Association Analysis

IE500 Data Mining

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Outline

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1. What is Association Analysis?
2. Frequent Itemset Generation
3. Rule Generation
4. Handling Continuous and Categorical Attributes
5. Interestingness Measures
6. Sequential Pattern Mining

Correlation Analysis

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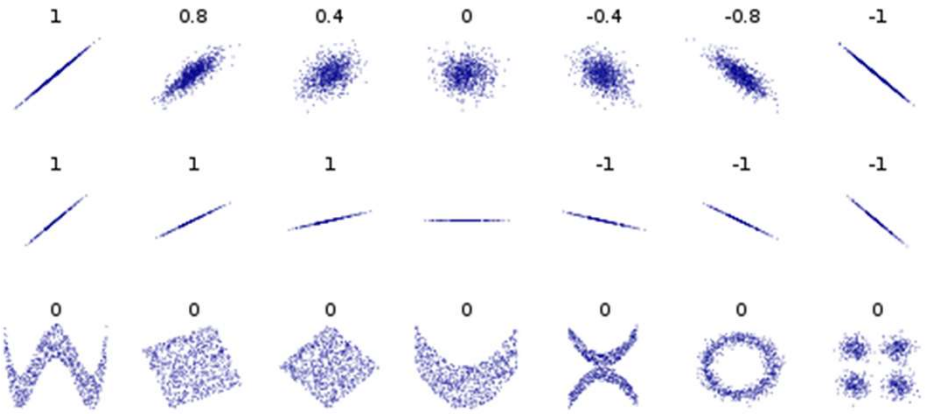
- Correlation analysis measures the degree of dependency between **two variables**
  - Continuous variables: Pearson's correlation coefficient (PCC)
  - Binary variables: Phi coefficient

$$PCC(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

$$Phi(x,y) = \frac{f_{11} f_{00} - f_{01} f_{10}}{\sqrt{f_{1+} f_{+1} f_{0+} f_{+0}}}$$

- Value range [-1,1]
  - 1 : positive correlation
  - 0 : variables independent
  - -1 : negative correlation



Example: Correlations between Products in Shopping Baskets

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	P1	P2	P3	P4	P5
Basket 1	1	1	0	1	1
Basket 2	1	0	0	1	1
Basket 3	1	0	0	0	1

1 : always bought together
0 : sometimes bought together
-1 : never bought together

Correlation Matrix (Correlation Matrix)

Table View | Pairwise Table | Plot View | Annotations

Attributes	ThinkPad X...	Asus EeePC	HP Laserjet...	2 GB DDR3...	8 GB DDR3...	Lenovo Tab...	Netbook-Sc...	HP CE50 T...	LT Laser M...	LT Minimaus
ThinkPad X2	1	-1	0.356	-0.816	0.612	0.583	-0.667	0.356	0.167	-0.408
Asus EeePC	-1	1	-0.356	0.816	-0.612	-0.583	0.667	-0.356	-0.167	0.408
HP Laserjet	0.356	-0.356	1	-0.218	-0.327	0.356	-0.535	1	-0.089	-0.655
2 GB DDR3	-0.816	0.816	-0.218	1	-0.500	-0.816	0.816	-0.218	0	0.200
8 GB DDR3	0.612	-0.612	-0.327	-0.500	1	0.102	-0.408	-0.327	0.102	0
Lenovo Tabl	0.583	-0.583	0.356	-0.816	0.102	1	-0.667	0.356	-0.250	0
Netbook-Sch	-0.667	0.667	-0.535	0.816	-0.408	-0.667	1	-0.535	0.167	0.408
HP CE50 To	0.356	-0.356	1	-0.218	-0.327	0.356	-0.535	1	-0.089	-0.655
LT Laser Ma	0.167	-0.167	-0.089	0	0.102	-0.250	0.167	-0.089	1	-0.408
LT Minimaus	-0.408	0.408	-0.655	0.200	0	0	0.408	-0.655	-0.408	1

Shortcoming: Measures correlation only between two items but not between multiple items, e.g. {ThinkPad, Cover} → {Minimaus}

1. Association Analysis

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- Association analysis can find **multiple item co-occurrence relationships** (descriptive method)
- first algorithms developed in the early 90s at IBM by Agrawal & Srikant
- initially used for **shopping basket analysis** to find out how items purchased are related
- later extended to more complex data structures
  - sequential patterns
  - subgraph patterns
- and other application domains
  - web usage mining, social science, life science

Association Analysis

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- Given a set of transactions, **find rules** that will predict the occurrence of an item based on the occurrences of other items in the transaction.

- Examples of **Association Rules**
  - {Diaper} → {Beer}
    {Beer, Bread} → {Milk}
    {Milk, Bread} → {Eggs, Coke}

Implication denotes co-occurence, not causality!

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Shopping Transactions

Definition: Frequent Itemset

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- **Itemset**
  - Collection of one or more items
  - Example: {Milk, Bread, Diaper}
  - k-itemset: An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - e.g. σ({Milk, Bread, Diaper}) = 2
- **Support (s)**
  - Fraction of transactions that contain an itemset
  - e.g. s({Milk, Bread, Diaper}) = 2/5 = 0.4
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a minimal support (minsup) threshold specified by the user

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Shopping Transactions

Definition: Association Rule

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- Association Rule
  - An implication of the form X → Y, where X and Y are itemsets
  - Interpretation: when X occurs, Y occurs with a certain probability

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Shopping Transactions

- More formally, it's a *conditional probability*
  - P(Y|X) given X, what is the probability of Y?

Definition: Association Rule

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- Association Rule
  - Example:
    {Milk, Diaper} → {Beer}
    Condition **Consequent**

- Rule Evaluation Metrics
  - **Support s:**
    Fraction of total transactions which contain both X and Y

$$s(X \rightarrow Y) = \frac{|X \cup Y|}{|T|}$$

$$s(\{\text{Milk, Diaper}\} \rightarrow \{Beer\}) = \frac{\sigma(\{Milk, Diaper, Beer\})}{|T|} = \frac{2}{5} = 0.4$$

  - **Confidence c:**
    Measures how often items in Y appear in transactions that contain X

$$c(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}$$

$$c(\{\text{Milk, Diaper}\} \rightarrow \{Beer\}) = \frac{\sigma(\{Milk, Diaper, Beer\})}{\sigma(\{Milk, Diaper\})} = \frac{2}{3} = 0.67$$

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Shopping Transactions

Main Challenges concerning Association Analysis

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1. Mining association rules from large amounts of data can be **computationally expensive**
   - algorithms need to apply smart pruning strategies
2. Algorithms often discover a **large number of associations**
   - many of them are uninteresting or redundant
   - the user needs to select the subset of the associations that is relevant given her task at hand

The Association Rule Mining Task

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- Given a set of **transactions T**, the goal of association rule mining is to **find all rules** having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold
- ***minsup*** and ***minconf*** are provided by the user

- Brute Force Approach:
  1. List all possible association rules
  2. Compute the support and confidence for each rule
  3. Remove rules that fail the *minsup* and *minconf* thresholds

