

# Web Ontology Language (OWL) Part II

## IE650 Knowledge Graphs



# Previously on “Knowledge Graphs”

- We have got to know
  - OWL, a more powerful ontology language than RDFS
  - Simple ontologies and some reasoning
  - Sudoku solving
- Today
  - New constructs in OWL2
  - Russell's paradox
  - Reasoning in OWL
  - Complexity of ontologies
  - A peek at rule languages for Knowledge Graphs



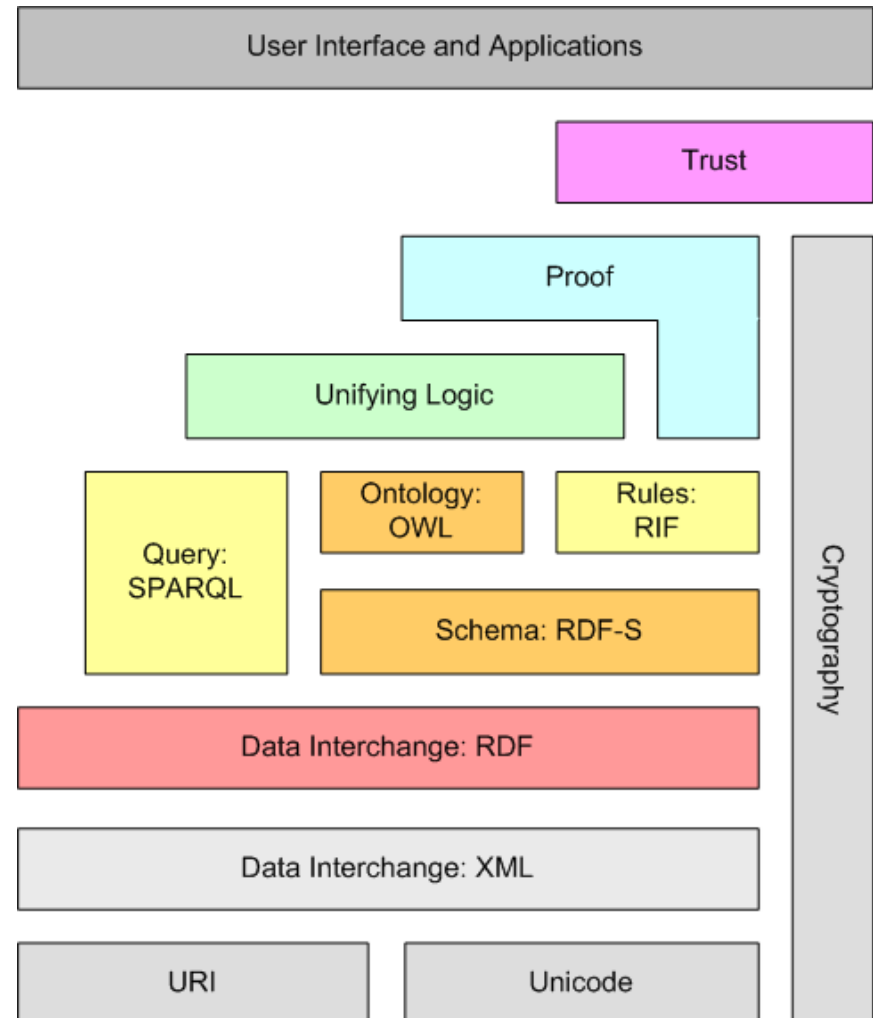
# Semantic Web Technology Stack



here be dragons...

