

# Knowledge Graphs Web Ontology Language (OWL) Part II



# Previously on “Knowledge Graphs”

- We have got to know
  - OWL, a more powerful ontology language than RDFS
  - Simple ontologies and some reasoning
  - Sudoku solving
- Today
  - New constructs in OWL2
  - Russell's paradox
  - Reasoning in OWL
  - Complexity of ontologies
  - A peek at rule languages for Knowledge Graphs



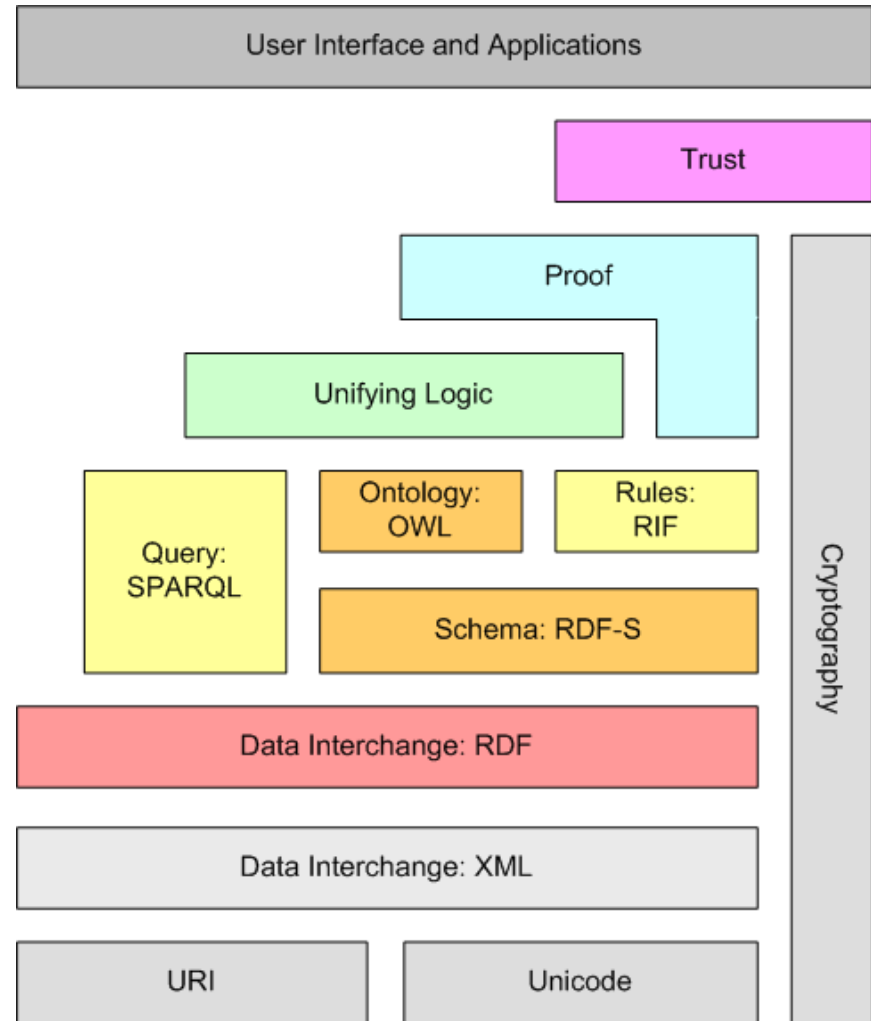
# Semantic Web Technology Stack



here be dragons...

Semantic Web  
Technologies  
(This lecture)

Technical  
Foundations



Berners-Lee (2009): *Semantic Web and Linked Data*  
<http://www.w3.org/2009/Talks/0120-campus-party-tbl/>

# OWL2 – New Constructs and More

- Five years after the first OWL standard
- OWL2: 2009
  - Syntactic sugar
  - New language constructs
  - OWL profiles
- We have already encountered some, e.g.,
  - Qualified relations
  - Reflexive, irreflexive, and antisymmetric properties



# OWL2: Syntactic Sugar

- Disjoint classes and disjoint unions

- OWL 1:

```
:Wine owl:equivalentClass [  
  a owl:Class ;  
  owl:unionOf (:RedWine :RoséWine :WhiteWine) ] .  
:RedWine owl:disjointWith :RoséWine, :WhiteWine .  
:RoséWine owl:disjointWith :WhiteWine .
```

- OWL 2:

```
:Wine owl:disjointUnionOf  
  (:RedWine :RoséWine :WhiteWine ) .
```

- Also possible:

```
_:x a owl:AllDisjointClasses ;  
  owl:members (:RedWine :RoséWine WhiteWine ) .
```

# OWL2: Syntactic Sugar

- Negative(Object|Data)PropertyAssertion
- Allow negated statements
- e.g.: Paul is not Peter's father

```
_x [ a owl:NegativeObjectPropertyAssertion;  
      owl:sourceIndividual :Paul ;  
      owl:targetIndividual :Peter ;  
      owl:assertionProperty :fatherOf ] .
```

- If that's syntactic sugar, it must also be possible differently
  - But how?

# OWL2: Syntactic Sugar

- Negative(Object|Data)PropertyAssertion
- Replaces less intuitive set constructs
- Paul is not Peter's father

```
Paul a [ owl:complementOf [  
          a owl:Restriction ;  
          owl:onProperty :fatherOf ;  
          owl:hasValue :Peter  
        ]  
      ].
```

# OWL2: Reflexive Class Restrictions

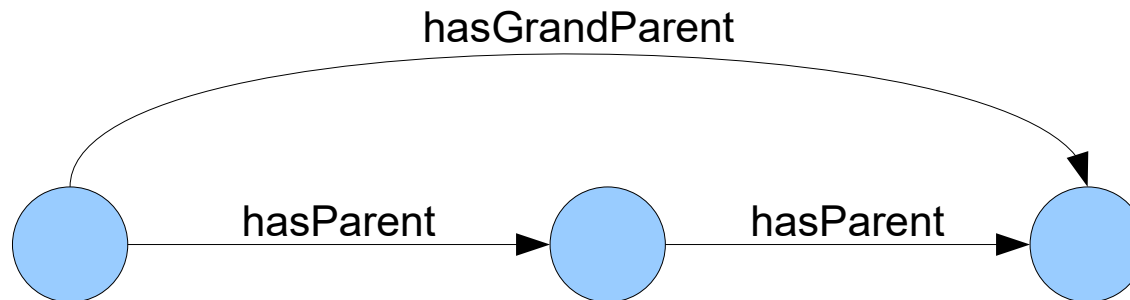
- Using `hasSelf`
- Example: defining the set of all autodidacts:

```
:AutoDidact owl:equivalentClass [  
  a owl:Restriction ;  
  owl:onProperty :teaches ;  
  owl:hasSelf "true"^^xsd:boolean ] .
```