⇒ **Computationally prohibitive due to large number of candidates!**

Mining Association Rules

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- Example rules:
  {Milk, Diaper} → {Beer} (s=0.4, c=0.67)
  {Milk, Beer} → {Diaper} (s=0.4, c=1.0)
  {Diaper, Beer} → {Milk} (s=0.4, c=0.67)
  {Beer} → {Milk, Diaper} (s=0.4, c=0.67)
  {Diaper} → {Milk, Beer} (s=0.4, c=0.5)
  {Milk} → {Diaper, Beer} (s=0.4, c=0.5)

TID	Items
1	Bread, **Milk**
2	Bread, **Diaper, Beer**, Eggs
3	**Milk, Diaper, Beer**, Coke
4	Break, **Milk, Diaper, Beer**
5	Bread, **Milk, Diaper**, Coke

Shopping Transactions

- Observations:
  - All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
  - Rules originating from the same itemset have identical support s
    - but can have different confidence
  - Thus, we may decouple the support and confidence requirements

$$s(X \rightarrow Y) = \frac{|X \cup Y|}{|T|}$$

Apriori Algorithm: Basic Idea

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- Two-step approach:

1. Frequent Itemset Generation
   - Generate all itemsets whose support $\geq$ minsup

2. Rule Generation
   - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

2. Frequent Itemset Generation

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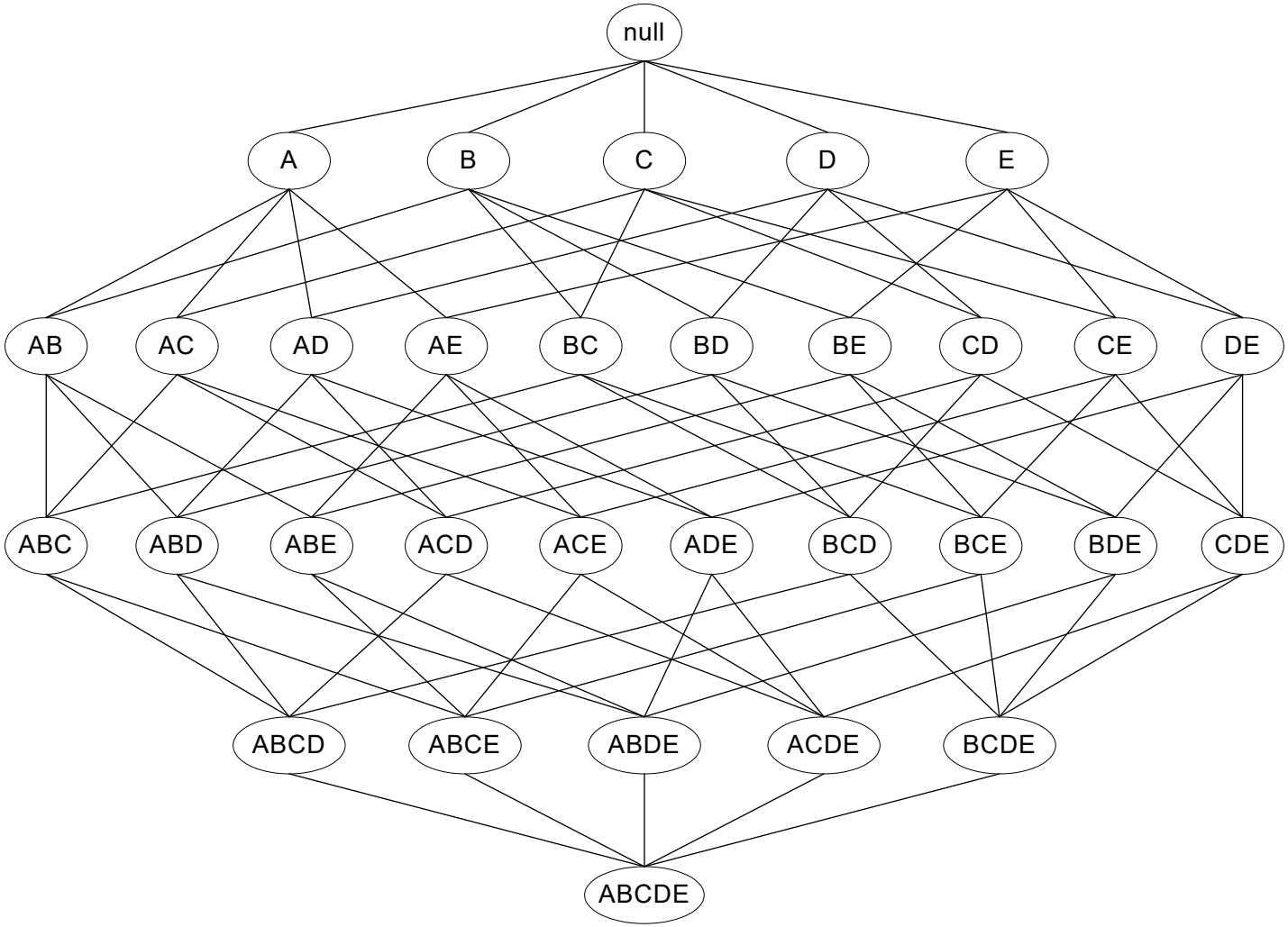
- Given d items, there are 2^d candidate itemsets!



Brute-force Approach

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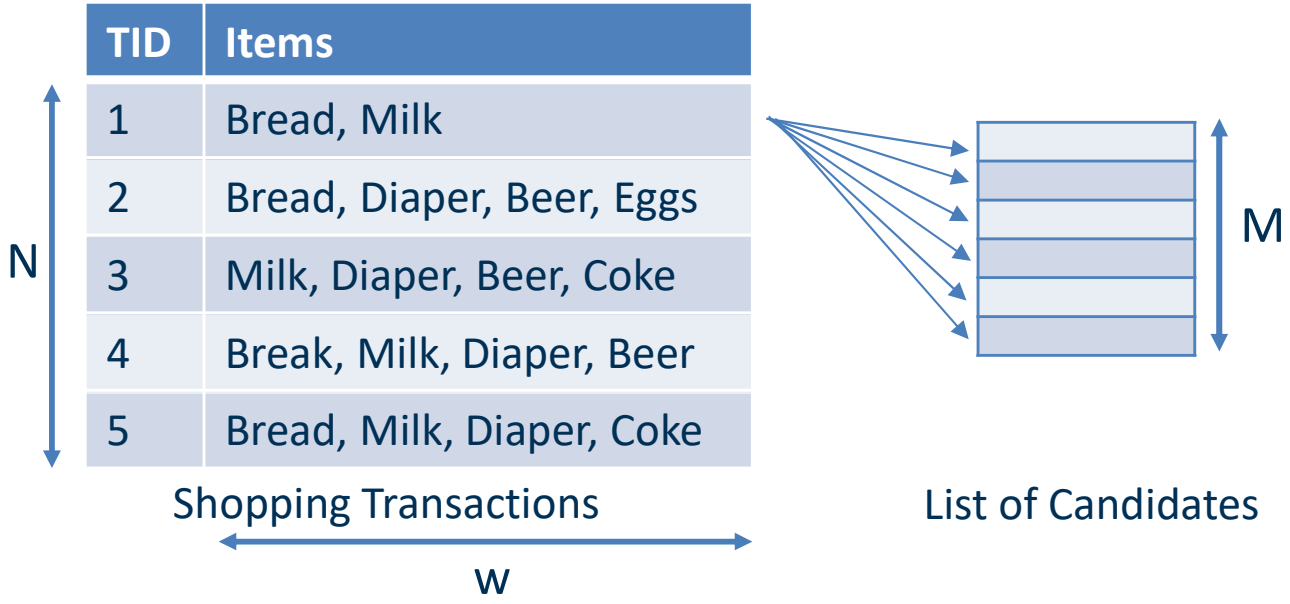
- Each itemset in the lattice is a **candidate** frequent itemset
- Count the support of each candidate by scanning the database
  = Match each transaction against every candidate



TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Break, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- Complexity ~ O(NMw) ➔ **Expensive since M = 2^d**

Brute-force Approach

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- Amazon sells 12M different products (as of 2023)
  - That is $2^{12.000.000}$ possible itemsets
    - That's a 3.6M digit number
  - Today's supercomputers: 1,200 Petaflops, i.e., 1.2x 10^{18} floating point operations per second
  - Even if an itemset could be checked with one single floating point operation this would take ~ $10^{3,612,334}$ years (age of universe: 1.4x10^{10} years)
- However:
  - Most itemsets will not be frequent at all, e.g., books on Chinese calligraphy, Inuit cooking, and data mining bought together
  - Thus, smarter algorithms should be possible
  - Intuition for smarter algorithms: All itemsets containing Inuit cooking are likely infrequent

Anti-Monotonicity of Support

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- What happens when an itemset gets larger?
  - s({Milk}) = 0.8
  - s({Milk,Diaper}) = 0.6
  - s({Milk,Diaper,Beer}) = 0.4

  - s({Bread}) = 0.8
  - s({Bread,Milk}) = 0.6
  - s({Bread,Milk,Diaper}) = 0.4

- There is a pattern here!

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Break, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Reducing the Number of Candidates

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- There is a pattern here!
  - It is called anti-monitonicity of support
- If X is a subset of Y
  - s(Y) is at most as large as s(X)

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Break, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

$$\forall X, Y: (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- Consequence for frequent itemset search (aka Apriori principle):
  - If Y is frequent, X also has to be frequent
  - i.e.: **all subsets of frequent itemsets are frequent**

Using the Apriori Principle for Pruning

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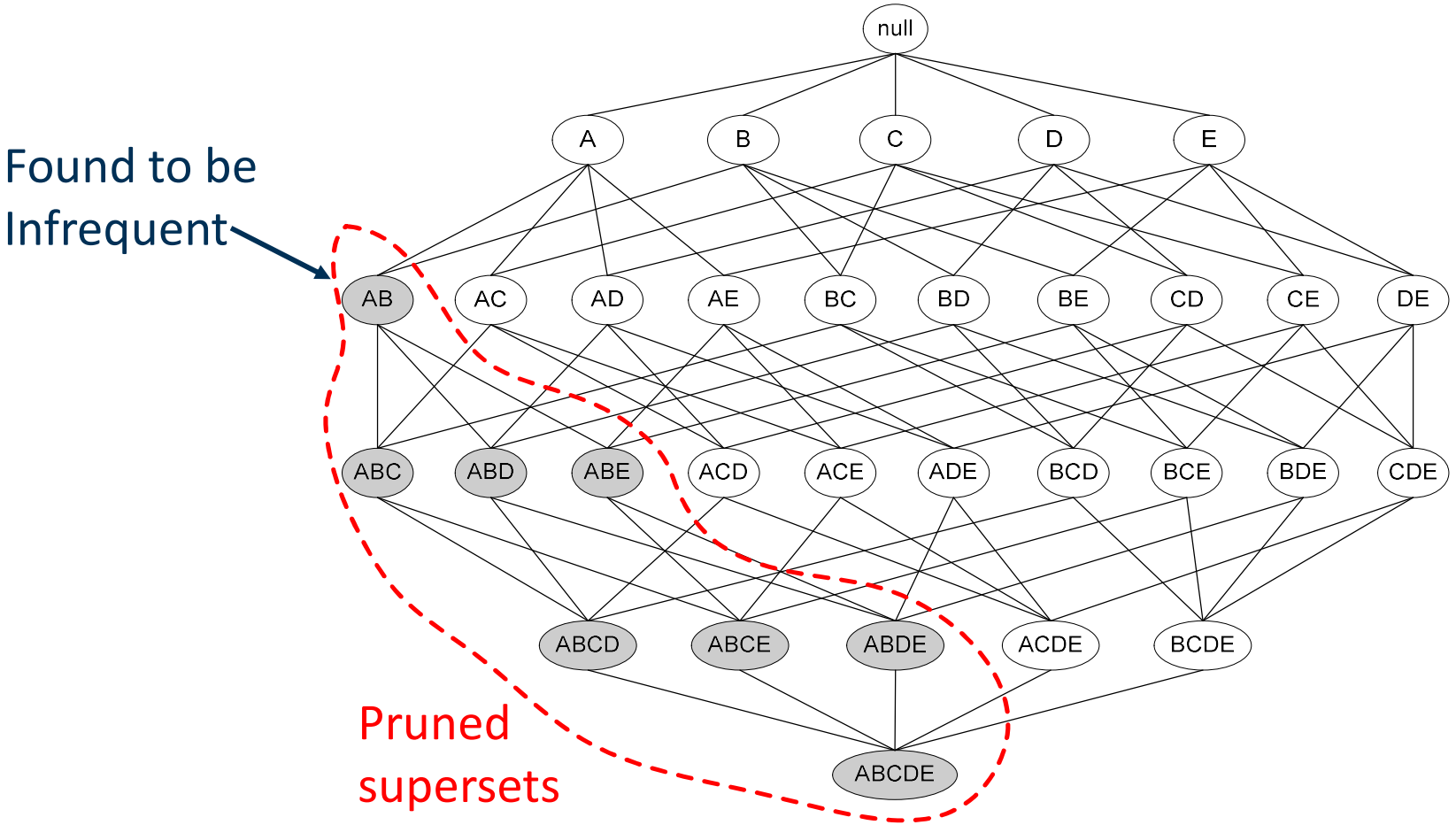
- If an itemset is infrequent,
  then all of its supersets must also be infrequent



The Apriori Algorithm

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- Let k=1
- Generate frequent itemsets of length 1
- Repeat until no new frequent itemsets are identified
  1. **Generate** length (k+1) candidate itemsets from length k frequent itemsets
  2. **Prune** candidate itemsets that can not be frequent because they contain subsets of length k that are infrequent (Apriori Principle)
  3. **Count** the support of each candidate by scanning the DB
  4. **Eliminate** candidates that are infrequent, leaving only those that are frequent

Illustrating the Apriori Principle

Minimum Support Count = 3

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Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

No need to generate candidates involving Coke or Eggs

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Break, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Pairs (2-itemsets)

Itemset	Count
{Bread, Milk}	3
{Bread, Beer}	2
{Bread, Diaper}	3
{Milk, Beer}	2
{Milk, Diaper}	3
{Beer, Diaper}	3

No need to generate candidate {Milk, Diaper, Beer} as count {Milk, Beer} = 2

Triplets (3-itemsets)