Knowledge Graph Technologies  
(This lecture)

Technical  
Foundations



# OWL2 – New Constructs and More

- Five years after the first OWL standard
- OWL2: 2009
  - Syntactic sugar
  - New language constructs
  - OWL profiles
- We have already encountered some, e.g.,
  - Qualified restrictions
  - Reflexive, irreflexive, and antisymmetric properties



# OWL2: Syntactic Sugar

- Disjoint classes and disjoint unions

- OWL 1:

```
:Wine owl:equivalentClass [  
  a owl:Class ;  
  owl:unionOf (:RedWine :RoséWine :WhiteWine) ].  
  
:RedWine owl:disjointWith :RoséWine, :WhiteWine.  
:RoséWine owl:disjointWith :WhiteWine .
```

- OWL 2:

```
:Wine owl:disjointUnionOf  
  (:RedWine :RoséWine :WhiteWine ) .
```

- Also possible:

```
_ :x a owl:AllDisjointClasses ;  
  owl:members (:RedWine :RoséWine WhiteWine ) .
```

# OWL2: Syntactic Sugar

- Negative(Object | Data)PropertyAssertion
- Allow negated statements
- e.g.: Paul is not Peter's father

```
_:x [  
  a owl:NegativeObjectPropertyAssertion;  
  owl:sourceIndividual :Paul ;  
  owl:targetIndividual :Peter ;  
  owl:assertionProperty :fatherOf  
] .
```

- If that's syntactic sugar, it must also be possible differently
  - But how?

# OWL2: Syntactic Sugar

- Negative(Object|Data)PropertyAssertion
- Replaces less intuitive set constructs
- Paul is not Peter's father

```
:Paul a [  
    owl:complementOf [  
        a owl:Restriction ;  
        owl:onProperty :fatherOf ;  
        owl:hasValue :Peter  
    ]  
].
```

# OWL2: Reflexive Class Restrictions

- Using `hasSelf`
- Example: defining the set of all autodidacts:

```
:AutoDidact owl:equivalentClass [  
  a owl:Restriction ;  
  owl:onProperty :teaches ;  
  owl:hasSelf "true"^^xsd:boolean  
] .
```



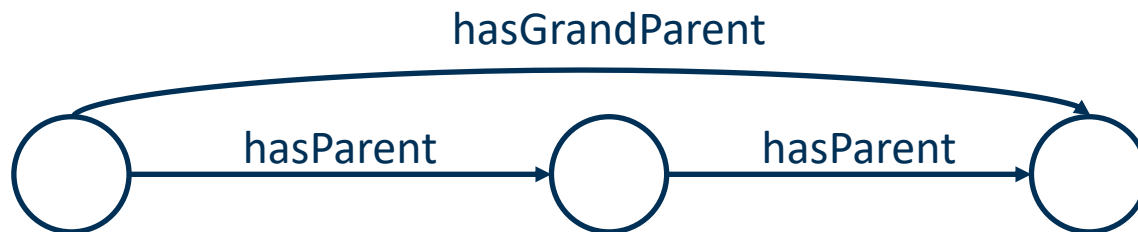
# OWL2: Property Chains

- Typically used for defining rule-like constructs, e.g.

`hasParent(X,Y) and hasParent(Y,Z) →  
hasGrandParent(X,Z)`

- OWL Syntax:

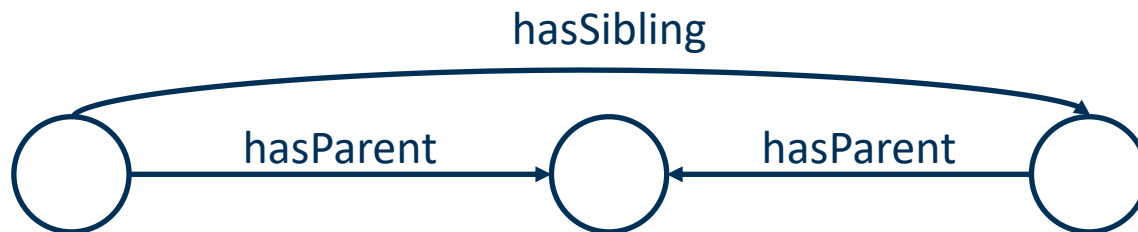
```
:hasGrandparent owl:propertyChainAxiom  
  ( :hasParent :hasParent ) .
```



