

Data Mining

Cluster Analysis



University of Mannheim – Prof. Bizer: Data Mining - FSS 2023 (Version 17.02.2023)

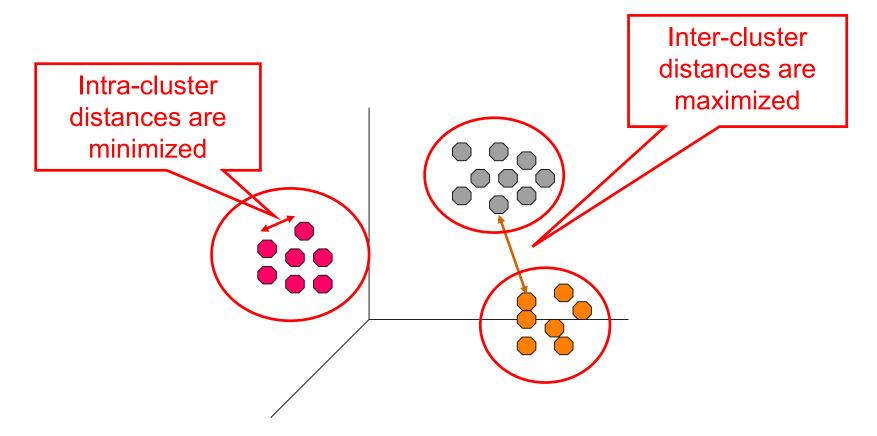
Slide 1

Outline

- 1. What is Cluster Analysis?
- 2. K-Means Clustering
- 3. Density-based Clustering
- 4. Hierarchical Clustering
- 5. Proximity Measures

1. What is Cluster Analysis?

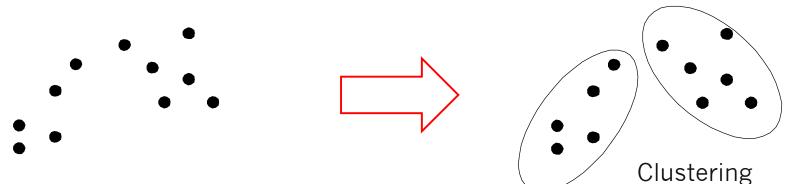
- Finding groups of objects such that
 - the objects in a group will be similar to one another
 - and different from the objects in other groups.
- Goal: Get a better understanding of the data



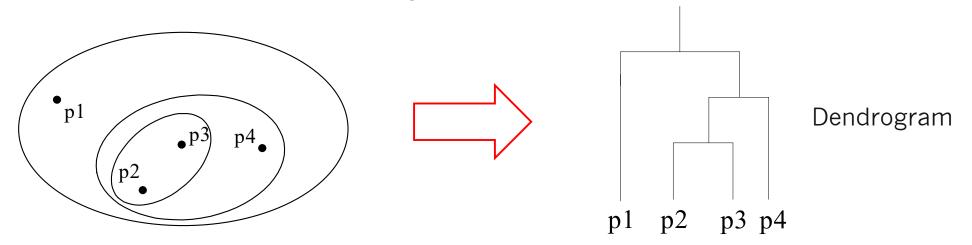
Types of Clusterings

Partitional Clustering

 A division of data objects into non-overlapping subsets (clusters) such that each data object is in exactly one subset



- Hierarchical Clustering
 - A set of nested clusters organized as a hierarchical tree

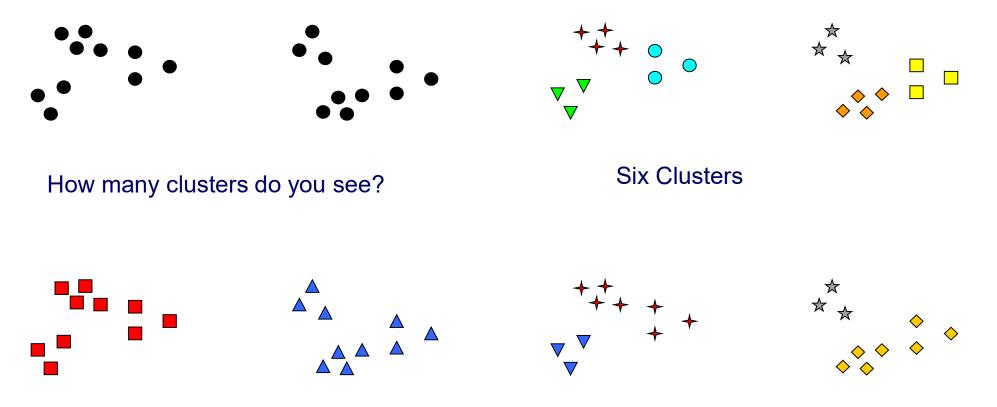


Aspects of Cluster Analysis

A clustering algorithm

- Partitional algorithms
- Density-based algorithms
- Hierarchical algorithms
- ...
- A proximity (similarity, or dissimilarity) measure
 - Euclidean distance
 - Cosine similarity
 - Data type-specific similarity measures
 - Domain-specific similarity measures
- Clustering quality
 - Intra-clusters distance \Rightarrow minimized
 - Inter-clusters distance \Rightarrow maximized
 - The clustering should be useful with regard to the goal of the analysis

The Notion of a Cluster is Ambiguous



Two Clusters

Four Clusters

The usefulness of a clustering depends on the goal of the analysis

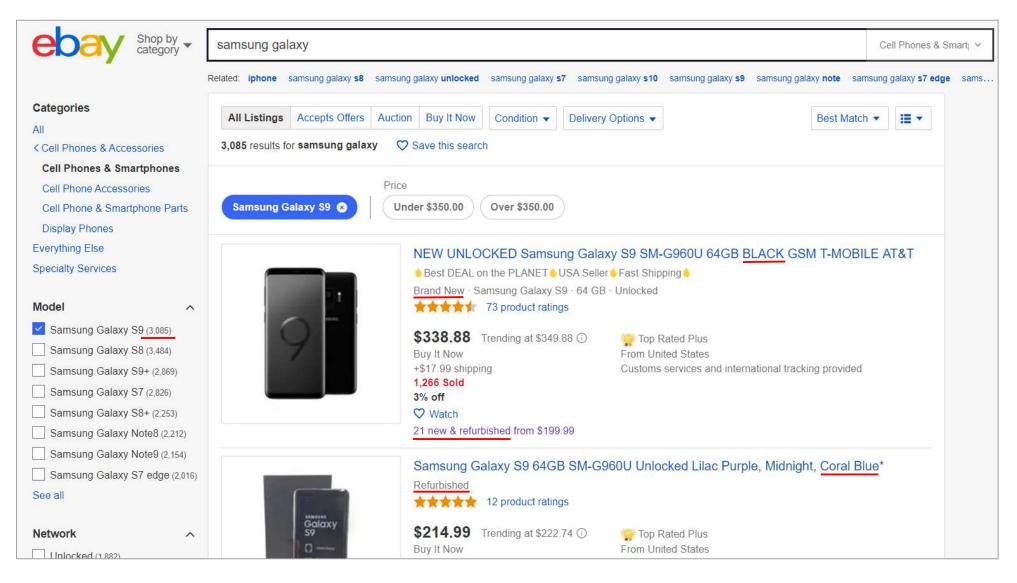
Example Application 1: Market Segmentation

- Goal: Identify groups of similar customers
- Level of granularity depends on the task at hand
- Relevant customer attributes depend on the task at hand