Itemset	Count
{Bread, Milk, Diaper}	3

Illustrating the Apriori Principle

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- In the example, we had six items, and examined
  - Six 1-itemsets
  - Six 2-itemsets
  - One 3-itemset
  - i.e., **13 in total**
- vs. possible itemsets: 2^6 = **64 in total**

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Break, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Pruning strongly reduced the number of itemsets for which we need to scan the DB

Example of Frequent Itemsets

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Result History | FrequentItemSets

Data

Annotations

No. of Sets: 83

Total Max. Size: 4

Min. Size: 1

Max. Size: 4

Contains Item:

Update View

Size	Support ↓	Item 1	Item 2	Item 3
1	0.600	Asus EeePC		
1	0.500	LT Minimaus		
1	0.500	2 GB DDR3 RAM		
2	0.500	Asus EeePC	2 GB DDR3 RAM	
1	0.400	ThinkPad X220		
1	0.400	Netbook-Schutzhülle		
1	0.400	Lenovo Tablet Sleeve		
1	0.400	LT Laser Maus		
2	0.400	Asus EeePC	LT Minimaus	
2	0.400	Asus EeePC	Netbook-Schutzhülle	
2	0.400	2 GB DDR3 RAM	Netbook-Schutzhülle	
3	0.400	Asus EeePC	2 GB DDR3 RAM	Netbook-Schutzhülle
1	0.300	HP Laserjet P2055		
1	0.300	HP CE50 Toner		
2	0.300	LT Minimaus	2 GB DDR3 RAM	
2	0.300	LT Minimaus	Netbook-Schutzhülle	
2	0.300	ThinkPad X220	Lenovo Tablet Sleeve	
2	0.300	HP Laserjet P2055	HP CE50 Toner	
3	0.300	Asus EeePC	LT Minimaus	2 GB DDR3 RAM

Application of Frequent Itemsets

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1. Take top-k frequent itemsets of size 2 containing item A
2. Rank second item according to
   - profit made by selling item
   - whether you want to reduce number of items B in stock
   - knowledge about customer preferences
3. Offer special price for combination with top-ranked second item

amazon

3. From Frequent Itemsets to Rules

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- Given a frequent itemset L,
  find all non-empty subsets f $\subset$ L
  such that f $\rightarrow$ L \ f satisfies
  the **minimum confidence** requirement

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Break, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- Example Frequent Itemset L:
  - {Milk,Diaper,Beer}
- Example Rule:
  - $\underbrace{\{Milk,Diaper\}}_{f} \rightarrow \{Beer\}$

$$c = \frac{\sigma(Milk, Diaper, Beer)}{\sigma(Milk, Diaper)} = \frac{2}{3}$$

Challenge: Combinatorial Explosion

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- Given a 4-itemset {A,B,C,D}, we can generate

{A} → {B,C,D},	{A,B} → {C,D},	{B,C} → {A,D},	{C,D} → {A,B},	{A,B,C} → {D}
{B} → {A,C,D},	{A,C} → {B,D},	{B,D} → {A,C},		{A,B,D} → {C}
{C} → {A,B,D},	{A,D} → {B,C},			{A,C,D} → {B}
{D} → {A,B,C},				{B,C,D} → {A}

  - i.e., a total of 14 rules for just one itemset!

- General number for a k-itemset: $2^k - 2$
  - It's not 2^k since we ignore Ø → {…} and {…} → Ø

Challenge: Combinatorial Explosion

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- Wanted:
  another pruning trick like Apriori

- However
  - **c**({Milk,Diaper} → {Beer}) =0.67
  - **c**({Milk} → {Beer}) =0.5
  - **c**({Diaper} → {Beer}) =0.8

  - c(ABC → D) can be larger or smaller than c(AB → D)
    - In general, confidence does not have an anti-monotone property

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Break, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Challenge: Combinatorial Explosion

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- But:
  confidence of rules generated from the **same itemset** has an anti-monotone property
  - E.g. L = {Milk,Diaper,Beer}
  - {Milk,Diaper,Beer} → Ø c=1.0
    - {Milk,Diaper} → {Beer} c=0.67
    - {Milk} → {Diaper,Beer} c=0.5
    - {Diaper} → {Milk,Beer} c=0.5
  - {Milk,Beer} → {Diaper} c=1.0
    - {Milk} → {Diaper,Beer} c=0.5
    - {Beer} → {Milk,Diaper} c=0.67
  - e.g., L = {A,B,C,D}:
    $c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Break, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Observation: moving elements from antecedent to consequence ("left to right") in the rule never increases confidence!

Explanation

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- Confidence is anti-monotone with respect to the number of items on the right-hand side (RHS) of the rule
  - i.e., “moving elements from left to right” cannot increase confidence
- Reason:

$$c(AB \rightarrow C) := \frac{s(ABC)}{s(AB)} \qquad c(A \rightarrow BC) := \frac{s(ABC)}{s(A)}$$

  - Due to anti-monotone property of support, we know

$$s(AB) \leq s(A)$$

  - Hence

$$c(AB \rightarrow C) \geq c(A \rightarrow BC)$$

Candidate Rule Pruning

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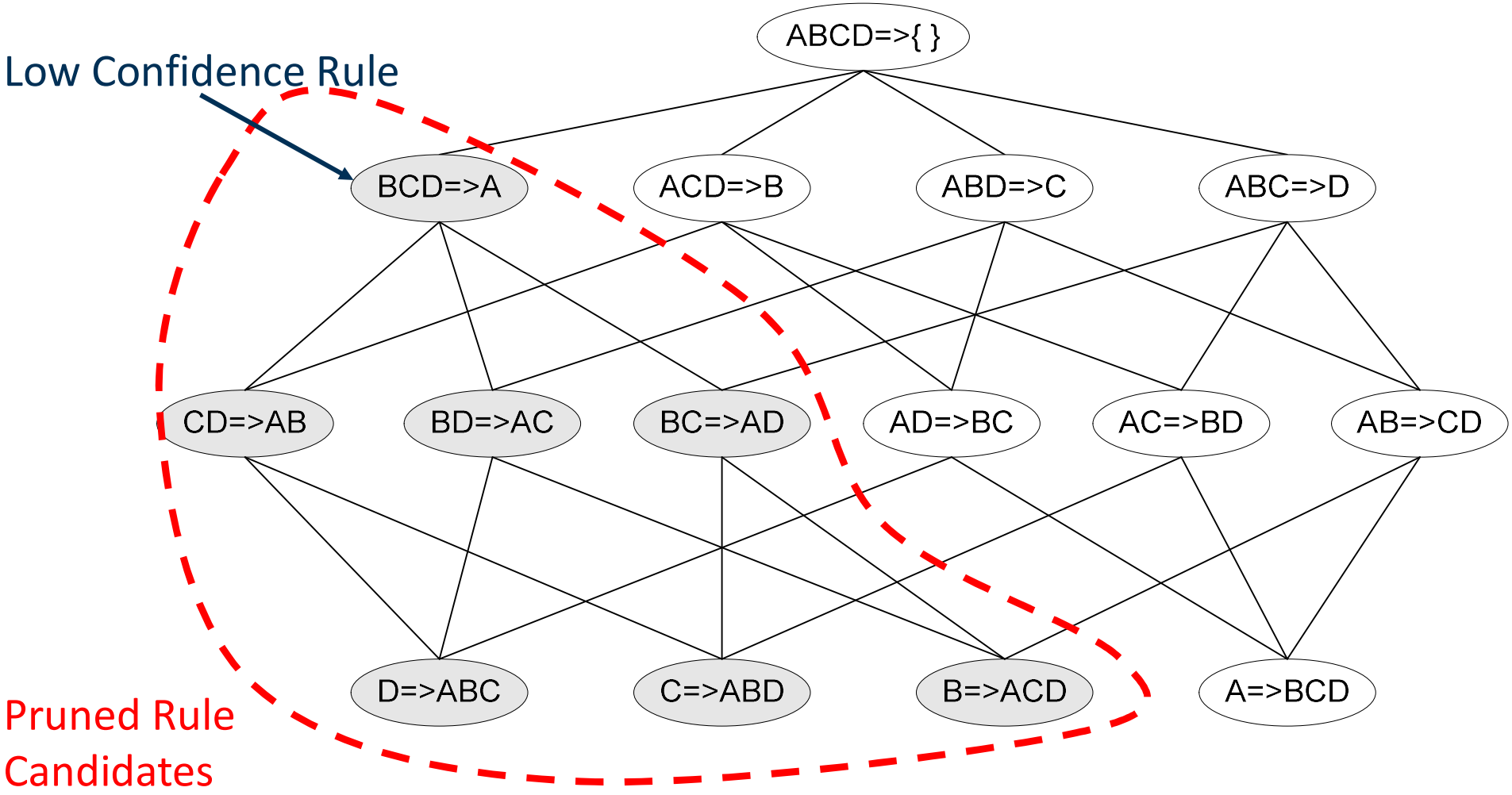
- Moving elements from left to right cannot increase confidence