# OWL2: Property Chains

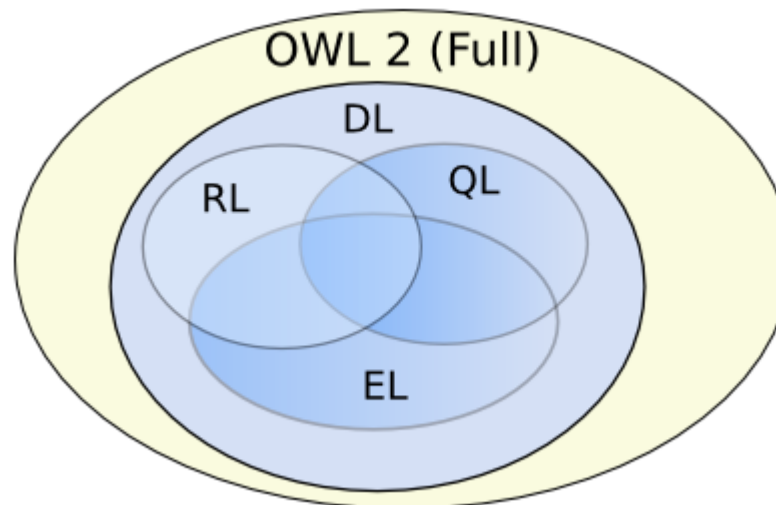
- Can be combined with inverse properties and others  
 $\text{hasParent}(X, Y) \text{ and } \text{hasParent}(Z, Y) \rightarrow \text{hasSibling}(X, Z)$
- This is not a proper chain yet, so we have to rephrase it to  
 $\text{hasParent}(X, Y) \text{ and } \text{hasParent}^{-1}(Y, Z) \rightarrow \text{hasSibling}(X, Z)$
- OWL Syntax:  

```
:hasSibling owl:propertyChainAxiom  
  ( :hasParent [ owl:inverseOf :hasParent ] ) .
```



# OWL2: Profiles

- Profiles are subsets of OWL2 DL
  - EL, RL und QL
  - Similar to complexity classes
- Different runtime and memory complexity
- Depending on requirements



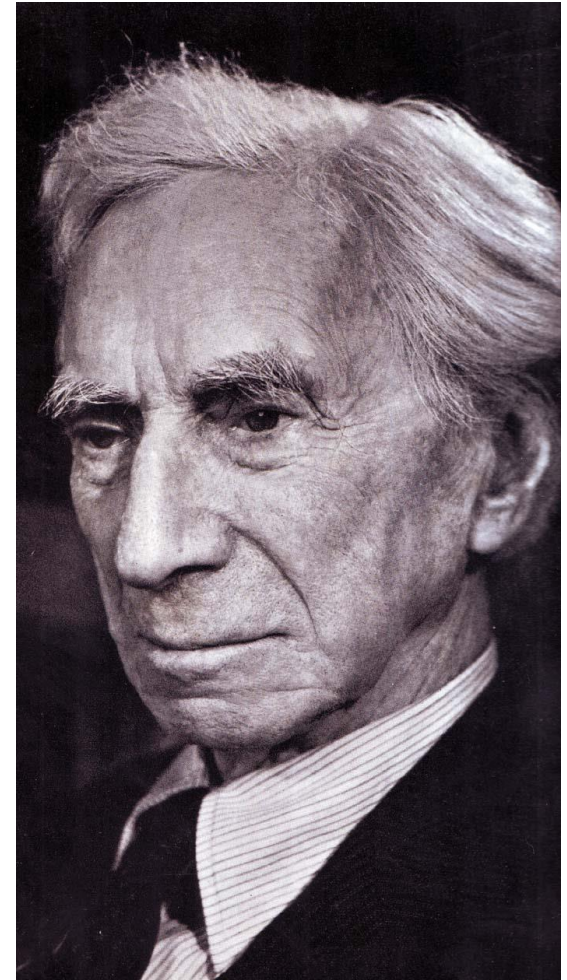
- OWL2 EL (Expressive Language)
  - Fast reasoning on many standard ontologies
  - Restrictions, e.g.:
    - someValuesFrom, but not allValuesFrom
    - No inverse and symmetric properties
    - No unionOf and complementOf
- OWL2 QL (Query Language)
  - Fast query answering on relational databases
  - Restrictions, e.g.:
    - No unionOf, allValuesFrom, hasSelf, ...
    - No cardinalities and functional properties