Example Application 2: E-Commerce

Identify offers of the same product on electronic markets



Example Application 3: Image Recognition

Identify parts of an image that belong to the same object



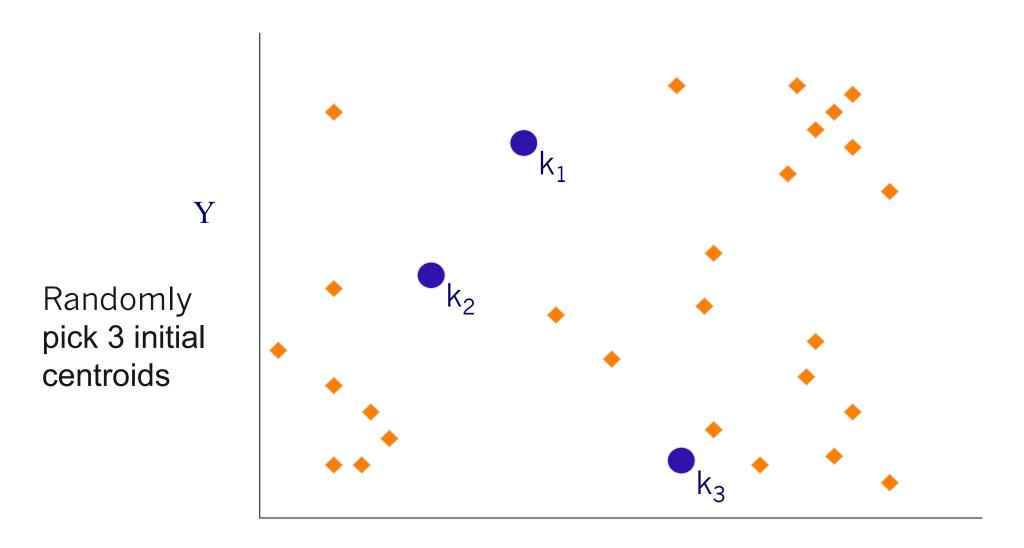


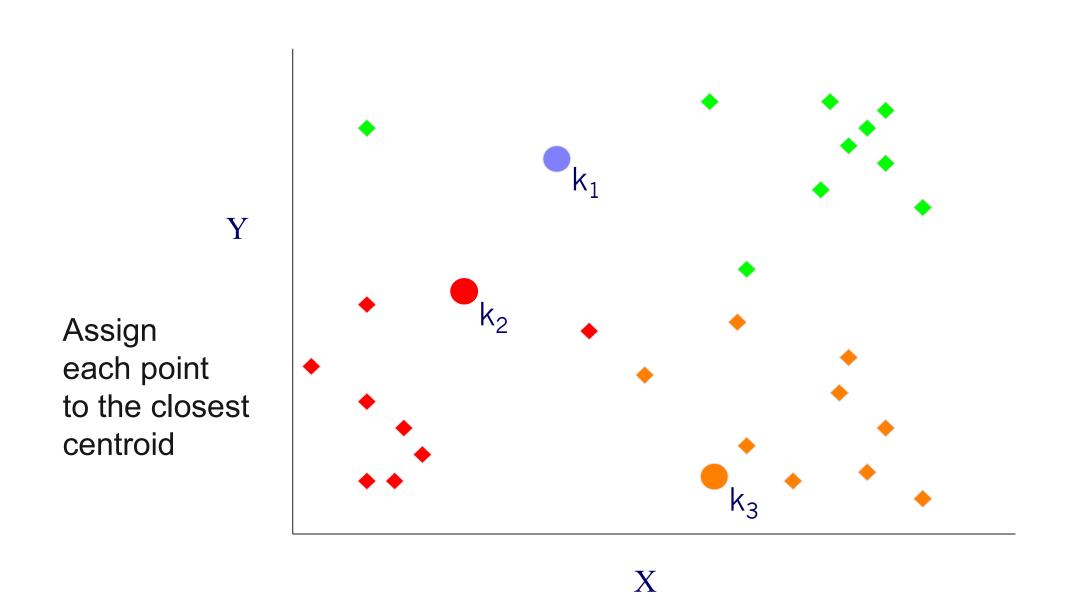
Cluster Analysis as Unsupervised Learning

- Supervised learning: Discover patterns in the data that relate data attributes with a target (class) attribute
 - these patterns are then utilized to predict the values of the target attribute in unseen data instances
 - the set of classes <u>is known</u> before
 - training data is often provided by human annotators
- Unsupervised learning: The data has no target attribute
 - we want to explore the data to find some intrinsic patterns in it
 - the set of classes/clusters is not known before
 - no training data is used
- Cluster Analysis is an unsupervised learning task

2. K-Means Clustering

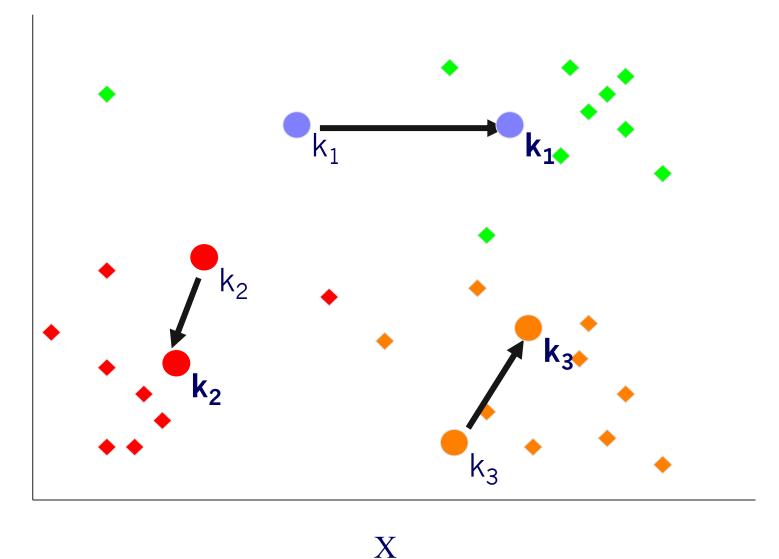
- Partitional clustering algorithm
- Each cluster is associated with a centroid (center point)
- Each point is assigned to the cluster with the closest centroid
- Number of clusters K must be specified manually
- The K-Means algorithm is very simple:
- 1: Select K points as the initial centroids.
- 2: repeat
- 3: Form K clusters by assigning all points to the closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: **until** The centroids don't change





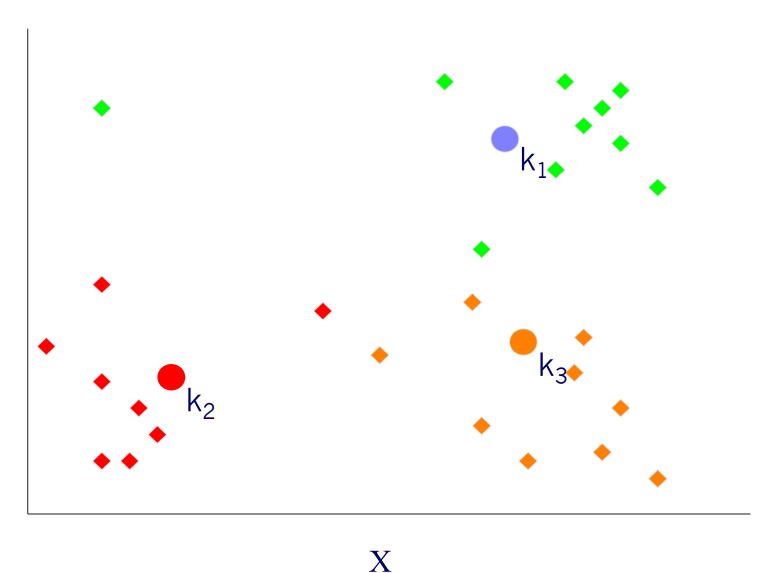
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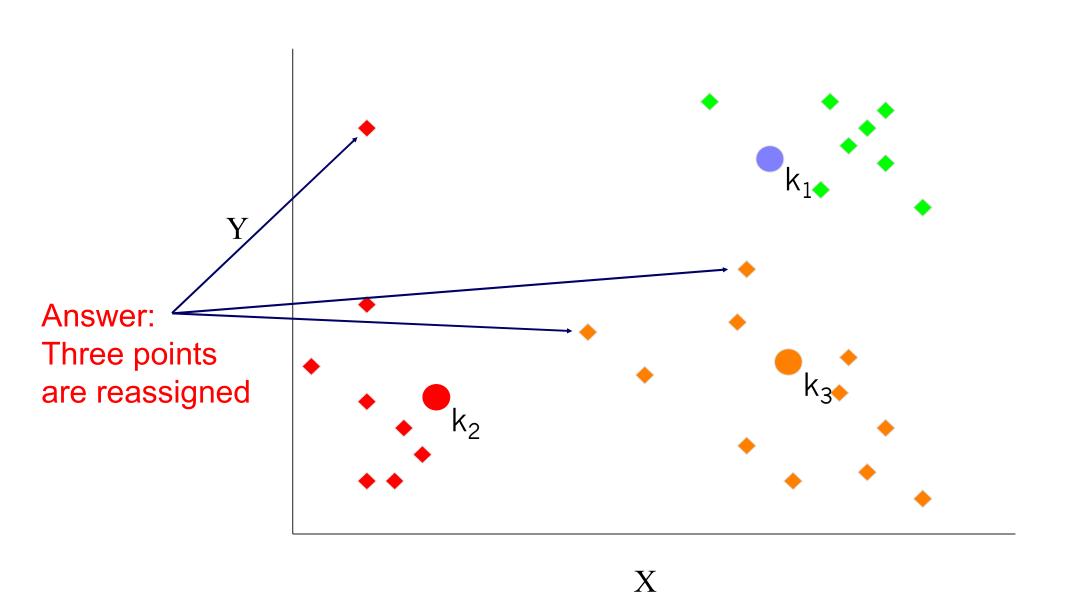
Move each centroid to the mean of each cluster

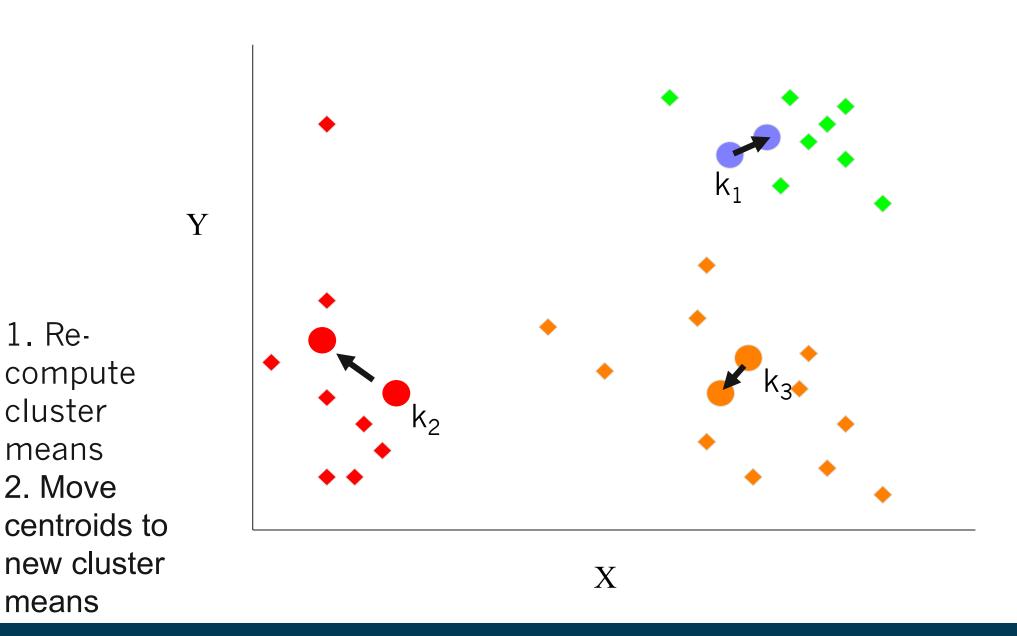


Reassign points if they are now closer to a Y different centroid

Question: Which points are reassigned?







