occupation = Prof-specialty	class = >50K	0.064	0.479	2.417
13	class = >50K	education = Bachelors	0.058	0.295	1.758
25	education = Bachelors	class = >50K	0.058	0.348	1.758
35	occupation = Exec-managerial	class = <=50K	0.053	0.554	0.691
3	education = HS-grad	occupation = Other-service	0.051	0.211	1.428
24	occupation = Other-service	education = HS-grad	0.051	0.346	1.428
49	occupation = Handlers-cleaners	class = <=50K	0.049	0.936	1.167

Lift close to 1



No.	Premises	Conclusion	Support	Confidence	Lift
25	education = Bachelors	class = >50K	0.058	0.348	1.758
29	occupation = Exec-managerial	class = >50K	0.043	0.446	2.249
30	occupation = Prof-specialty	class = >50K	0.064	0.479	2.417
31	education = Masters	class = >50K	0.030	0.484	2.441

Solid lift

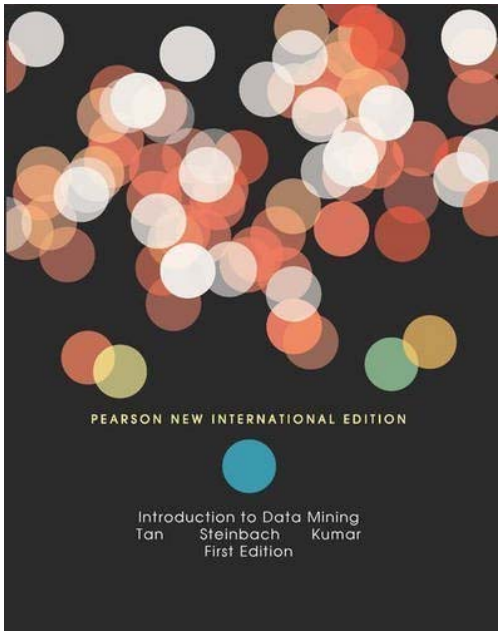
Conclusion

- The algorithm does the counting for you and finds patterns in the data.

- You need to do the interpretation based on your knowledge about the application domain.
  - Which patterns are meaningful?
  - Which patterns are surprising?

Literature Reference for this Slideset

Pang-Ning Tan, Michael Steinbach, Vipin Kumar:
Introduction to Data Mining.
Pearson / Addison Wesley.



Chapter 6: Association Analysis:
Basic Concepts and Algorithms

Chapter 7: Association Analysis:
Advanced Concepts