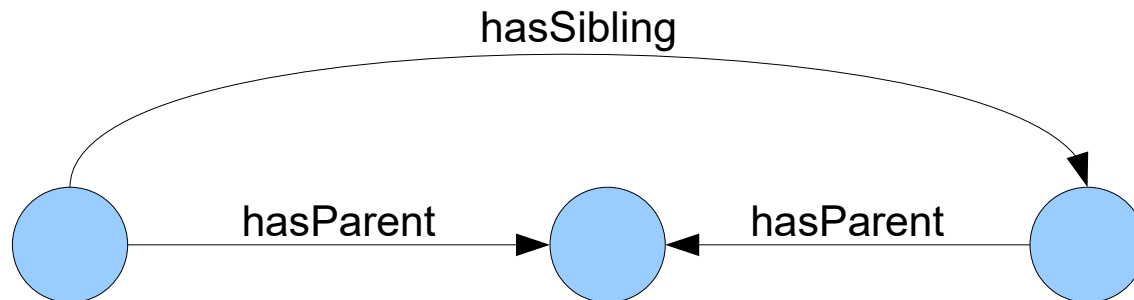
# OWL2: Property Chains

- Typically used for defining rule-like constructs, e.g.
  - `hasParent(X,Y)` and `hasParent(Y,Z) → hasGrandParent(X,Z)`
- OWL Syntax:
  - `:hasGrandparent owl:propertyChainAxiom ( :hasParent :hasParent ) .`



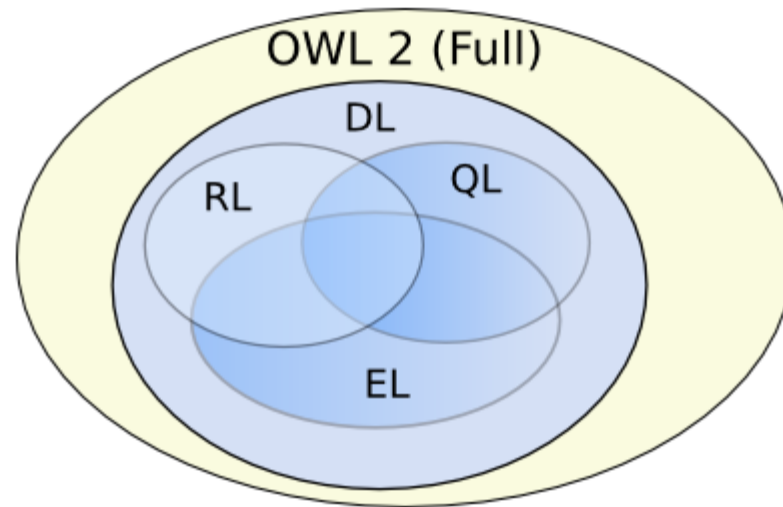
# OWL2: Property Chains

- Can be combined with inverse properties and others
  - $\text{hasParent}(X, Y)$  and  $\text{hasParent}(Z, Y) \rightarrow \text{hasSibling}(X, Z)$
- This is not a proper chain yet, so we have to rephrase it to
  - $\text{hasParent}(X, Y)$  and  $\text{hasParent}^{-1}(Y, Z) \rightarrow \text{hasSibling}(X, Z)$
- OWL Syntax:
  - `:hasSibling owl:propertyChainAxiom ( :hasParent [ owl:inverseOf :hasParent ] ) .`



# OWL2: Profiles

- Profiles are subsets of OWL2 DL
  - EL, RL und QL
  - Similar to complexity classes
- Different runtime and memory complexity
- Depending on requirements



# OWL2 Profile

- OWL2 EL (Expressive Language)
  - Fast reasoning on many standard ontologies
  - Restrictions, e.g.:
    - someValuesFrom, but not allValuesFrom
    - No inverse and symmetric properties
    - No unionOf and complementOf
- OWL2 QL (Query Language)
  - Fast query answering on relational databases
  - Restrictions, e.g.:
    - No unionOf, allValuesFrom, hasSelf, ...
    - No cardinalities and functional properties

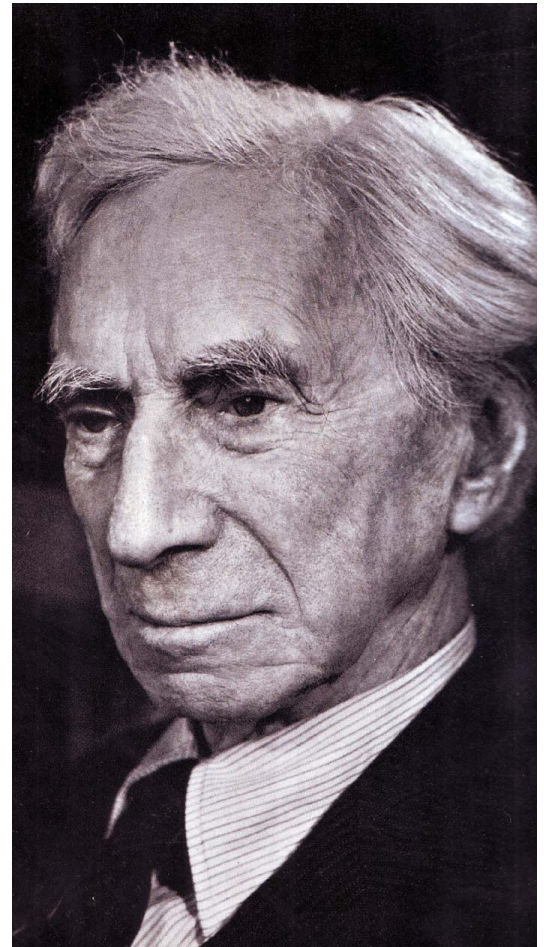
# OWL2 Profile

- OWL2 RL (Rule Language)
  - Subset similar to rule languages such as datalog
    - subClassOf is translated to a rule (Person  $\leftarrow$  Student)
  - Restrictions, e.g.:
    - Only qualified restrictions with 0 or 1
    - Some restrictions for head and body
- The following holds for all three profiles:
  - Reasoning can be implemented in polynomial time for each of the three
  - Reasoning on the union of two profiles only possible in exponential time

# OWL2 Example: Russell's Paradox

- A classic paradox by Bertrand Russell, 1918
- In a city, there is exactly one barber who shaves everybody who does not shave themselves.

Who shaves the barber?