Convergence Criteria

Default convergence criterion

- no (or minimum) change of centroids

Alternative convergence criteria

- 1. no (or minimum) re-assignments of data points to different clusters
- 2. stop after x iterations
- 3. minimum decrease in the sum of squared error (SSE)
 - see next slide

Evaluating K-Means Clusterings

- Widely used cohesion measure: Sum of Squared Error (SSE)
 - For each point, the error is the distance to the nearest centroid
 - To get SSE, we square these errors and sum them

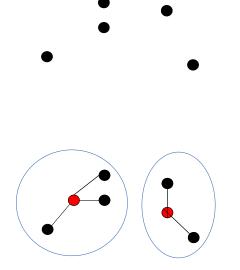
$$SSE = \sum_{j=1}^{k} \sum_{\mathbf{x} \in C_j} dist(\mathbf{x}, \mathbf{m}_j)^2$$

- C_j is the *j*-th cluster
- m_j is the centroid of cluster C_j (the mean vector of all the data points in C_j)
- $dist(x, m_j)$ is the distance between data point x and centroid m_j
- Given several clusterings (= groupings), we should prefer the one with the smallest SSE

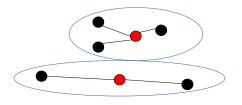
Illustration: Sum of Squared Error

- Cluster analysis problem

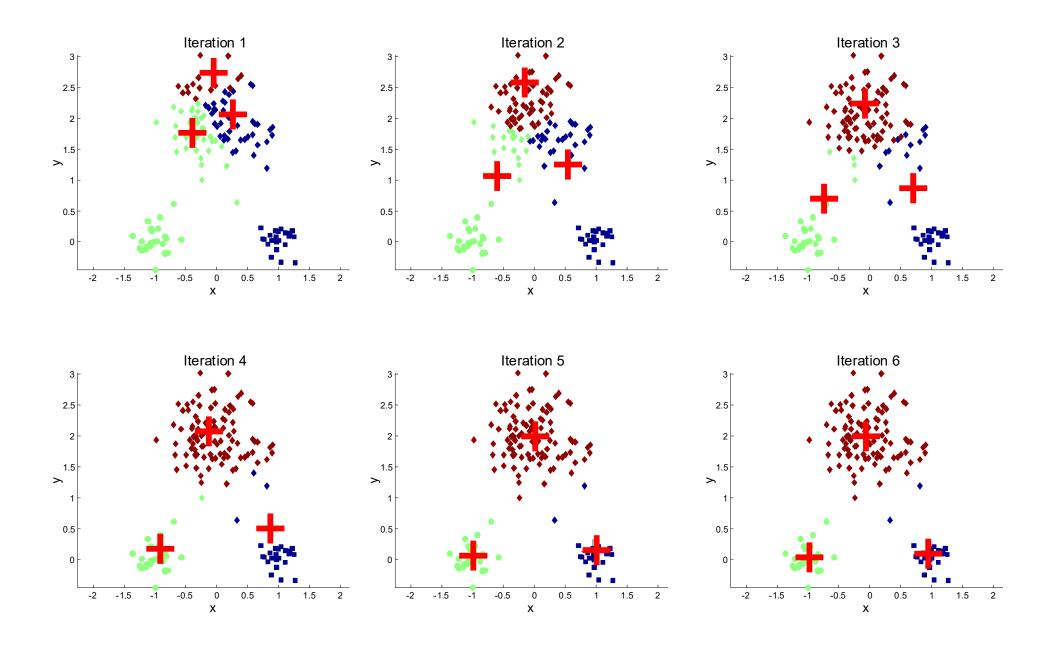
- Good clustering
 - small distances to centroids



- Not so good clustering
 - larger distances to centroids

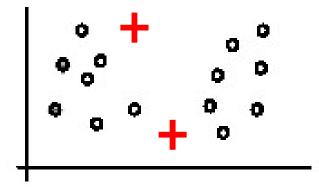


K-Means Clustering – Second Example

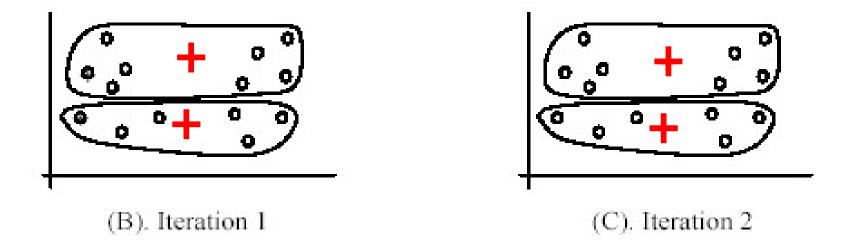


Weaknesses of K-Means: Initial Seeds

Clustering results may vary significantly depending on initial choice of seeds (number and position of seeds)

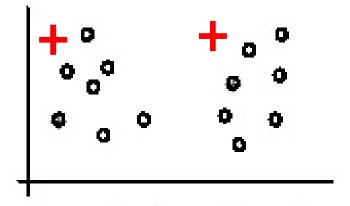


(A). Random selection of seeds (centroids)

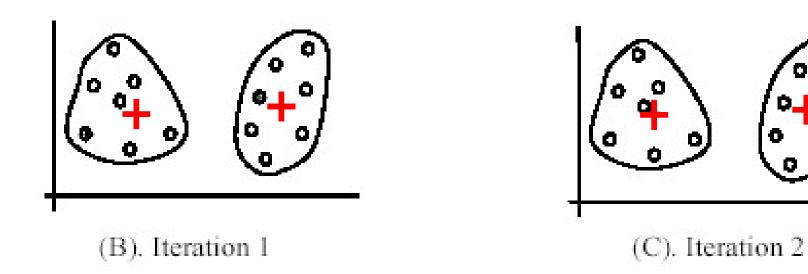


Weaknesses of K-Means: Initial Seeds

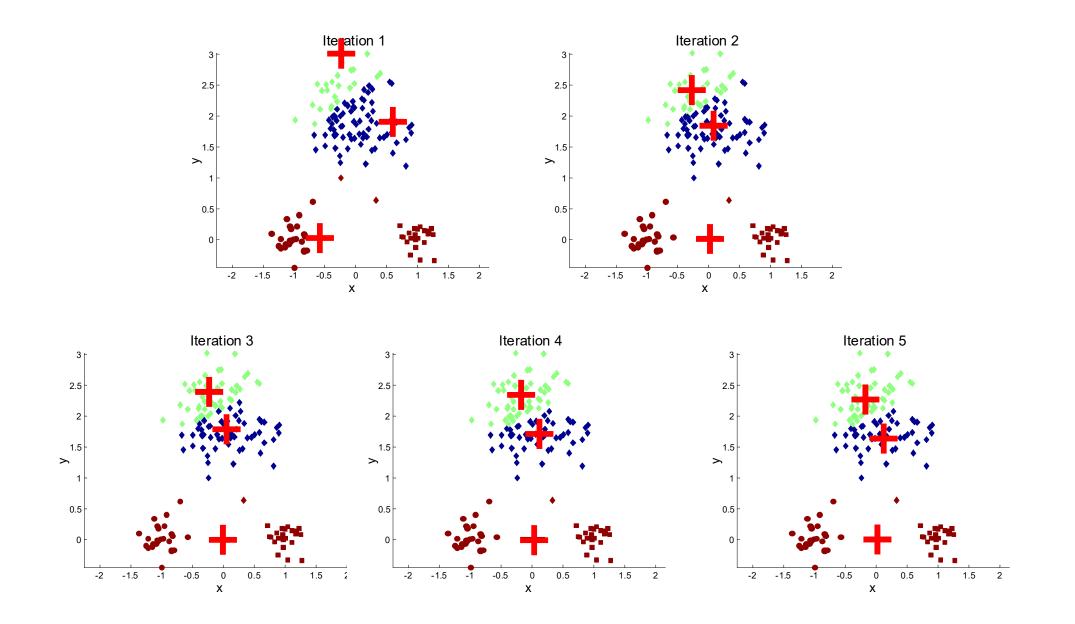
If we use different seeds, we get good results



(A). Random selection of k seeds (centroids)



Bad Initial Seeds – Second Example

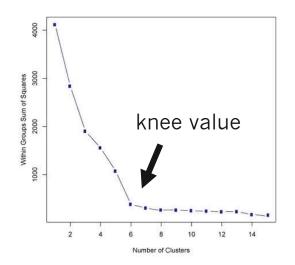


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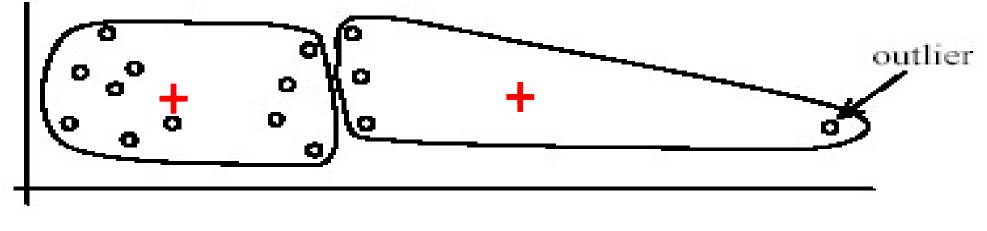
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Increasing the Chance of Finding Good Clusters