Rule Generation for Apriori Algorithm

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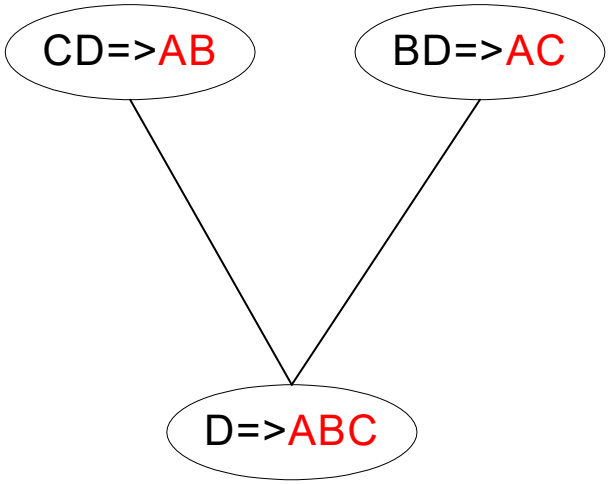
- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent
  - join(CD → AB, BD → AC) would produce the candidate rule D → ABC
  - Prune rule D → ABC if one of its parent rules does not have high confidence (e.g. AD → BC)



- All the required information for confidence computation has already been recorded during itemset generation
  - Thus, there is no need to see the data any more

$$c(X \rightarrow Y) = \frac{s(X \cup Y)}{s(X)}$$

4. Handling Continuous and Categorical Attributes

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- How to apply association analysis to attributes that are not asymmetric binary variables?

Session Id	Country	Session Length (sec)	Number of Web Pages viewed	Gender	Browser Type	Buy
1	USA	982	8	Male	Chrome	No
2	China	811	10	Female	Chrome	No
3	USA	2125	45	Female	Firefox	Yes
4	Germany	596	4	Male	IE	Yes
5	Australia	123	9	Male	Firefox	No
…	…	…	…	…	…	…

- Example Rule:

  $\{\text{Number of Pages} \in [5,10) \wedge (\text{Browser=Firefox})\} \rightarrow \{\text{Buy = No}\}$

Handling Categorical Attributes

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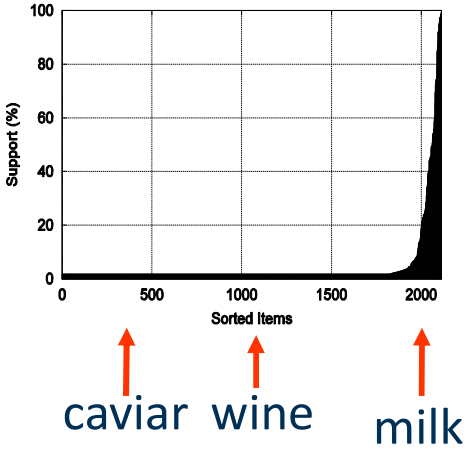
- Transform categorical attribute into asymmetric binary variables
- Introduce a new “item” for each distinct attribute-value pair
  - e.g. replace “Browser Type” attribute with
    - attribute: “Browser Type = Chrome”
    - attribute: “Browser Type = Firefox”
    - …..
  - Python: OneHotEncoder
- Issues
  - What if attribute has many possible values?
    - Many of the attribute values may have very low support
    - Potential solution: aggregate low-support attribute values
  - What if distribution of attribute values is highly skewed?
    - Example: 95% of the visitors have Buy = No
    - Most of the items will be associated with (Buy=No) item
    - Potential solution: drop the highly frequent item



Handling Continuous Attributes

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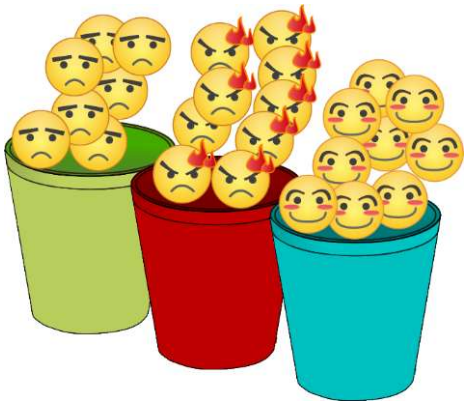

- Transform continuous attribute into binary variables using discretization
  - Values of the attribute, e.g., age of a person:
    - 0, 4, 12, 16, 16, 18, 24, 26, 30
  - **Equal-interval binning** – for bin width of e.g., 10:
    - Bin 1: 0, 4 [-∞,10) bin
    - Bin 2: 12, 16, 16, 18 [10,20) bin
    - Bin 3: 24, 26, 30 [20,+∞) bin
  - **Equal-frequency binning** – for bin density of e.g., 3:
    - Bin 1: 0, 4, 12 [-, 14) bin
    - Bin 2: 16, 16, 18 [14, 21) bin
    - Bin 3: 24, 26, 30 [21,+] bin

Handling Continuous Attributes

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- Issue: Size of the discretization intervals affects support & confidence
  - {Refund=No, (Income=$51,251)} → {Cheat=No}
  - {Refund=No, (60K<= Income <=80K)} → {Cheat=No}
  - {Refund=No, (0K<= Income <=1B)} → {Cheat=No}

  - If intervals are too small
    - Itemsets may not have enough support
  - If intervals too large
    - Rules may not have enough confidence
    - e.g. combination of different age groups compared to a specific age group

Association Analysis in Python

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- Various packages exist
  - In the exercise, we'll use the Orange3 package
  - Frequent Itemset Generation