# OWL2 Example: Russell's Paradox

- Class definitions

```
:People owl:disjointUnionOf  
  (:PeopleWhoShaveThemselves  
   :PeopleWhoDoNotShaveThemselves ) .
```

- Relation definitions:

```
:shavedBy rdfs:domain :People .  
:shavedBy rdfs:range :People .  
:shaves owl:inverseOf :shavedBy .
```

- Every person is shaved by exactly one person:

```
:People rdfs:subClassOf [  
  a owl:Restriction ;  
  owl:onProperty :shavedBy ;  
  owl:cardinality "1"^^xsd:integer ] .
```

# OWL2 Example: Russell's Paradox

- Then, we define the barber:

```
:Barbers rdfs:subClassOf :People ;  
    owl:equivalentClass [  
        rdf:type owl:Class ;  
        owl:oneOf ( :theBarber )  
    ] .
```



# OWL2 Example: Russell's Paradox

- Definition of people shaving themselves:

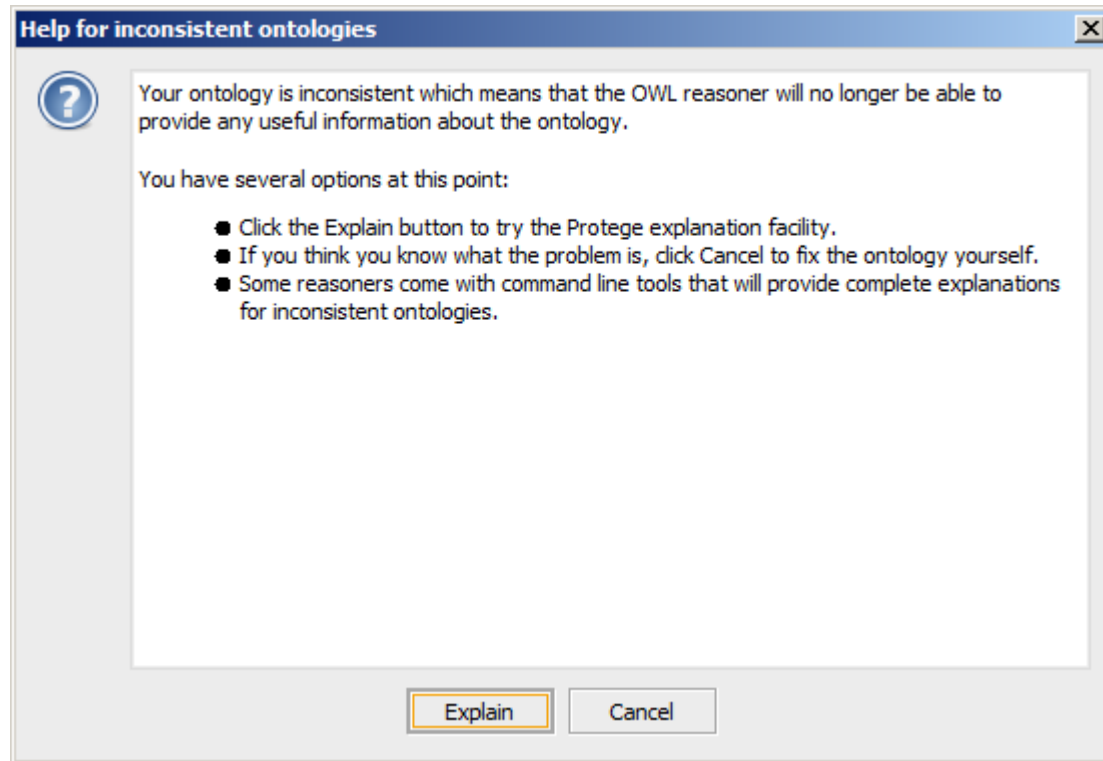
```
:PeopleWhoShaveThemselves owl:equivalentClass [  
  rdf:type owl:Class ;  
  owl:intersectionOf  
    ( :People  
      [  
        a owl:Restriction ;  
        owl:onProperty :shavedBy ;  
        owl:hasSelf "true"^^xsd:boolean  
      ]  
    )  
  ] .
```

# OWL2 Example: Russell's Paradox

- Definition of people who do not shave themselves:

```
:PeopleWhoDoNotShaveThemselves owl:equivalentClass [  
  a owl:Class ;  
  owl:intersectionOf (  
    :People  
    [ a owl:Restriction  
      owl:onProperty :shavedBy ;  
      owl:allValuesFrom :Barbers  
    ]  
  )  
] .
```

# OWL2 Example: Russell's Paradox



# OWL2 Example: Russell's Paradox

**Inconsistent ontology explanation**

☒ Show regular justifications    ☐ All justifications  
☐ Show laconic justifications    ☒ Limit justifications to

Explanation 1    ☐ Display laconic explanation

Explanation for: Thing SubClassOf Nothing

1)	PersonsWhoDoNotShaveThemselves(?x) -> shaves(the-barber, ?x)	In 1 other justifications	?
2)	PersonsWhoDoNotShaveThemselves DisjointWith PersonsWhoShaveThemselves	In ALL other justifications	?
3)	Barber SubClassOf Person	In ALL other justifications	?
4)	shaves(?x, ?x) -> PersonsWhoShaveThemselves(?x)	In ALL other justifications	?
5)	shaves(the-barber, ?x) -> PersonsWhoDoNotShaveThemselves(?x)	In 1 other justifications	?
6)	PersonsWhoShaveThemselves(?x) -> shaves(?x, ?x)	In ALL other justifications	?
7)	Person EquivalentTo PersonsWhoDoNotShaveThemselves or PersonsWhoShaveThemselves	In ALL other justifications	?
8)	the-barber Type Barber	In ALL other justifications	?

OK

# Reasoning in OWL DL

- We have seen reasoning for RDFS
  - Forward chaining algorithm
  - Derive axioms from other axioms
- Limitations of forward chaining
  - `:Motorbike owl:intersectionOf (:TwoWheeledVehicle :MotorVehicle)`  
`:x a :Motorbike`  
→  
`:x a TwoWheeledVehicle, :MotorVehicle .`
  - `:TwoWheeledVehicle owl:unionOf (:Bicycle :Motorbike)`  
`:x a :Motorbike`  
→ ?

# Reasoning in OWL DL

- Reasoning for OWL DL is more difficult
  - Forward chaining may have scalability issues
  - Conjunction (e.g., `unionOf`) is not supported by forward chaining
    - same holds for some other constructs
    - no negation
  - Different approach: Tableau Reasoning
  - Underlying idea: find contradictions in ontology
    - i.e., both a statement and its opposite can be derived from the ontology

# Typical Reasoning Tasks

- What do we want to know from a reasoner?
  - Subclass relations
    - e.g., Are all birds flying animals?
  - Equivalent classes
    - e.g., Are all birds flying animals *and vice versa*?
  - Disjoint classes
    - e.g., Are there animals that are mammals and birds at the same time?
  - Class consistency
    - e.g., Can there be mammals that lay eggs?
  - Class instantiation
    - e.g., Is Flipper a dolphin?
  - Class enumeration
    - e.g., List all dolphins

# Example: A Simple Contradiction

- Given:

```
:Human a owl:Class .
```

```
:Animal a owl:Class .
```

```
:Human owl:disjointWith :Animal .
```