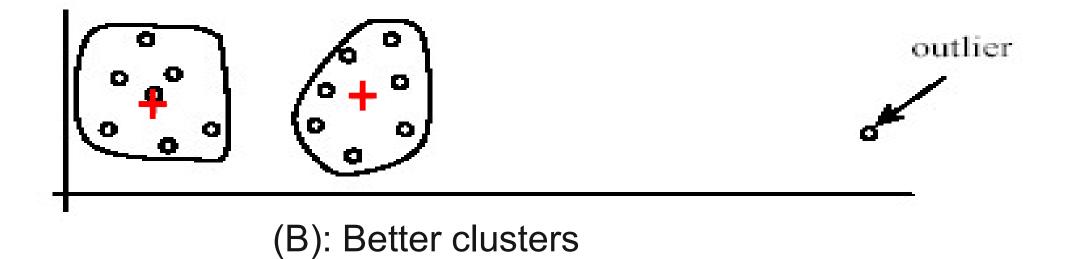
- 1. Restart a number of times with different random seeds
 - chose the resulting clustering with the smallest sum of squared error (SSE)
- 2. Run k-means with different values of k
 - The SSE for different values of k cannot directly be compared
 - think: what happens for $k \rightarrow$ number of examples?
 - Workarounds
 - Choose k where SSE improvement decreases (knee value of k)
 - 2. Employ X-Means
 - variation of K-Means algorithm that automatically determines k
 - starts with small k, then splits large clusters until improvement decreases



Weaknesses of K-Means: Problems with Outliers



(A): Undesirable clusters



Approaches to deal with outliers:

- 1. K-Medoids
 - K-Medoids is a K-Means variation that uses the median of each cluster instead of the mean
 - Medoids are the most central existing data points in each cluster
 - K-Medoids is more robust against outliers as the median is less affected by extreme values:
 - Mean and Median of 1, 3, 5, 7, 9 is 5
 - Mean of 1, 3, 5, 7, 1009 is 205
 - Median of 1, 3, 5, 7, 1009 is 5

2. DBSCAN

- Density-based clustering method that removes outliers
 - see next section

Advantages

- Simple, understandable
- Efficient time complexity:
 O(n * K * I * d)

where

- *n* = *number* of points
- *K* = number of clusters
- *I* = number of iterations
- *d* = *number* of *attributes*

Disadvantages

- Need to determine number of clusters
- All items are forced into a cluster
- Sensitive to outliers
- Does not work for non-globular clusters

K-Means Clustering in RapidMiner and Python

	RapidMiner	Parameters ×						
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		add cluster attribute	Ð	^				
	Retrieve Data Clustering	add as label		٢				
	clu res	remove unlabeled		٢				
		k	4	٩				
Python		max runs	10	Ð				
<i>import KMeans</i> rom sklearn.clu	ster import KMeans	determine good star	tvalues	٢				
create cluster stimator = KMea	er ns(n_clusters = 3)	measure types	MixedMeasures •	•				
<i>create cluster</i> luster_ids = es	<pre>ing timator.fit_predict(dataset[['Att1', 'Att2']])</pre>	mixed measure	MixedEuclideanDis •	•				
		max optimization steps	100	Ð	~			

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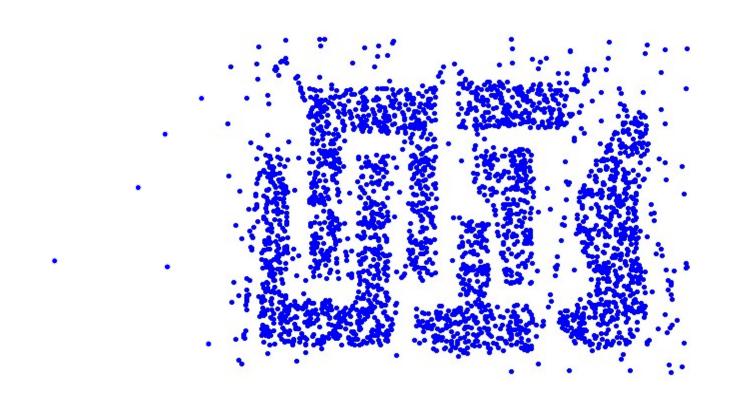
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K-Means Clustering Results

Data	Open in Turbo Prep 🛱 Auto Model						Filter (150 / 150 examples):		New cluster attribute		
	Row No.	id	label	cluster	a1	a2	a3			attributo	
Σ	1	id_1	Iris-setos	a cluster_1	5,100	0.000	1.400	0.200			
	2	id_2	Iris-setos	a cluster_1	4.900	3	1.400	0.200			
Statistics	3	id_3	Iris-se	Desult Listers		EvenueleC	Set (Clustering)	Chuster B	Nodel (Clustering) 🛛 🗙		
	4	id_4	Iris-se	Result History		Examples	set (Clustering)	Cluster N	nodel (Clustering) 🛛 🛪		
Visualizations	5	id_5	Iris-se	_							
	6	id_6	Iris-se					- 0		about a D	
	7	id_7	Iris-se	Description	Attribute		cluste		cluster_1	cluster_2	
	8	id_8	Iris-se	Folder View	a1		6.913		5.006	6.252	
	9	id_9	Iris-se		a2		3.100		3.418	2.855	
	10	id_10	Iris-se		a3		Result History	Ever	mpleSet (Clustering) 🛛 🗙	📓 Cluster Model	l (Clustering) 🛛 🖂
	11	id_11	Iris-se				Result History	E CXar	npieser (clustering)		r (crustering)
	12	id_12	Iris-se		a4		_				
	13	id_13	Iris-se					Cluster N	lodel		
	14	id_14	Iris-se	Z			Description	Cluster 0: 32	itama		
				Graph				Cluster 1: 50			
				_				Cluster 2: 40			
								Cluster 3: 28 Total number of			
							Folder View	ioour number (51 100m5. 100		
				Centroid			View				
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3. Density-based Clustering



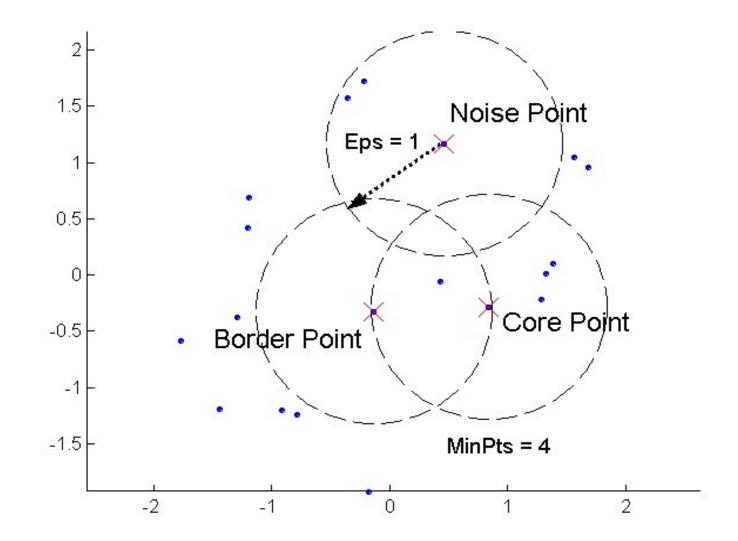
Challenging use case for K-Means because

- Problem 1: Non-globular shapes
- Problem 2: Outliers / noise points

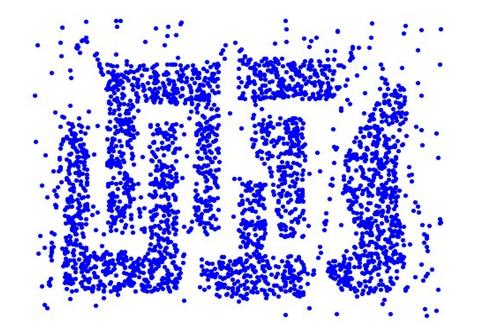
DBSCAN

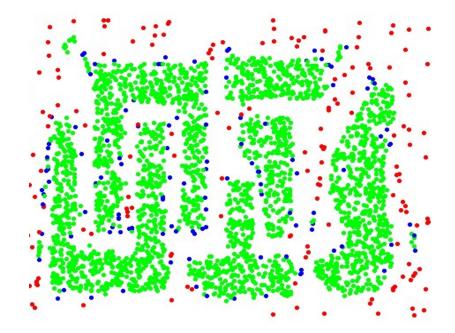
- DBSCAN is a density-based algorithm
 - Density = number of points within a specified radius Epsilon (Eps)
- Divides data points into three classes:
 - 1. A point is a core point if it has at least a specified number of neighboring points (MinPts) within the specified radius Eps
 - the point itself is counted as well
 - these points form the interior of a dense region (cluster)
 - 2. A border point has fewer points than MinPts within Eps, but is in the neighborhood of a core point
 - 3. A noise point is any point that is not a core point or a border point