Python

```python
from orangecontrib.associate.fpgrowth import frequent_itemsets

# Calculate frequent itemsets
itemsets = dict(frequent_itemsets(dataset.values, 0.20))
```

Min support

  - Creating Association Rules

Python

```python
from orangecontrib.associate.fpgrowth import association_rules

# Calculate association rules from itemsets
rules = association_rules(itemsets, 0.70)
```

Min confidence

5. Interestingness Measures

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- Association rule algorithms tend to produce too many rules
  - Many of them are uninteresting or redundant
  - Redundant if {A,B,C} → {D} and {A,B} → {D} have same support & confidence
- Interestingness measures can be used to prune or rank the derived rules
- In the original formulation of association rules, support & confidence were the only interestingness measures used
- Later, various other measures have been proposed
  - We will have a look at one: Lift
  - See Tan/Steinbach/Kumar, Chapter 6.7

Drawback of Confidence

Contingency table

	Coffee	$\overline{\text{Coffee}}$	
Tea	3	1	4
$\overline{\text{Tea}}$	15	1	16
	18	2	20

TID	Items
1	Coffee
2	Coffee
3	Coffee
4	Coffee
5	Coffee
6	Coffee
7	Coffee
8	Coffee
9	Coffee
10	Coffee
11	Coffee
12	Coffee
13	Coffee
14	Coffee
15	Coffee
16	Tea, Coffee
17	Tea, Coffee
18	Tea, Coffee
19	Tea
20	Bread

- Association Rule:
  Tea → Coffee
- confidence(Tea → Coffee) = $\frac{3}{4} = 0.75$
- **but** support(Coffee) = $\frac{18}{20} = 0.9$

- Although confidence is high, rule is misleading as the fraction of coffee drinkers is higher than the confidence of the rule
  - We want confidence(X → Y) > support(Y)
  - otherwise rule is misleading as X reduces probability of Y

Lift

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- We discover a high confidence rule for tea → coffee
  - 75% of all people who drink tea also drink coffee
  - Hypothesis: people who drink tea are likely to drink coffee
    - Implicitly: **more likely than all people**
- Test: Compare the confidence of the two rules
  - Rule: Tea → coffee $c(tea \rightarrow coffee) = \frac{s(\{tea\} \cup \{coffee\})}{s(\{tea\})}$
  - Default rule: all → coffee $c(all \rightarrow coffee) = \frac{s(\{all\} \cup \{coffee\})}{s(\{all\})} = \frac{s(\{coffee\})}{1} = s(\{coffee\})$
- We accept a rule iff its confidence is higher than the default rule

$$\frac{c(tea \rightarrow coffee)}{c(all \rightarrow coffee)} = \frac{c(tea \rightarrow coffee)}{s(\{coffee\})} > 1$$

Lift

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- The lift of an association rule $X \rightarrow Y$ is defined as:

$$c(X \rightarrow Y) = \frac{s(X \cup Y)}{s(X)}$$

$$Lift = \frac{c(X \rightarrow Y)}{s(Y)} = \frac{s(X \cup Y)}{s(X) * s(Y)}$$

  - Confidence normalized by support of consequent

- Interpretation
  - if **lift > 1**, then X and Y are positively correlated
  - if **lift = 1**, then X and Y are independent
  - if **lift < 1**, then X and Y are negatively correlated

Lift Example

Contingency table

	Coffee	$\overline{\text{Coffee}}$	
Tea	3	1	4
$\overline{\text{Tea}}$	15	1	16
	18	2	20

TID	Items
1	Coffee
2	Coffee
3	Coffee
4	Coffee
5	Coffee
6	Coffee
7	Coffee
8	Coffee
9	Coffee
10	Coffee
11	Coffee
12	Coffee
13	Coffee
14	Coffee
15	Coffee
16	Tea, Coffee
17	Tea, Coffee
18	Tea, Coffee
19	Tea
20	Bread

- Association Rule:
  Tea → Coffee
- confidence(Tea → Coffee) = $\frac{3}{4} = 0.75$
- **but** support(Coffee) = $\frac{18}{20} = 0.9$

$$Lift(Tea \rightarrow Coffee) = \frac{c(tea \rightarrow coffee)}{c(all \rightarrow coffee)} = \frac{c(tea \rightarrow coffee)}{s(\{coffee\})}$$

$$= \frac{0.75}{0.9} = 0.833 \ < 1$$

  - lift < 1, therefore is negatively correlated and removed

Conclusion

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- The algorithm does the counting for you and finds patterns in the data

- You need to do the interpretation based on your knowledge about the application domain.
  - Which patterns are meaningful?
  - Which patterns are surprising?

6. Sequential Pattern Mining

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- Association rule mining does not consider **the order of transactions**
- In many applications such orderings are relevant

Sequence Database	Sequence	Element (Transaction)	Event (Item)
Customer Data	Purchase history of a given customer	A set of items bought by a customer at time t	Books, diary products, CDs, etc
Web Server Logs	Browsing activity of a particular visitor	A collection of files viewed by a Web visitor after a single mouse click	Home page, index page, contact info, etc
Sensor Data	History of events generated by a given sensor	Events triggered by a sensor at time t	Types of alarms generated by sensors

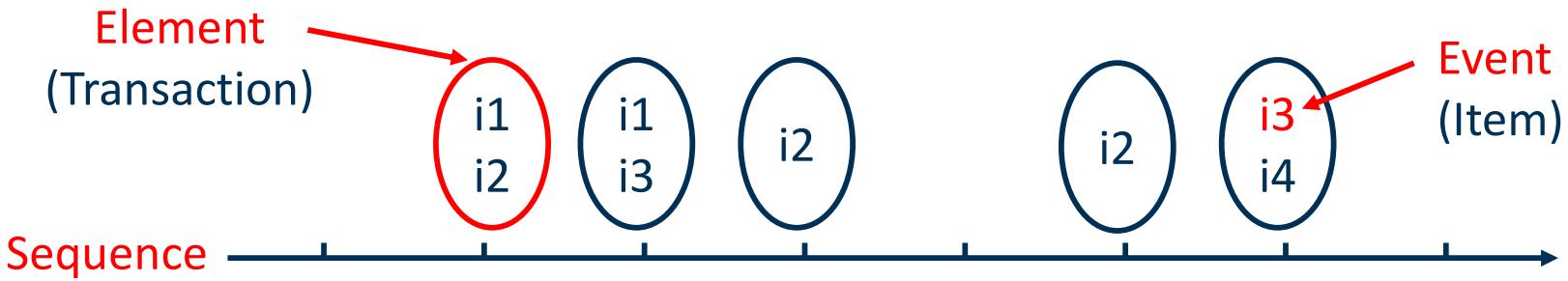
Definition of a Sequence

- A **sequence** is an ordered list of elements (transactions)

$$s = < e_1\ e_2\ e_3\ \ldots >$$

- Each **element** contains a collection of **events** (items)