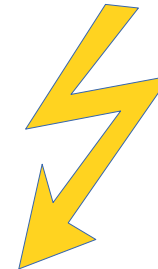
```
:Jimmy a :Animal .
```

```
:Jimmy a :Human .
```



# Example: A Simple Contradiction

- We can derive:
  - $:Human \cap :Animal = \emptyset$   
`owl:Nothing owl:intersectionOf (:Human :Animal) .`
  - $:Jimmy \in (:Human \cap :Animal)$   
`:Jimmy a [ a owl:Class; owl:intersectionOf  
(:Human :Animal)] .`
- i.e.:
  - $:Jimmy \in \emptyset$   
`:Jimmy a owl:Nothing .`
  - That means: the instance must not exist
  - but it does



# Reasoning Tasks Revisited

- Subclass Relations

$\text{Student} \subseteq \text{Person} \Leftrightarrow \text{„Every student is a person“}$

- Proof method: Reductio ad absurdum

- "Invent" an instance  $i$
- Define  $\text{Student}(i)$  and  $\neg \text{Person}(i)$
- Check for contradictions
  - If there is one:  $\text{Student} \subseteq \text{Person}$  has to hold
  - If there is none:  $\text{Student} \subseteq \text{Person}$  cannot be derived
    - Note: it may still hold!

# Example: Subclass Relations

- **Ontology:**

```
:Student owl:subClassOf :UniversityMember .  
:UniversityMember owl:subClassOf :Person .
```

- **Invented instance:**

```
:i a :Student .  
:i a [ owl:complementOf :Person ] .
```

- **We have**

```
:i a :Student .  
:Student owl:subClassOf :UniversityMember .
```

Thus

```
:i a :UniversityMember .
```

- **And from**

```
:UniversityMember owl:subClassOf :Person .
```

- **We further derive that**

```
:i a Person .
```

# Example: Subclass Relations

- Now, we have

```
:i a :Person .  
:i a [ owl:complementOf :Person ] .
```

i.e.,

```
:i a [ owl:intersectionOf  
      (:Person [ owl:complementOf :Person ])] .
```

- from which we derive

```
:i a owl:Nothing .
```



# Reasoning Tasks Revisited

- Class equivalence
  - $\text{Person} \equiv \text{Human}$
- Split into
  - $\text{Person} \subseteq \text{Human}$  and
  - $\text{Human} \subseteq \text{Person}$
- i.e., show subclass relation twice
  - We have seen that
- Class disjointness
  - Are C and D disjoint?
  - "Invent" an instance i
  - Define C(i) and D(i)
    - We have done set (the Jimmy example)

# Class Consistency

- Can a class have instances?

- e.g., married bachelors

```
:Bachelor owl:subClassOf :Man .
```

```
:Bachelor owl:subClassOf
```

```
  [ a owl:Restriction;
```

```
    owl:onProperty :marriedTo;
```

```
    owl:cardinality 0 ] .
```

```
:MarriedPerson owl:subClassOf [
```

```
  a owl:Restriction;
```

```
    owl:onProperty :marriedTo;
```

```
    owl:cardinality 1 ] .
```

```
:MarriedBachelor owl:intersectionOf
```

```
  (:Bachelor :MarriedPerson) .
```

- Now: invent an instance of the class

- And check for contradictions

# Reasoning Tasks Revisited

- Class Instantiation
  - Is Flipper a dolphin?
- Check:
  - define  $\neg \text{Dolphin}(\text{Flipper})$
  - Check for contradiction
- Class enumeration
  - Repeat class instantiation for all known instances

# Typical Reasoning Tasks Revisited

- What do we want to know from a reasoner?
  - Subclass relations
    - e.g., Are all birds flying animals?
  - Equivalent classes
    - e.g., Are all birds flying animals *and vice versa*?
  - Disjoint classes
    - e.g., Are there animals that are mammals and birds at the same time?
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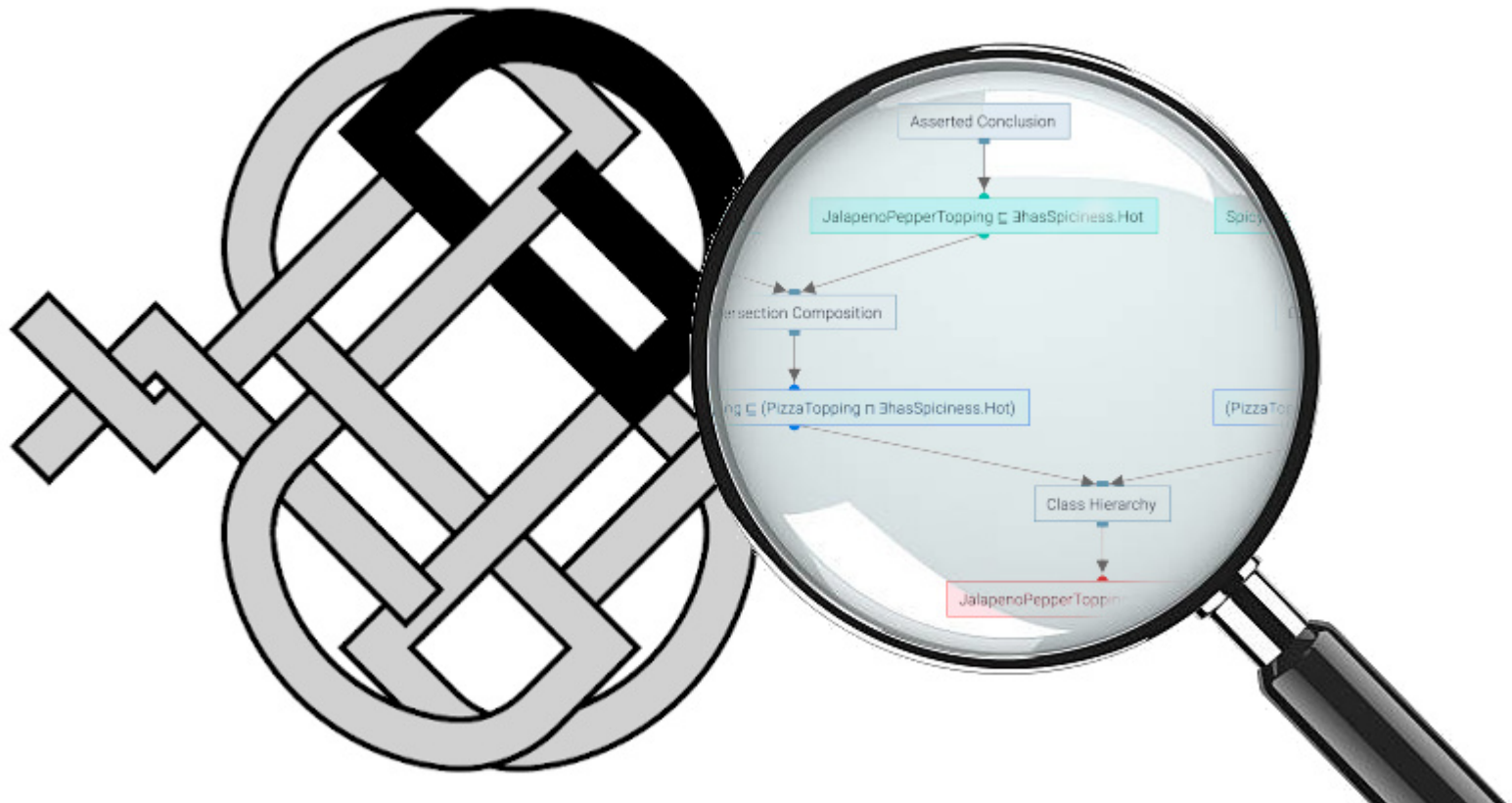