Examples of Core, Border, and Noise Points 1



Examples of Core, Border, and Noise Points 2





Original Points

Point types: core, border and noise

Eliminates noise points and returns clustering of the remaining points:

- 1. Label all points as core, border, or noise points
- 2. Eliminate all noise points
- 3. Put an edge between all core points that are within Eps of each other
- 4. Make each group of connected core points into a separate cluster
- 5. Assign each border point to one of the clusters of its associated core points
 - as a border point can be at the border of multiple clusters
 - use voting if core points belong to different clusters
 - if equal vote, than assign border point randomly

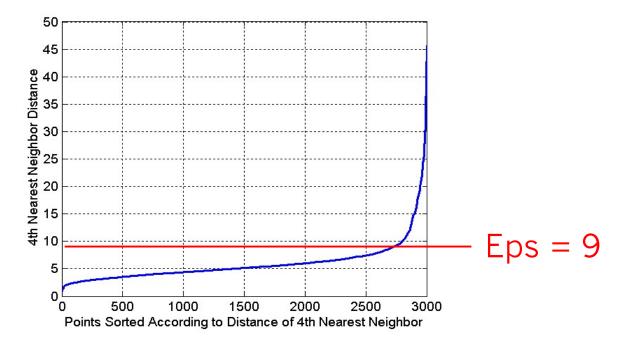
Time complexity: O(n log n)

• dominated by neighborhood search for each point using an index

How to Determine Suitable Eps and MinPts Values?

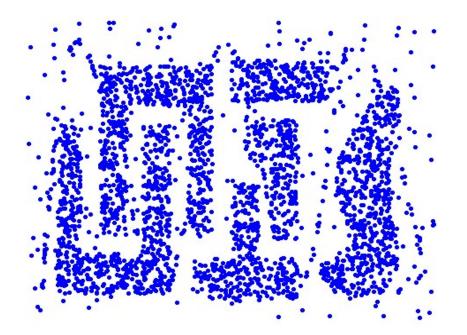
For points in a cluster, their kth nearest neighbor (single point) should be at roughly the same distance. Noise points have their kth nearest neighbor at farther distance

- 1. Start with setting MinPts = 4 (rule of thumb)
- 2. Plot sorted distance of every point to its kth nearest neighbor:

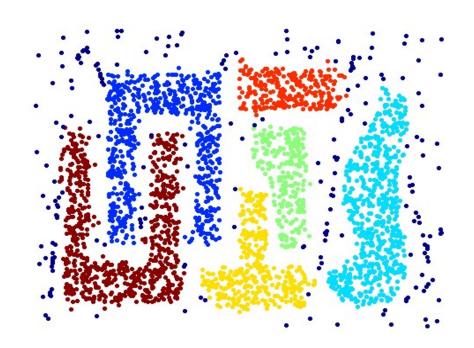


- 3. Set Eps to the sharp increase of the distances (start of noise points)
- 4. Decrease k if small clusters are labeled as noise (subjective decision)
- 5. Increase k if outliers are included into the clusters (subjective decision)

When **DBSCAN** Works Well



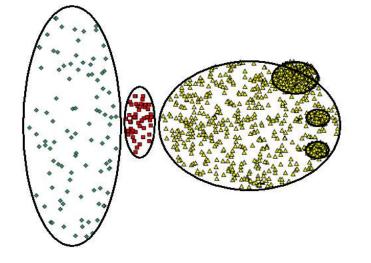
Original Points



Clusters

- Resistant to noise
- Can handle clusters of different shapes and sizes

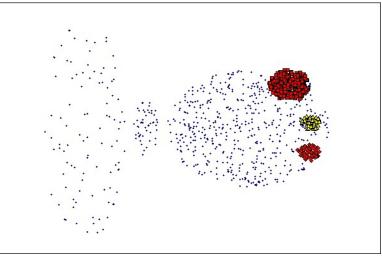
When DBSCAN Does NOT Work Well



Original Points

DBSCAN has problems with datasets of varying densities.

(MinPts=4, Eps=9.92)



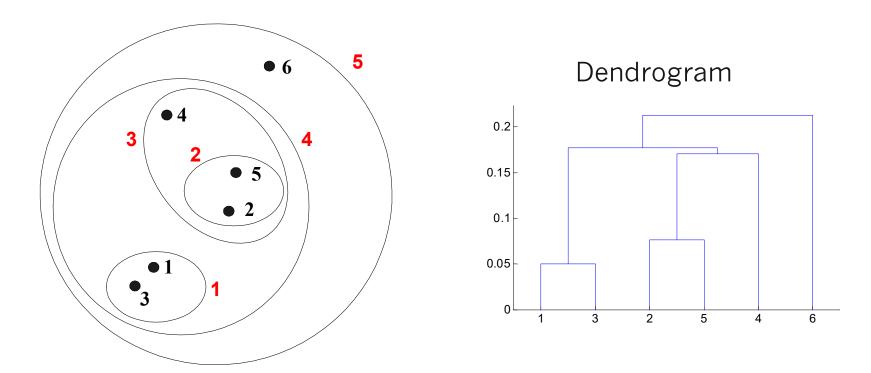
(MinPts=4, Eps=9.75)

DBSCAN in RapidMiner and Python

	RapidMiner		Parameters ×					
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				epsilon		1.5		١
	re ciu re	exa 🎆 clu		min points		3		0
		res (add cluster attribute add as label remove unlabeled 				٩	
							1	
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				measure types		MixedMeasures	•	0
Python						MixedEuclideanDistance	•	٩
# import DBSCAN from sklearn.cluster import DBSCAN				003)	L			
# create t	he clusterer							
clusterer	= DBSCAN(min_sa	amples=3, e	ps=1.5, metri	<pre>c='euclidean')</pre>				
# create the clusters								
<pre>clusters = clusterer.fit_predict(dataset[['Att1', 'Att2']])</pre>								

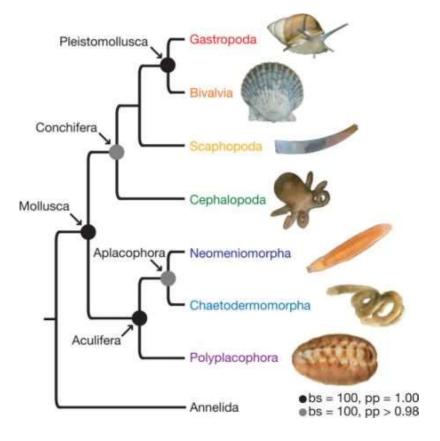
4. Hierarchical Clustering

- Produces a set of nested clusters organized as a hierarchical tree
- Can be visualized as a dendrogram
 - A tree like diagram that records the sequences of merges or splits
 - The y-axis displays the former distance between merged clusters



Strengths of Hierarchical Clustering

- We do not have to assume any particular number of clusters
 - any desired number of clusters can be obtained by 'cutting' the dendogram at the proper level
- May be used to discover meaningful taxonomies
 - taxonomies of biological species
 - taxonomies of different customer groups



- Agglomerative

- start with the points as individual clusters
- at each step, merge the closest pair of clusters until only one cluster (or k clusters) is left

Divisive

- start with one, all-inclusive cluster
- at each step, split a cluster until each cluster contains a single point (or there are k clusters)
- Agglomerative Clustering is more widely used

The basic algorithm is straightforward:

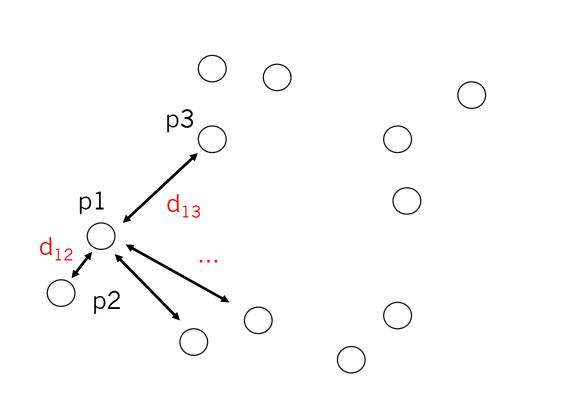
- 1. Compute the proximity matrix
- 2. Let each data point be a cluster
- 3. Repeat
 - 1. Merge the two closest clusters
 - 2. Update the proximity matrix

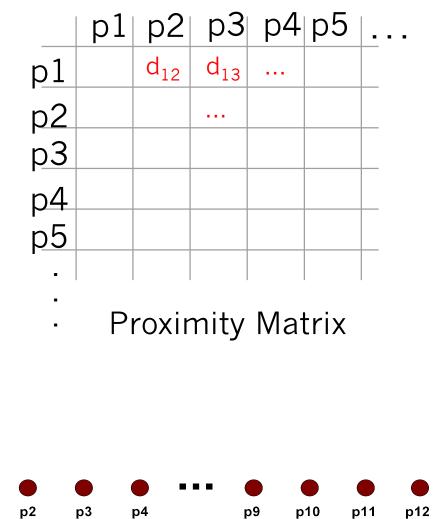
Until only a single cluster remains

- The key operation is the computation of the proximity of two clusters
- The different approaches to defining the distance between clusters distinguish the different algorithms