$$e_i = \{i_1, i_2, \ldots, i_k\}$$



Sequence Data

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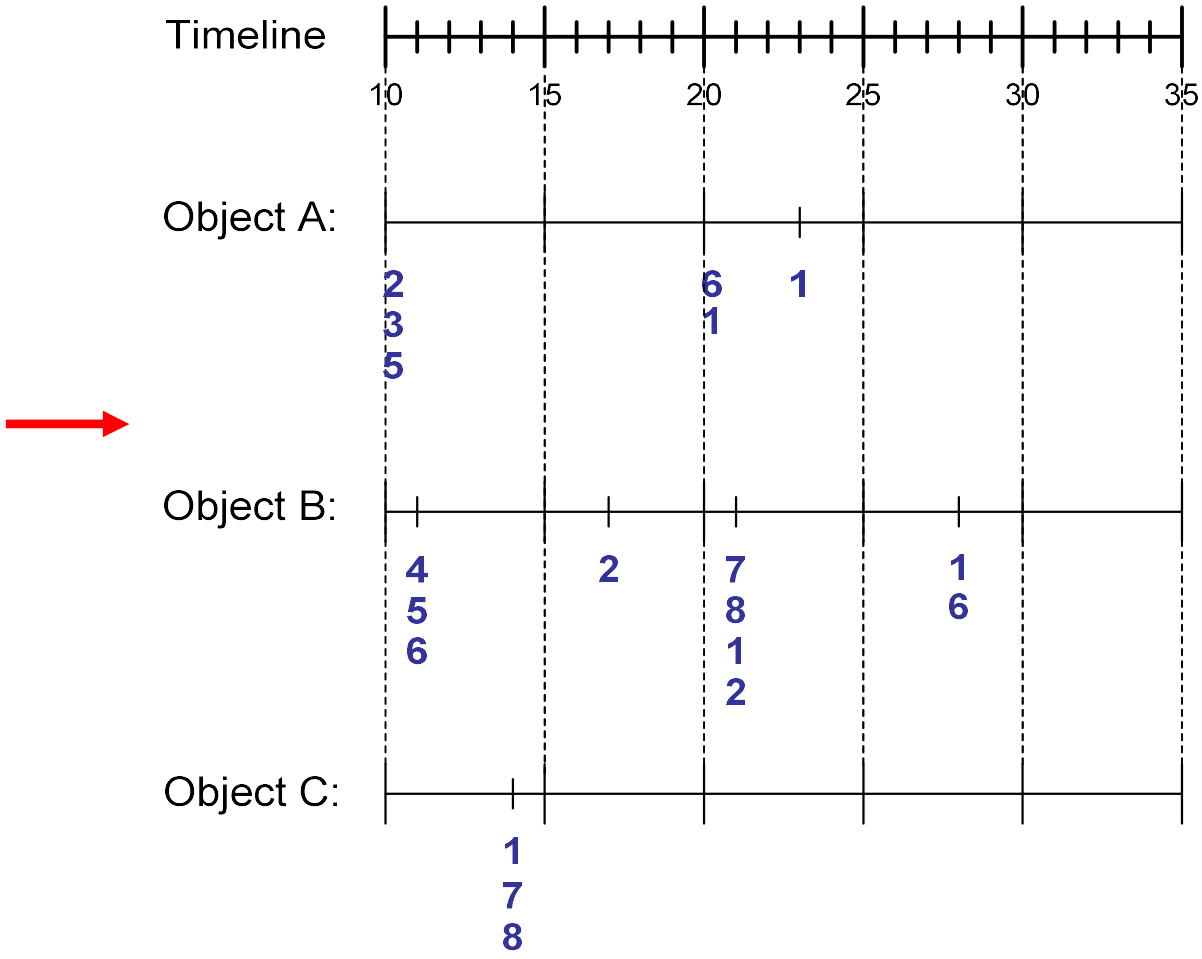
Sequence Database:

Object	Timestamp	Events
A	10	2, 3, 5
A	20	6, 1
A	23	1
B	11	4, 5, 6
B	17	2
B	21	7, 8, 1, 2
B	28	1, 6
C	14	1, 8, 7



Sequential Pattern Mining

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- Formal Definition of a Subsequence
  - A sequence $\langle a_1\ a_2 \ldots a_n \rangle$ is contained in another sequence $\langle b_1\ b_2 \ldots b_m \rangle$ ($m \geq n$) if there exist integers $i_1 < i_2 < \ldots < i_n$ such that $a_1 \subseteq b_{i_1}$, $a_2 \subseteq b_{i_2}$, …, $a_n \subseteq b_{i_n}$

Data sequence <b>	Subsequence <a>	Contain?
< {2,4} {3,5,6} {8} >	< {2} {3,5} >	Yes
< {1,2} {3,4} >	< {1} {2} >	No
< {2,4} {2,4} {2,5} >	< {2} {4} >	Yes

- The **support** of a subsequence w is defined as the fraction of data sequences that contain w
- A **sequential pattern** is a frequent subsequence (i.e., a subsequence whose support is ≥ minsup)

Sequential Pattern Mining

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- Given:
  - a database of sequences
  - a user-specified minimum support threshold, *minsup*
- Task:
  - Find all subsequences with support ≥ *minsup*
- Challenge:
  - Very large number of candidate subsequences that need to be checked against the sequence database.
  - By applying the Apriori principle, the number of candidates can be pruned significantly.

Determining the Candidate Subsequences

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- Given n events: $i_1, i_2, i_3, \ldots, i_n$
- Candidate 1-subsequences:

  $\langle\{i_1\}\rangle, \langle\{i_2\}\rangle, \langle\{i_3\}\rangle, \ldots, \langle\{i_n\}\rangle$
- Candidate 2-subsequences:

  $\langle\{i_1, i_2\}\rangle, \langle\{i_1, i_3\}\rangle, \ldots, \langle\{i_1\}\ \{i_1\}\rangle, \langle\{i_1\}\ \{i_2\}\rangle, \ldots, \langle\{i_{n-1}\}\ \{i_n\}\rangle$
- Candidate 3-subsequences:

  $\langle\{i_1, i_2, i_3\}\rangle, \langle\{i_1, i_2, i_4\}\rangle, \ldots, \langle\{i_1, i_2\}\ \{i_1\}\rangle, \langle\{i_1, i_2\}\ \{i_2\}\rangle, \ldots,$

  $\langle\{i_1\}\ \{i_1, i_2\}\rangle, \langle\{i_1\}\ \{i_1, i_3\}\rangle, \ldots, \langle\{i_1\}\ \{i_1\}\ \{i_1\}\rangle, \langle\{i_1\}\ \{i_1\}\ \{i_2\}\rangle, \ldots$

Generalized Sequential Pattern Algorithm (GSP)

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- Make the first pass over the sequence database D to yield all the 1-element frequent subsequences
- Repeat until no new frequent subsequences are found
  1. Candidate **Generation**:
     Merge pairs of frequent subsequences found in the (k-1)*th* pass to generate candidate sequences that contain k items
  2. Candidate **Pruning**:
     Prune candidate *k*-sequences that contain infrequent (*k-1)*-subsequences (**Apriori principle**)
  3. Support **Counting**:
     Make a new pass over the sequence database D to find the support for these candidate sequences
  4. Candidate **Elimination**:
     Eliminate candidate *k*-sequences whose actual support is less than *minsup*