# Typical Reasoning Tasks Revisited

- We have seen
  - All reasoning tasks can be reduced to the same basic task
  - i.e., consistency checking
- This means: for building a reasoner that can solve those tasks,
  - we only need a reasoner capable of consistency checking

# OWL DL

- The DL stands for “Description Logics”
- A logic formalism dating back to the 1980s



# Ontologies in Description Logics Notation

- Classes and Instances

- $C(x) \leftrightarrow x \text{ a } C .$
- $R(x,y) \leftrightarrow x R y .$
- $C \sqsubseteq D \leftrightarrow C \text{ rdfs:subClassOf } D$
- $C \equiv D \leftrightarrow C \text{ owl:equivalentClass } D$
- $C \sqsubseteq \neg D \leftrightarrow C \text{ owl:disjointWith } D$
- $C \equiv \neg D \leftrightarrow C \text{ owl:complementOf } D$
- $C \equiv D \sqcap E \leftrightarrow C \text{ owl:intersectionOf } (D \ E) .$
- $C \equiv D \sqcup E \leftrightarrow C \text{ owl:unionOf } (D \ E) .$
- $T \leftrightarrow \text{owl:Thing}$
- $\perp \leftrightarrow \text{owl:Nothing}$

# Ontologies in Description Logics Notation

- Domains, ranges, and restrictions
  - $\exists R.T \sqsubseteq C \leftrightarrow R \text{ rdfs:domain } C \text{ .}$
  - $\forall R.C \sqsubseteq D \leftrightarrow R \text{ rdfs:range } D \text{ .}$
  - $C \sqsubseteq \forall R.D \leftrightarrow C \text{ owl:subClassOf } [ \text{ a owl:Restriction; owl:onProperty } R; \text{ owl:allValuesFrom } D ] \text{ .}$
  - $C \sqsubseteq \exists R.D \leftrightarrow C \text{ owl:subClassOf } [ \text{ a owl:Restriction; owl:onProperty } R; \text{ owl:someValuesFrom } D ] \text{ .}$
  - $C \sqsubseteq \geq nR \leftrightarrow C \text{ owl:subClassOf } [ \text{ a owl:Restriction; owl:onProperty } R; \text{ owl:minCardinality } n ] \text{ .}$

# Global Statements in Description Logic

- So far, we have seen mostly statements about single classes
  - e.g.,  $C \sqsubseteq D$
- In Description Logics, we can also make global statements
  - e.g.,  $D \sqcup E$
  - This means: every single instance is a member of D or E (or both)
- Those global statements are heavily used in the reasoning process

# Negation Normal Form (NNF)

- Transforming ontologies to Negation Normal Form:
  - $\sqsubseteq$  und  $\equiv$  are not used
  - Negation only for atomic classes and axioms
- A simplified notation of ontologies
- Used by tableau reasoners

# Negation Normal Form (NNF)

- Eliminating  $\sqsubseteq$ :
  - Replace  $C \sqsubseteq D$  by  $\neg C \sqcup D$
  - Note: this is a shorthand notation for  $\forall x: \neg C(x) \vee D(x)$
- Why does this hold?
  - $C \sqsubseteq D$  is equivalent to  $C(x) \rightarrow D(x)$

$C(x)$	$D(x)$	$C(x) \rightarrow D(x)$	$\neg C(x) \vee D(x)$
true	true	true	true
true	false	false	false
false	true	true	true
false	false	true	true

# Negation Normal Form (NNF)

- Eliminating  $\equiv$ :
  - Replace  $C \equiv D$  by  $C \sqsubseteq D$  and  $D \sqsubseteq C$
  - Proceed as before
- i.e.:  $C \equiv D$  becomes
$$C \sqsubseteq D$$
$$D \sqsubseteq C$$
  - and thus
$$\neg C \sqcup D$$
$$\neg D \sqcup C$$

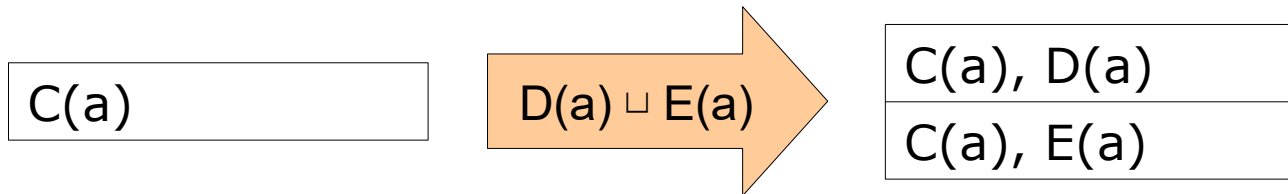


# Negation Normal Form (NNF)

- Further transformation rules
  - $\text{NNF}(C) = C$  (for atomic  $C$ )
  - $\text{NNF}(\neg C) = \neg C$  (for atomic  $C$ )
  - $\text{NNF}(\neg \neg C) = C$
  - $\text{NNF}(C \sqcup D) = \text{NNF}(C) \sqcup \text{NNF}(D)$
  - $\text{NNF}(C \sqcap D) = \text{NNF}(C) \sqcap \text{NNF}(D)$
  - $\text{NNF}(\neg(C \sqcap D)) = \text{NNF}(\neg C) \sqcup \text{NNF}(\neg D)$
  - $\text{NNF}(\neg(C \sqcup D)) = \text{NNF}(\neg C) \sqcap \text{NNF}(\neg D)$
  - $\text{NNF}(\forall R.C) = \forall R.\text{NNF}(C)$
  - $\text{NNF}(\exists R.C) = \exists R.\text{NNF}(C)$
  - $\text{NNF}(\neg \forall R.C) = \exists R.\text{NNF}(\neg C)$
  - $\text{NNF}(\neg \exists R.C) = \forall R.\text{NNF}(\neg C)$

# The Basic Tableau Algorithm

- Tableau: Collection of derived axioms
  - Is subsequently extended
  - As for forward chaining
- In case of conjunction
  - Split the tableau