Starting Situation

Start with clusters of individual points and a proximity matrix



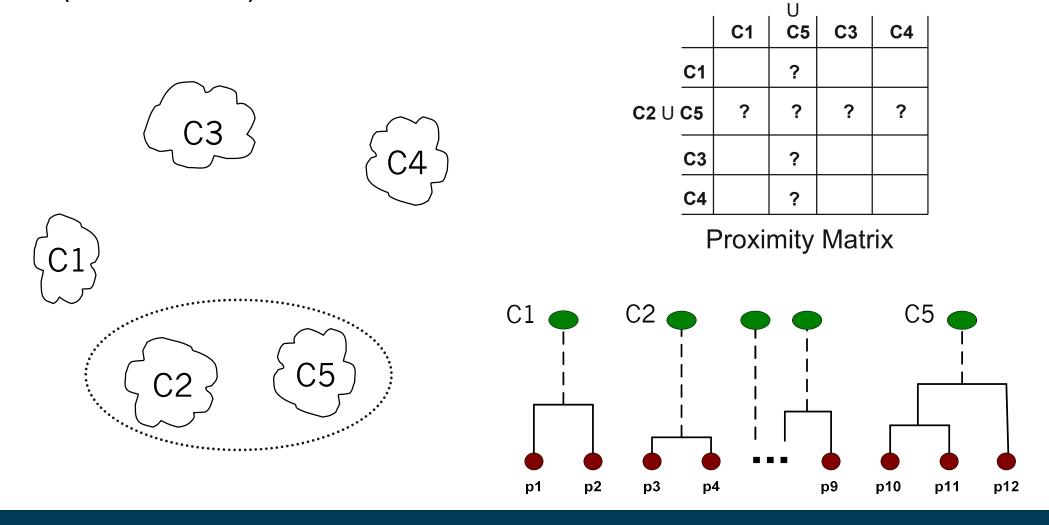


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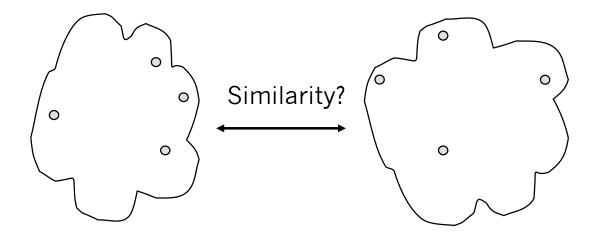
Intermediate Situation

- After some merging steps, we have larger clusters.
- We want to keep on merging the two closest clusters (C2 and C5?)



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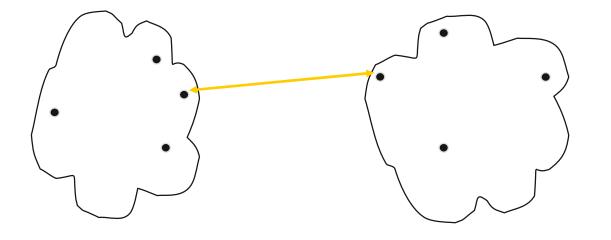
How to Define Inter-Cluster Similarity?



Different approaches are used:

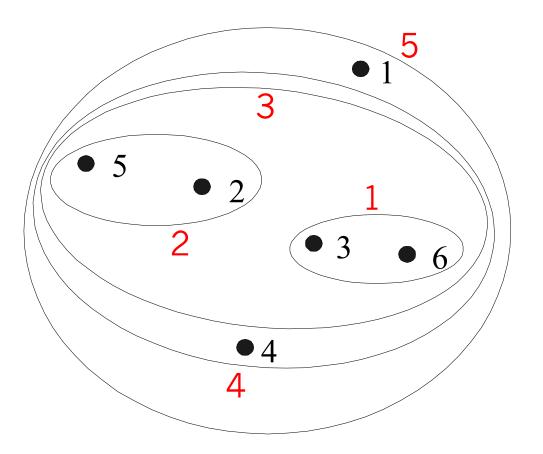
- 1. Single Link
- 2. Complete Link
- 3. Group Average
- 4. Distance Between Centroids

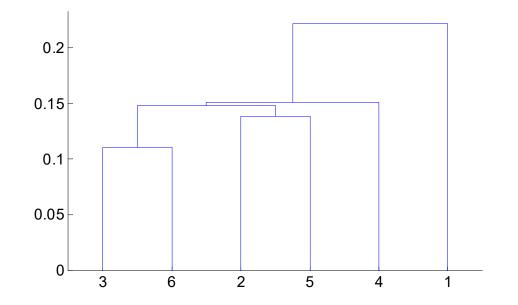
Cluster Similarity: Single Link



- Similarity of two clusters is based on the two most similar (closest) points in the different clusters
- Determined by one pair of points,
 i.e. by one link in the proximity graph

Example: Single Link



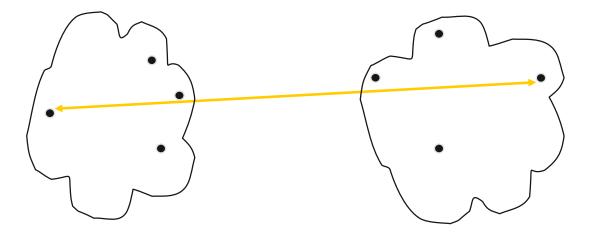


Nested Clusters

Dendrogram

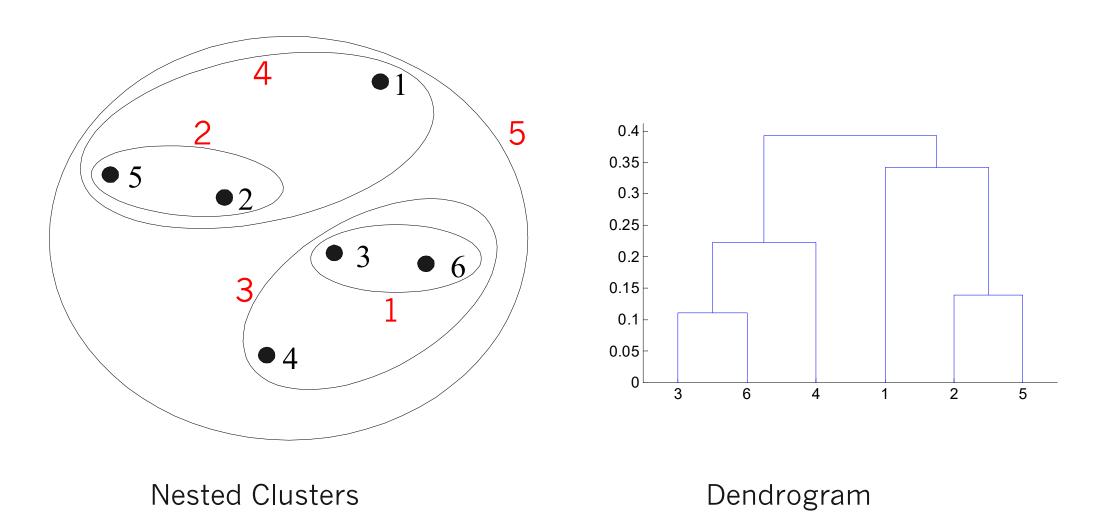
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Cluster Similarity: Complete Linkage



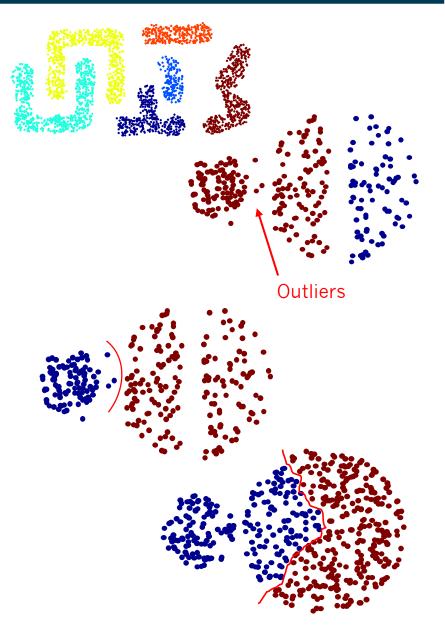
- Similarity of two clusters is based on the two least similar (most distant) points in the different clusters
- Determined by all pairs of points in the two clusters

Example: Complete Linkage

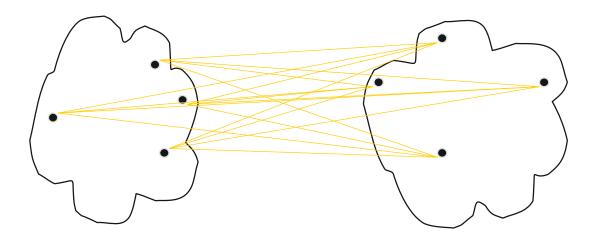


Single Link vs. Complete Linkage

- Single Link
 - Strength: Can handle non-elliptic shapes
 - Limitation: Sensitive to noise and outliers
- Complete Linkage
 - Strength: Less sensitive to noise and outliers
 - Limitation: Biased towards globular clusters
 - Limitation: Tends to break large clusters, as decisions can not be undone.