Candidate Generation

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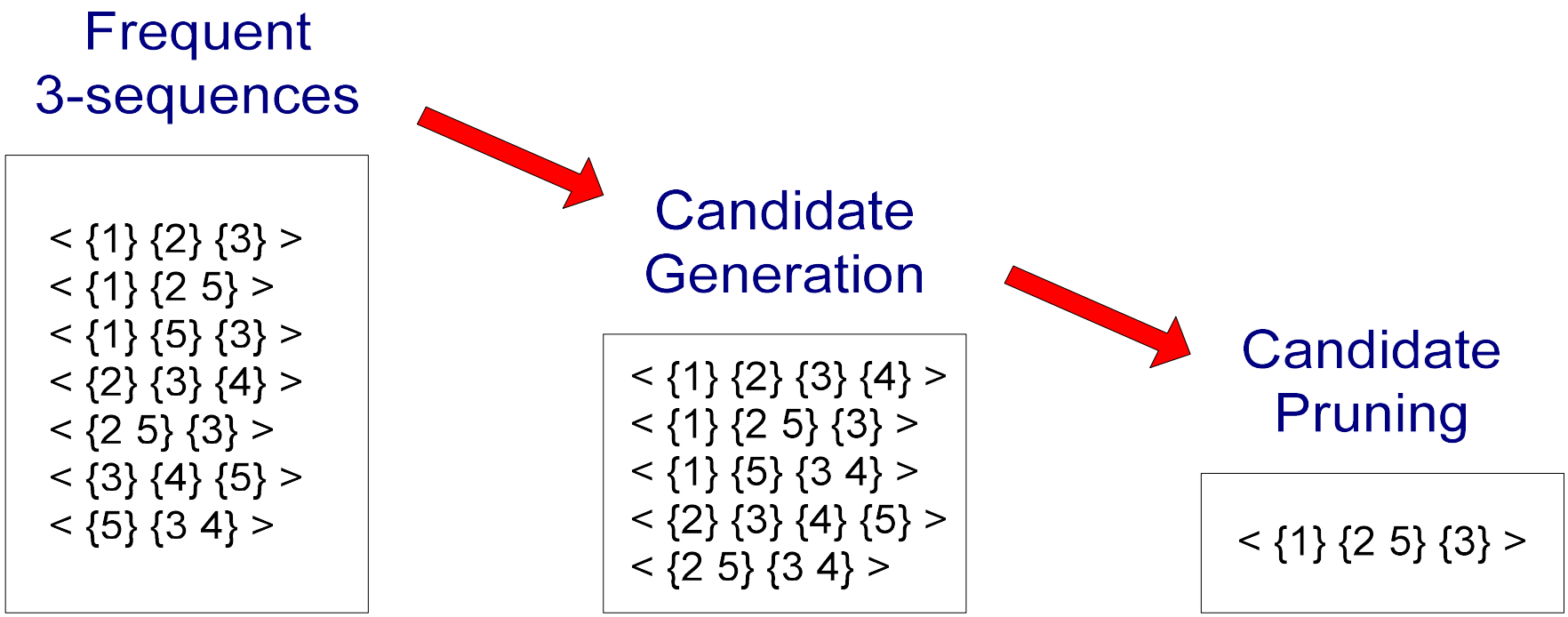
- Base case (k=2):
  - Merging two frequent 1-sequences $<\{i_1\}>$ and $<\{i_2\}>$ will produce two candidate 2-sequences: $<\{i_1\}\ \{i_2\}>$ and $<\{i_1\ i_2\}>$
- General case (k>2):
  - A frequent (*k-1*)-sequence w_1 is merged with another frequent (*k-1*)-sequence w_2 to produce a candidate *k*-sequence if the subsequence obtained by **removing the first event in w_1** is the same as the subsequence obtained by **removing the last event** in w_2
  - The resulting candidate after merging is given by the sequence w_1 extended with the last event of w_2.
    - If the last two events in w_2 belong to the same element, then the last event in w_2 becomes part of the last element in w_1
    - Otherwise, the last event in w_2 becomes a separate element appended to the end of w_1

GSP Example



- Only one 4-sequence survives the candidate pruning step
- All other 4-sequences are removed because they contain subsequences that are not part of the set of frequent 3-sequences.

Timing Constraints

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- Timing constraints allow us to pose **additional restrictions** on whether a sequence is counted to support a pattern or not.
- Motivating Example:

  < {Statistics} {Database Systems} {Data Mining} >

  < {Database Systems} {Statistics} {Data Mining} >
- We are interested in students that support the pattern

  < {Database Systems, Statistics} {Data Mining} >
- We don’t care about the order of Database Systems and Statistics.
- We care about that the gap between these courses and Data Mining is not too long.

Window Size

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- Spezifies a time window in the data sequence in which all events will be considered to belong to the same element.
- Given a candidate pattern: <{a, c}>
- Any data sequences that contain

  <… {a c} … >,
  <… {a} … {c}…> (where time({c}) – time({a}) $\leq$ ws)
  <…{c} … {a} …> (where time({a}) – time({c}) $\leq$ ws)

  will contribute to the support count of the candidate pattern
- max_g = 2, min_g = 0, ws = 1

Data sequence	Subsequence	Contain?
< {2,4} {3,5,6} {4,7} {4,6} {8} >	< {3} {5} >	No
< {1} {2} {3} {4} {5}>	< {1,2} {3} >	Yes
< {1,2} {2,3} {3,4} {4,5}>	< {1,2} {3,4} >	Yes

Max-Gap, Min-Gap

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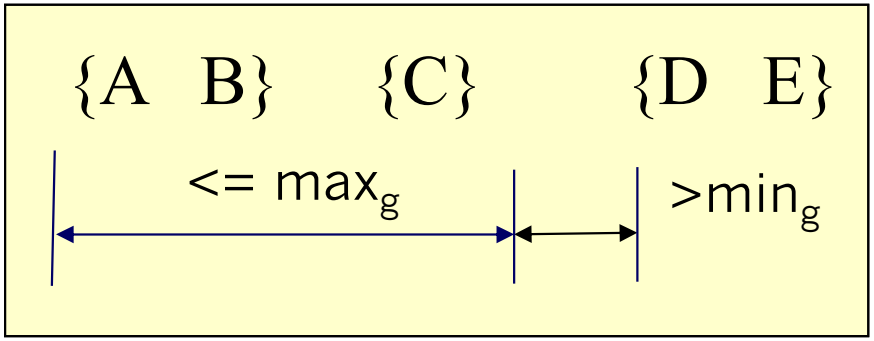
- Max-Gap: Sequence is counted if gap between consecutive elements is smaller than max_g
- Min-Gap: Sequence is counted if gap between consecutive elements is bigger than min_g
- $max_g = 2$, $min_g = 0$



Data sequence	Subsequence	Contain?
< {2,4} {3,5,6} {4,7} {4,5} {8} >	< {6} {5} >	Yes
< {1} {2} {3} {4} {5}>	< {1} {4} >	No
< {1} {2,3} {3,4} {4,5}>	< {2} {3} {5} >	Yes
< {1,2} {3} {2,3} {3,4} {2,4} {4,5}>	< {1,2} {5} >	No

Literature for this Slideset

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- Pang-Ning Tan, Michael Steinbach, Karpatne,Vipin Kumar: Introduction to Data Mining. 2nd Edition. Pearson.

- Chapter 4: Association Analysis: Basic Concepts and Algorithms
- Chapter 7: Association Analysis: Advanced Concepts