# When is an Ontology Free of Contradictions?

- Tableau is continuously extended and split
- Free of contradictions if...
  - No further axioms can be created
  - At least one partial tableau is free of contradictions
  - A partial tableau has a contradiction if it contains both an axiom and its negation
    - e.g..  $\text{Person}(\text{Peter})$  und  $\neg\text{Person}(\text{Peter})$
    - The partial tableau is then called *closed*

# The Basic Tableau Algorithm

- Given: an ontology  $O$  in NNF

While not all partial tableaux are closed

- \* Choose a non-closed partial tableau  $T$  and an  $A \in O \cup T$

  If  $A$  is not contained in  $T$

    If  $A$  is an atomic statement

      add  $A$  to  $T$

      back to \*

    If  $A$  is a non-atomic statement

      Choose an individual  $i \in O \cup T$

      Add  $A(i)$  to  $T$

      back to \*

  else

    Extend the tableau with consequences from  $A$

    back to \*

# The Basic Tableau Algorithm

- Extending a tableau with consequences

Nr	Axiom	Action
1	$C(a)$	Add $C(a)$
2	$R(a,b)$	Add $R(a,b)$
3	$C$	Choose an individual $a$ , add $C(a)$
4	$(C \sqcap D)(a)$	Add $C(a)$ and $D(a)$
5	$(C \sqcup D)(a)$	Split tableau into $T1$ and $T2$ . Add $C(a)$ to $T1$ , $D(a)$ to $T2$
6	$(\exists R.C)(a)$	Add $R(a,b)$ and $C(b)$ for a <i>new</i> Individual $b$
7	$(\forall R.C)(a)$	For all $b$ with $R(a,b) \in T$ : add $C(b)$

# A Simple Example

- Given the following ontology:
  - :Animal owl:unionOf (:Mammal :Bird :Fish :Insect :Reptile) .
  - :Animal owl:disjointWith :Human .
  - :Seth a :Human .
  - :Seth a :Insect .
- Is this knowledge graph consistent?

# A Simple Example

- Given the following ontology:

:Animal owl:unionOf (:Mammal :Bird :Fish :Insect :Reptile) .

:Animal owl:disjointWith :Human .

:Seth a :Human .

:Seth a :Insect .

- The same ontology in DL-NNF:

$\neg \text{Animal} \sqcup \neg \text{Human}$

$\text{Animal} \sqcup (\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect} \sqcap \neg \text{Reptile})$

$\neg \text{Animal} \sqcup (\text{Mammal} \sqcup \text{Bird} \sqcup \text{Fish} \sqcup \text{Insect} \sqcup \text{Reptile})$

Human(Seth)

Insect(Seth)

- Let's try how reasoning works now!

# A Simple Example

Human(Seth), Insect(Seth)

Nr	Axiom	Action
1	C(a)	Add C(a)



# A Simple Example

Human(Seth), Insect(Seth),  
 $(\neg \text{Animal} \sqcup \neg \text{Human})(\text{Seth})$

Nr	Axiom	Action
3	C	Choose an individual a, add C(a)

# A Simple Example

Human(Seth), Insect(Seth),  
( $\neg$ Animal  $\sqcup$   $\neg$ Human)(Seth)  
 $\neg$ Animal(Seth)

Human(Seth), Insect(Seth),  
( $\neg$ Animal  $\sqcup$   $\neg$ Human)(Seth)  
 $\neg$ Human(Seth)

Nr	Axiom	Action
5	(C $\sqcup$ D)(a)	Split the tableau into T1 and T2. Add C(a) to T1, D(a) to T2

# A Simple Example

Human(Seth), Insect(Seth),  
( $\neg$ Animal  $\sqcup$   $\neg$ Human)(Seth)  
 $\neg$ Animal(Seth)  
Animal  $\sqcup$  ( $\neg$ Mammal  $\sqcap$   $\neg$ Bird  $\sqcap$   $\neg$ Fish  $\sqcap$   $\neg$ Insect)(Seth)

Human(Seth), Insect(Seth),  
( $\neg$ Animal  $\sqcup$   $\neg$ Human)(Seth)  
 $\neg$ Human(Seth)

Nr	Axiom	Action
3	C	Choose an individual a, add C(a)

# A Simple Example

Human(Seth), Insect(Seth),  
 $(\neg \text{Animal} \sqcup \neg \text{Human})(\text{Seth})$

$\neg \text{Animal}(\text{Seth})$

$\text{Animal} \sqcup (\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect})(\text{Seth})$

$\text{Animal}(\text{Seth})$

Human(Seth), Insect(Seth),  
 $(\neg \text{Animal} \sqcup \neg \text{Human})(\text{Seth})$

$\neg \text{Animal}(\text{Seth})$

$\text{Animal} \sqcup (\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect})(\text{Seth})$

$(\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect})(\text{Seth})$

Human(Seth), Insect(Seth),  
 $(\neg \text{Animal} \sqcup \neg \text{Human})(\text{Seth})$

$\neg \text{Human}(\text{Seth})$

Nr	Axiom	Action
5	$(C \sqcup D)(a)$	Split the tableau into T1 and T2. Add $C(a)$ to T1, $D(a)$ to T2

# A Simple Example

Human(Seth), Insect(Seth),  
 $(\neg \text{Animal} \sqcup \neg \text{Human})(\text{Seth})$   
 $\neg \text{Animal}(\text{Seth})$   
 $\text{Animal} \sqcup (\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect})(\text{Seth})$   
 $\text{Animal}(\text{Seth})$

Human(Seth), Insect(Seth),  
 $(\neg \text{Animal} \sqcup \neg \text{Human})(\text{Seth})$   
 $\neg \text{Animal}(\text{Seth})$   
 $\text{Animal} \sqcup (\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect})(\text{Seth})$   
 $(\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect} \sqcap \neg \text{Reptile})(\text{Seth})$   
 $\neg \text{Mammal}(\text{Seth})$   
 $\neg \text{Bird}(\text{Seth})$   
 $\neg \text{Fish}(\text{Seth})$   
 $\neg \text{Insect}(\text{Seth})$

Nr	Axiom	Action
4	$(C \sqcap D)(a)$	Add C(a) and D(a)
	$(\neg \text{Animal} \sqcup \neg \text{Human})(\text{Seth})$	
	$\neg \text{Human}(\text{Seth})$	

# Another Example

- Again, a simple ontology:

```
:Woman rdfs:subClassOf :Person .  
:Man rdfs:subClassOf :Person .  
:hasChild rdfs:domain :Person .  
:hasChild rdfs:range :Person .  
:Peter :hasChild :Julia .  
:Julia a :Woman .  
:Peter a :Man .
```

# Another Example

- in DL NNF:
  - $\neg \text{Man} \sqcup \text{Person}$
  - $\neg \text{Woman} \sqcup \text{Person}$
  - $\neg \exists \text{hasChild.T} \sqcup \text{Person}$
  - $\forall \text{hasChild.Person}$
  - $\text{hasChild}(\text{Peter}, \text{Julia})$
  - $\text{Woman}(\text{Julia})$
  - $\text{Man}(\text{Peter})$