Cluster Similarity: Group Average

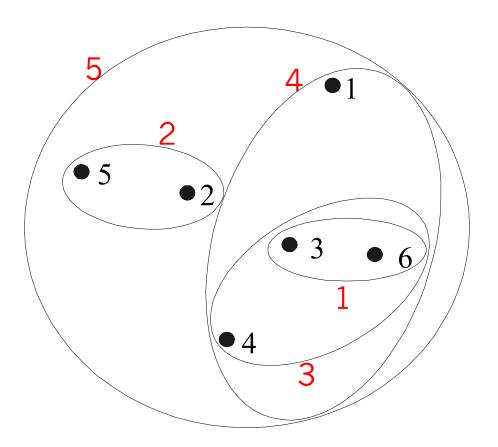


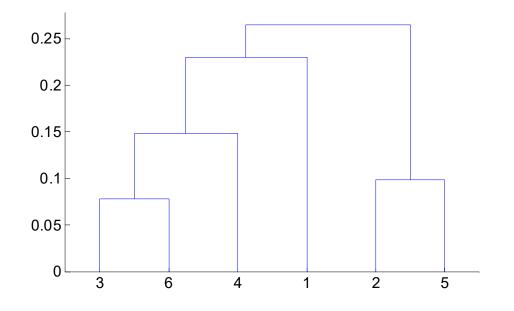
 Proximity of two clusters is the average of pair-wise proximity between all points in the two clusters.

proximity(Cluster_i, Cluster_j) =
$$\frac{\sum_{\substack{p_i \in \text{Cluster}_i \\ p_j \in \text{Cluster}_j}}{\sum_{\substack{p_i \in \text{Cluster}_i \\ p_j \in \text{Cluster}_j}} | * | \text{Cluster}_i | *$$

- Compromise between single and complete link
 - Strength: Less sensitive to noise and outliers than single link
 - Limitation: Biased towards globular clusters

Example: Group Average





Nested Clusters

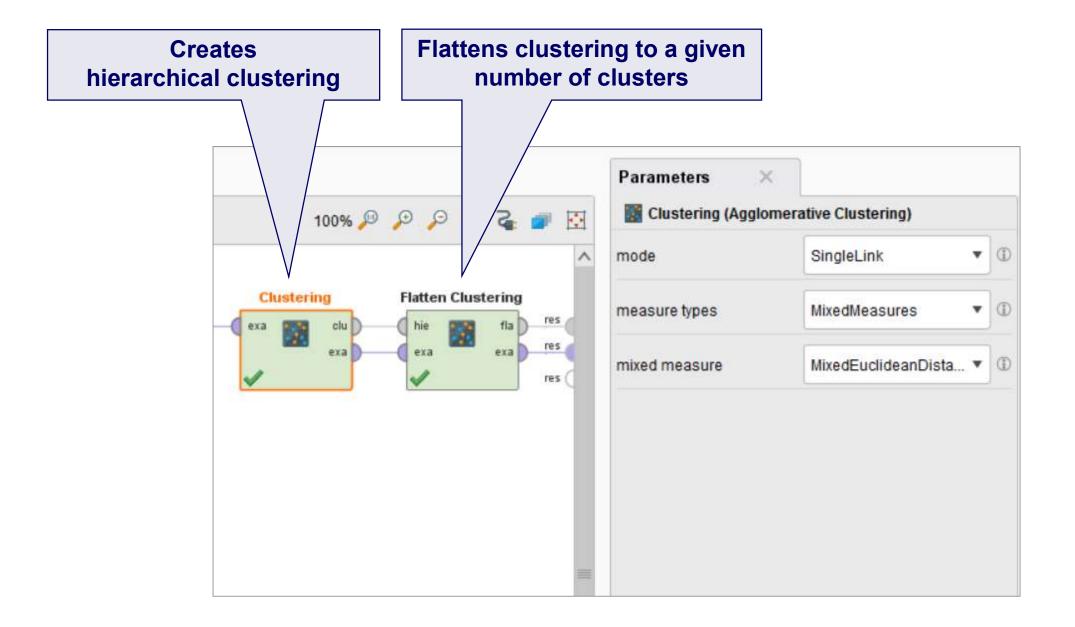
Dendrogram

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Hierarchical Clustering: Problems and Limitations

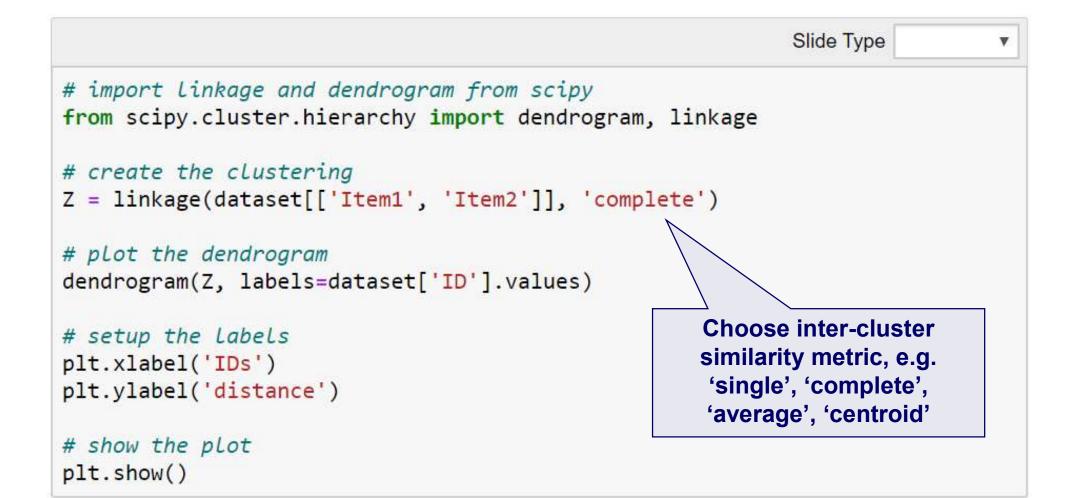
- Different schemes have problems with one or more of the following:
 - 1. sensitivity to noise and outliers
 - 2. difficulty handling non-elliptic shapes
 - 3. breaking large clusters
- High space and time complexity
 - O(N²) space since it uses the proximity matrix
 - N is the number of points
 - O(N³) time in many cases
 - there are N steps and at each step the size N² proximity matrix must be searched and updated
 - complexity can be reduced to $O(N^2 \log(N))$ time in some cases
 - Workaround: Apply hierarchical clustering to a random sample of the original data (<10,000 examples)

Agglomerative Hierarchical Clustering in RapidMiner



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Agglomerative Hierarchical Clustering in Python



5. Proximity Measures

- So far, we have seen different clustering algorithms all of which rely on proximity (distance, similarity, ...) measures
- Now, we discuss proximity measures in more detail
- A wide range of different measures is used depending on the requirements of the application
- Similarity
 - Numerical measure of how <u>alike</u> two data objects are
 - Often falls in the range [0,1]
- Dissimilarity / Distance
 - Numerical measure of how <u>different</u> are two data objects
 - Minimum dissimilarity is often 0, upper limit varies
- We distinguish proximity measures for single attributes and measures for multidimensional data points (records)

Attribute	Dissimilarity	Similarity
Type		
Nominal	$d = \left\{ egin{array}{ccc} 0 & ext{if} \; p = q \ 1 & ext{if} \; p eq q \end{array} ight.$	$s = \left\{egin{array}{ccc} 1 & ext{if } p = q \ 0 & ext{if } p eq q \end{array} ight.$
Ordinal	$d = \frac{ p-q }{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{ p-q }{n-1}$
Interval or Ratio	d = p - q	$s = -d, s = \frac{1}{1+d}$ or
		$s = -d, s = \frac{1}{1+d}$ or $s = 1 - \frac{d-min_d}{max_d-min_d}$

Similarity and dissimilarity for simple attributes

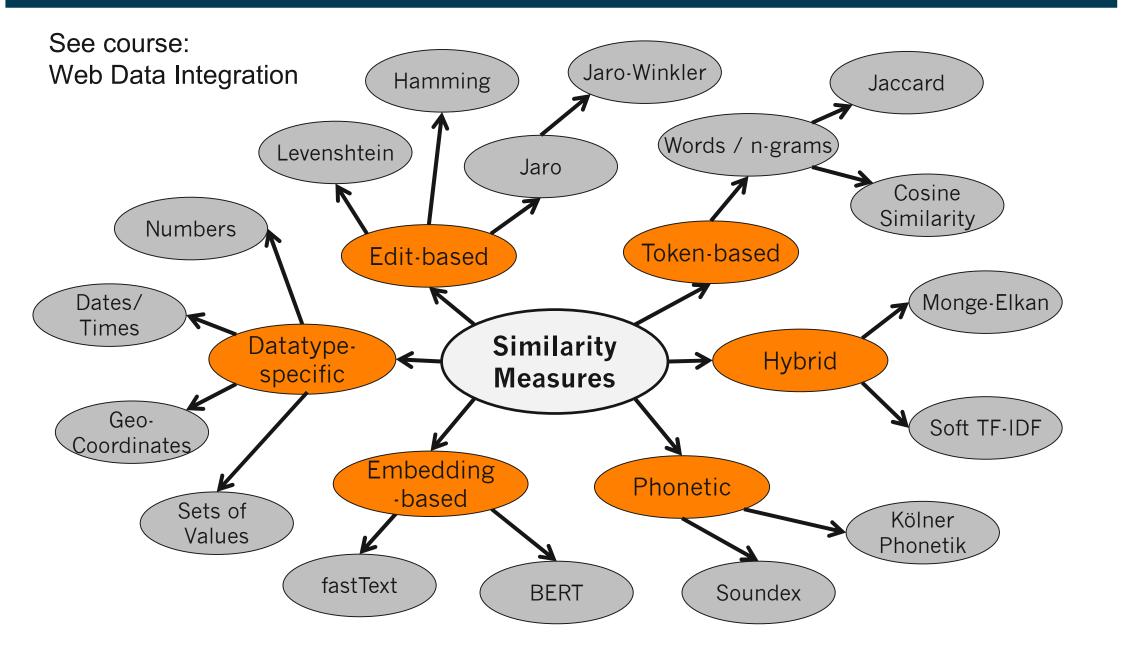
p and q are attribute values for two data objects