# Another Example

hasChild(Peter,Julia)

Nr	Axiom	Action
2	$R(a,b)$	Add $R(a,b)$



# Another Example

hasChild(Peter,Julia), Woman(Julia)

Nr	Axiom	Action
1	C(a)	Add C(a)

# Another Example

hasChild(Peter,Julia), Woman(Julia),  
 $(\neg \exists \text{hasChild.T} \sqcup \text{Person})(\text{Peter})$

Nr	Axiom	Action
3	C	Choose an individual a, add C(a)

# Another Example

hasChild(Peter,Julia), Woman(Julia),  
( $\neg\exists\text{hasChild.T} \sqcup \text{Person}$ )(Peter),  
 $\neg\exists\text{hasChild.T}(\text{Peter})$

hasChild(Peter,Julia), Woman(Julia),  
( $\neg\exists\text{hasChild.T}$ )(Peter),  
Person(Peter)

Nr	Axiom	Action
5	$(C \sqcup D)(a)$	Split the tableau into T1 and T2. Add C(a) to T1, D(a) to T2

# Another Example

hasChild(Peter,Julia), Woman(Julia),  
( $\neg\exists\text{hasChild.T} \sqcup \text{Person}$ )(Peter),  
 $\neg\exists\text{hasChild.T}(\text{Peter})$

hasChild(Peter,Julia), Woman(Julia),  
( $\neg\exists\text{hasChild.T}$ )(Peter),  
Person(Peter),  
 $\neg\text{hasChild}(\text{Peter},b0),T(b0)$

Nr	Axiom	Action
6	$(\exists R.C)(a)$	Add $R(a,b)$ und $C(b)$ for a <i>new</i> Individual $b$

# Another Example

hasChild(Peter,Julia), Woman(Julia),  
( $\neg \exists \text{hasChild.T} \sqcup \text{Person}$ )(Peter),  
 $\neg \exists \text{hasChild.T}(\text{Peter})$

hasChild(Peter,Julia), Woman(Julia),  
( $\neg \exists \text{hasChild.T}$ )(Peter),  
Person(Peter),  
 $\neg \text{hasChild}(\text{Peter},b_0),T(b_0),$   
 $\neg \text{hasChild}(\text{Peter},b_1),T(b_1),$   
...

Nr	Axiom	Action
6	$(\exists R.C)(a)$	Add $R(a,b)$ und $C(b)$ for a <i>new</i> Individual $b$

# Introducing Rule Blocking

- Observation
  - The tableau algorithm does not necessarily terminate
  - We can add arbitrarily many new axioms

Nr	Axiom	Action
6	$(\exists R.C)(a)$	Add $R(a,b)$ und $C(b)$ for a <i>new</i> Individual $b$

- Idea: avoid rule 6 if no new information is created
  - i.e., if we already created one instance  $b_a$  for instance  $a$ , then block using rule 6 for  $a$ .

# Tableau Algorithm with Rule Blocking

- Given: an ontology  $O$  in NNF

While not all partial tableaux are closed

and further axioms can be created

- \* Choose a non-closed partial tableau  $T$  and a **non-blocked**  $A \in O \cup T$

If  $A$  is not contained in  $T$

If  $A$  is an atomic statement

add  $A$  to  $T$

back to \*

If  $A$  is a non-atomic statement

Choose an individual  $i \in O \cup T$

Add  $A(i)$  to  $T$

back to \*

else

Extend the tableau with consequences from  $A$

**If rule 6 was used, block  $A$  for  $T$**

back to \*

# Example with Rule Blocking

hasChild(Peter,Julia), Woman(Julia),  
( $\neg\exists\text{hasChild.T} \sqcup \text{Person}$ )(Peter),  
 $\neg\exists\text{hasChild.T}(\text{Peter})$

hasChild(Peter,Julia), Woman(Julia),  
( $\neg\exists\text{hasChild.T}$ )(Peter),  
Person(Peter),  
 $\neg\text{hasChild}(\text{Peter},b0),T(b0)$

now it will  
terminate  
ultimately

Nr	Axiom	Action
6	$(\exists R.C)(a)$	Add $R(a,b)$ und $C(b)$ for a <i>new</i> Individual $b$ , <b>block rule 6 for <math>a</math></b>



# Tableau Algorithm: Wrap Up

- An algorithm for description logic based ontologies
  - works for OWL Lite and DL
- We have seen examples for some OWL expressions
  - Other OWL DL expressions can be “translated” to DL as well
  - And they come with their own expansion rules
  - Reasoning may become more difficult
    - e.g., dynamic blocking and unblocking

# Optimizing Tableau Reasoners

- Given: an ontology  $O$  in NNF

While not all partial tableaux are closed

and further axioms can be created

\* Choose a non-closed partial tableau  $T$  and a non-blocked  $A \in O \cup T$

If  $A$  is not contained in  $T$

If  $A$  is an atomic statement

add  $A$  to  $T$

back to \*

If  $A$  is a non-atomic statement

Choose an individual  $i \in O \cup T$

Add  $A(i)$  to  $T$

back to \*

else

Extend the tableau with consequences from  $A$

If rule 6 was used, block  $A$  for  $T$

back to \*

# OWL Lite vs DL Revisited

- Recap: OWL Lite has some restrictions
  - Those are meant to allow for faster reasoning
- Restrictions only with cardinalities 0 and 1
  - Higher cardinalities make blocking more complex
- `unionOf`, `disjointWith`, `complementOf`, closed classes, ...
  - they all introduce more disjunctions
  - i.e., more splitting operations

# Complexity of Ontologies

- Reasoning is usually expensive
- Reasoning performance depends on ontology complexity
  - Rule of thumb: the more complexity, the more costly
- Most useful ontologies are in OWL DL
  - But there are differences
  - In detail: complexity classes

# Simple Ontologies: ALC

- ALC: Attribute Language with Complement
- Allowed:
  - subClassOf, equivalentClass
  - unionOf, complementOf, disjointWith
  - Restrictions: allValuesFrom, someValuesFrom
  - domain, range
  - Definition of individuals

# SHIQ, SHOIN & co

- Complexity classes are noted as letter sequences
- Using
  - S = ALC plus transitive properties (basis for most ontologies)
  - H = Property hierarchies (subPropertyOf)
  - O = closed classes (oneOf)
  - I = inverse properties (inversePropertyOf)
  - N = numeric restrictions (min/maxCardinality)
  - F = functional properties
  - Q = qualified numerical restrictions (OWL2)
  - (D) = Usage of datatype properties

# Some Tableau Reasoners

- Fact
  - University of Manchester, free
  - SHIQ
- Fact++/JFact
  - Extension of Fact, free
  - SHOIQ(and a little D), OWL-DL + OWL2
- Pellet
  - Clark & Parsia, free for academic use
  - SHOIN(D), OWL-DL + OWL2
- RacerPro
  - Racer Systems, commercial
  - SHIQ(D)

# Sudoku Revisited

- Recap: we used a closed class
  - Plus some disjointness
- Resulting complexity: SO
- Which reasoners do support that?
  - Fact: SHIQ :-()
  - RacerPro: SHIQ(D) :-()
  - Pellet: SHOIN(D) :-()
  - HermiT: SHOIQ :-()

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9



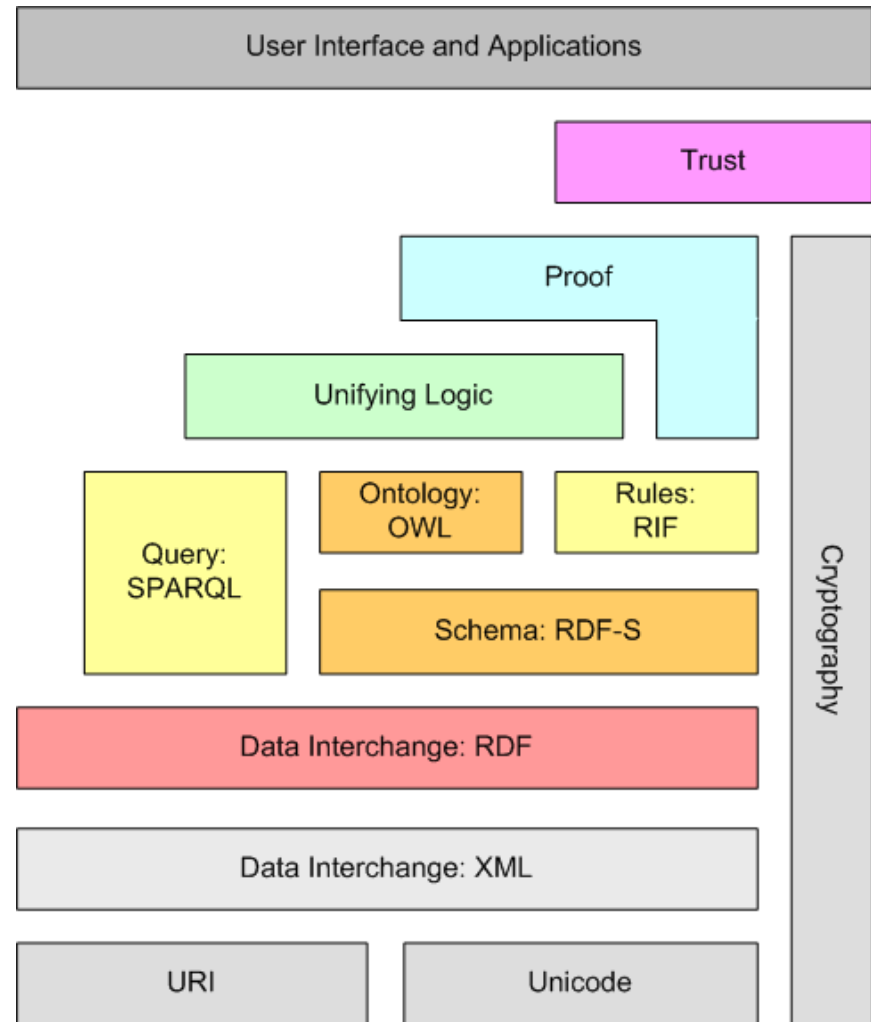
# Rules: Beyond OWL



here be dragons...

Semantic Web  
Technologies  
(This lecture)

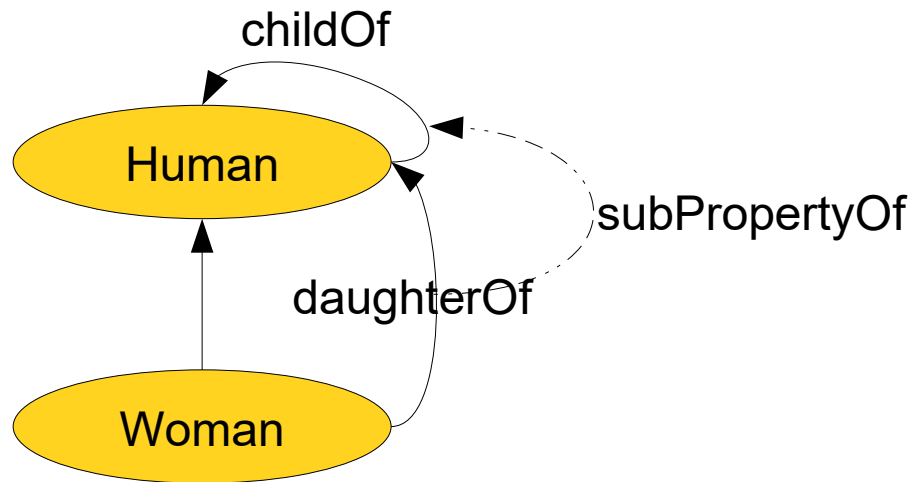
Technical  
Foundations



Berners-Lee (2009): *Semantic Web and Linked Data*  
<http://www.w3.org/2009/Talks/0120-campus-party-tbl/>

# Limitations of OWL

- Some things are hard or impossible to express in OWL
- Example:
  - If A is a woman and the child of B  
then A is the daughter of B



# Limitations of OWL

- Let's try this in OWL:

```
:Woman rdfs:subClassOf :Human .  
:childOf      a owl:ObjectProperty ;  
               rdfs:domain :Human ;  
               rdfs:range  :Human .  
:daughterOf a owl:ObjectProperty ;  
            rdfs:subPropertyOf :childOf ;  
            rdfs:domain :Woman .
```

# Limitations of OWL

- What can a reasoner conclude with this ontology?

- Example:

`:Julia :daughterOf :Peter .`

`→ :Julia a :Woman .`

- What we would like to have instead:

`:Julia :childOf :Peter .`

`:Julia a :Woman .`

`→ :Julia :daughterOf :Peter .`

# Limitations of OWL

- What we would like to have:  
$$\text{daughterOf}(X,Y) \leftarrow \text{childOf}(X,Y) \wedge \text{Woman}(X) .$$
- Rules are flexible
- There are rules in the Semantic Web, e.g.
  - Semantic Web Rule Language (SWRL)
  - Rule Interchange Format (RIF)
  - See lecture in a few weeks
- Some reasoners do (partly) support rules

# Wrap Up

- OWL comes in many flavours
  - OWL Lite, OWL DL, OWL Full
  - Detailed complexity classes of OWL DL
  - Additions and profiles from OWL2
  - However, there are still some things that cannot be expressed...
- Reasoning is typically done using the Tableau algorithm

# Questions?