Levenshtein Distance

- Measures the dissimilarity of two strings
- Measures the minimum number of edits needed to transform one string into the other
- Allowed edit operations:
 - 1. insert a character into the string
 - 2. delete a character from the string
 - 3. replace one character with a different character
- Examples:
 - levensthein('table', 'cable') = 1 (1 substitution)
 - levensthein('Bizer, Chris', 'Chris Bizer') = 11 (10 substitution,

1 deletion)

Further String Similarity Measures



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5.2 Proximity of Multidimensional Data Points

- All measures discussed so far cover the proximity of single attribute values
- But we usually have data points with many attributes
 - e.g., age, height, weight, sex...
- Thus, we need proximity measures for data points
 - taking multiple attributes/dimensions into account

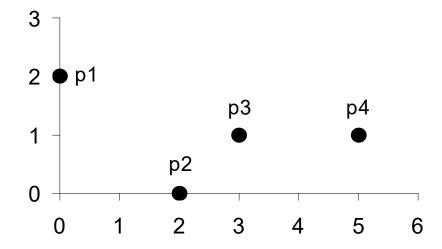
Definition

$$dist = \sqrt{\sum_{k=1}^{n} (p_k - q_k)^2}$$

Where *n* is the number of dimensions (attributes) and p_k and q_k are the kth attributes of data points *p* and *q*

- $p_k q_k$ is squared to increase impact of long distances
- All dimensions are weighted equality

Example: Euclidean Distance



point	X	У
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	р3	р4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

Normalization

- Attributes should be normalized so that all attributes can have equal impact on the computation of distances
- Consider the following pair of data points
 - x_{j} : (0.1, 20) and x_{j} : (0.9, 720).

dist (
$$\mathbf{x}_i, \mathbf{x}_j$$
) = $\sqrt{(0.9 - 0.1)^2 + (720 - 20)^2} = 700.000457$

- The distance is almost completely dominated by (720-20) = 700
- Solution: Scale attributes to have a common value range, for instance [0,1]

Scaling in RapidMiner and Python

RapidMiner ×			
100% 🔑 🔑 📮 🥃 🗃 🔯			
	create view		
Retrieve Dataset Select Attributes Normalize Clustering	attribute filter type	subset 💌	Ð
ori ori clu res	attributes	Select Attributes	0
	invert selection		٩
	include special attributes		٢
	method	range transformation	•
	min	0.0	1
Python	1.0	1	
<pre># import min-max scaler from sklearn import preprocessing.MinMaxScaler() # create scaler scaler = MinMaxScaler() # normalize the relevant attributes dataset[['Att1', 'Att2']] = scaler.fit_transform(dataset[['</pre>	'Att1', 'Att2']])		

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Similarity of Binary Attributes

- Common situation is that objects, p and q, have only binary attributes
 - products in shopping basket
 - courses attended by students
- We compute similarities using the following quantities:

 M_{11} = the number of attributes where p was 1 and q was 1 M_{00} = the number of attributes where p was 0 and q was 0 M_{01} = the number of attributes where p was 0 and q was 1 M_{10} = the number of attributes where p was 1 and q was 0

- A binary attribute is symmetric if both of its states (0 and 1) have equal importance, and carry the same weights, e.g., male and female
- Similarity measure: Simple Matching Coefficient

$$SMC(\mathbf{x}_{i}, \mathbf{x}_{j}) = \frac{M_{11} + M_{00}}{M_{01} + M_{10} + M_{11} + M_{00}}$$

Number of matches / number of all attributes values

- Asymmetric: If one of the states is more important than the other
 - by convention, state 1 represents the more important state
 - 1 is typically the rare or infrequent state
 - examples: Shopping baskets, word vectors
- Similarity measure: Jaccard Coefficient

$$J(\mathbf{x}_{i}, \mathbf{x}_{j}) = \frac{M_{11}}{M_{01} + M_{10} + M_{11}}$$

Number of 11 matches / number of not-both-zero attributes values

p = 1000000000q = 0000001001

Interpretation of the example: Customer p bought item 1 Customer q bought item 7 and 10

- $M_{11} = 0$ (the number of attributes where p was 1 and q was 1)
- $M_{00} = 7$ (the number of attributes where p was 0 and q was 0)
- $M_{01} = 2$ (the number of attributes where p was 0 and q was 1)
- $M_{10} = 1$ (the number of attributes where p was 1 and q was 0)

SMC = $(M_{11} + M_{00})/(M_{01} + M_{10} + M_{11} + M_{00}) = (0+7) / (2+1+0+7) = 0.7$

$$J = (M_{11}) / (M_{01} + M_{10} + M_{11}) = 0 / (2 + 1 + 0) = 0$$

SMC versus Jaccard: Question

- Which of the two measures would you use ...
- ...for a dating agency?
 - hobbies
 - favorite bands
 - favorite movies
 - ...
- ...for the Wahl-O-Mat?
 - (dis-)agreement with political statements
 - recommendation for voting



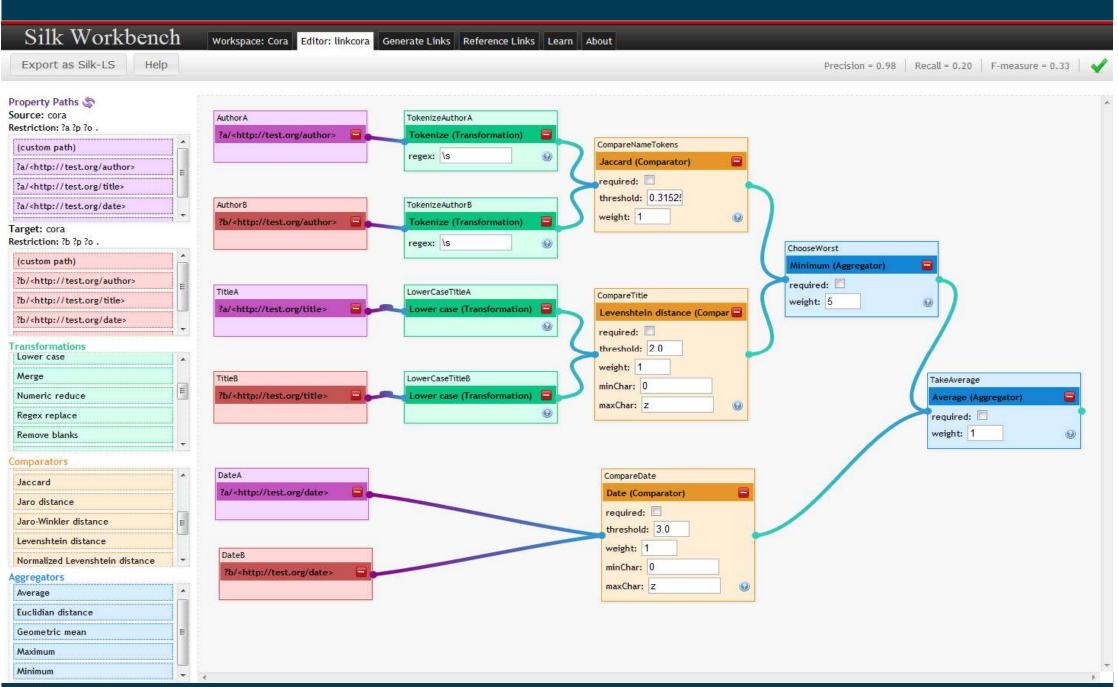


Using Weights to Combine Similarities

- You may not want to treat all attributes the same
 - use weights w_k which are between 0 and 1 and sum up to 1
 - weights are set according to the importance of the attributes
- Example: Weighted Euclidean Distance

$$dist(\mathbf{x}_{i}, \mathbf{x}_{j}) = \sqrt{w_{1}(x_{i1} - x_{j1})^{2} + w_{2}(x_{i2} - x_{j2})^{2} + \dots + w_{r}(x_{ir} - x_{jr})^{2}}$$

Combining Different Similarity Measures



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- "Best" algorithm depends on
 - 1. the analytical goals of the specific use case
 - 2. the distribution of the data
- Normalization, feature selection, distance measure, and parameter settings have equally high influence on results
- Due to these complexities, the common practice is to
 - 1. run several algorithms using different distance measures, feature subsets and parameter settings, and
 - 2. then visualize and interpret the results based on knowledge about the application domain as well as the goals of the analysis

Pang-Ning Tan, Michael Steinbach, Anuj Karpatne, Vipin Kumar: **Introduction to Data Mining.** 2nd Edition. Pearson.

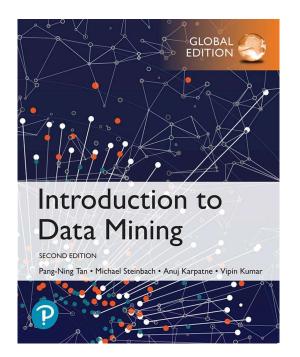
Chapter 5: Cluster Analysis

Chapter 5.2: K-Means

Chapter 5.3: Agglomerative Hierarchical Clustering

Chapter 5.4: DBSCAN

Chapter 2.4: Measures of Similarity and Dissimilarity



The respective chapters of the 1st Edition are available as PDF in ILIAS

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