- OWL2 RL (Rule Language)
  - Subset similar to rule languages such as datalog
    - subClassOf is translated to a rule (Person  $\leftarrow$  Student)
  - Restrictions, e.g.:
    - Only qualified restrictions with 0 or 1
    - Some restrictions for head and body
- The following holds for all three profiles:
  - Reasoning can be implemented in polynomial time for each of the three
  - Reasoning on the union of two profiles only possible in exponential time

# OWL2 Example: Russell's Paradox

- A classic paradox by Bertrand Russell, 1918
- In a city, there is exactly one barber who shaves everybody who does not shave themselves.

Who shaves the barber?



# OWL2 Example: Russell's Paradox

- In a city, there is exactly one barber who shaves everybody who does not shave themselves.

Who shaves the barber?

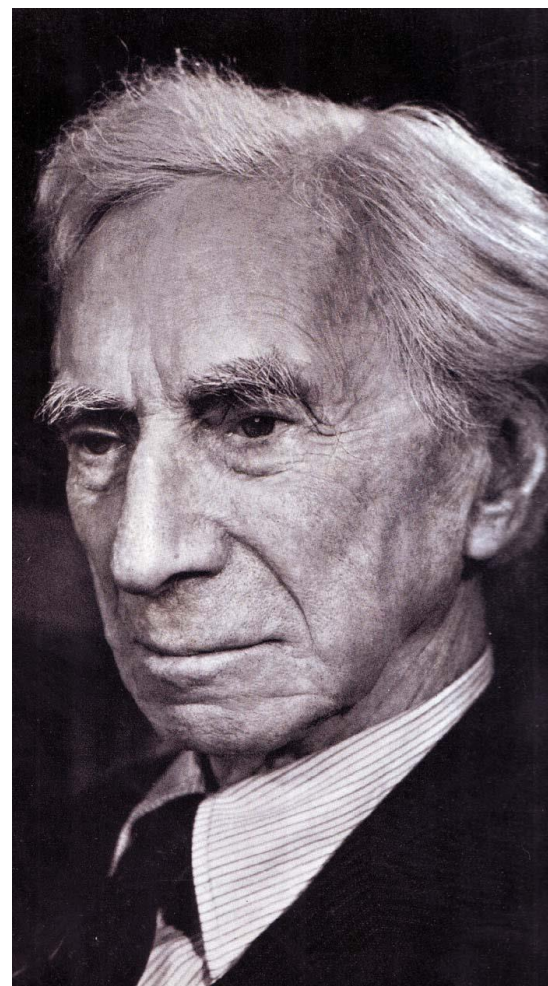
- Assume:

– The barber shave himself:

- he ceases to be the barber specified because he only shaves those who do not shave themselves

– The barber **does not** shave himself:

- then he fits into the group of people who would be shaved by the specified barber, and thus, as that barber, he must shave himself.



# OWL2 Example: Russell's Paradox

- Class definitions

```
:People owl:disjointUnionOf  
  (:PeopleWhoShaveThemselves  
   :PeopleWhoDoNotShaveThemselves ) .
```

- Relation definitions:

```
:shavedBy rdfs:domain :People .  
:shavedBy rdfs:range :People .  
:shaves owl:inverseOf :shavedBy .
```

- Every person is shaved by exactly one person:

```
:People rdfs:subClassOf [  
  a owl:Restriction ;  
  owl:onProperty :shavedBy ;  
  owl:cardinality "1"^^xsd:integer  
] .
```



# OWL2 Example: Russell's Paradox

- Then, we define the barber:

```
:Barbers rdfs:subClassOf :People ;  
    owl:equivalentClass [  
        rdf:type owl:Class ;  
        owl:oneOf ( :theBarber )  
    ] .
```

# OWL2 Example: Russell's Paradox

- Definition of people shaving themselves:

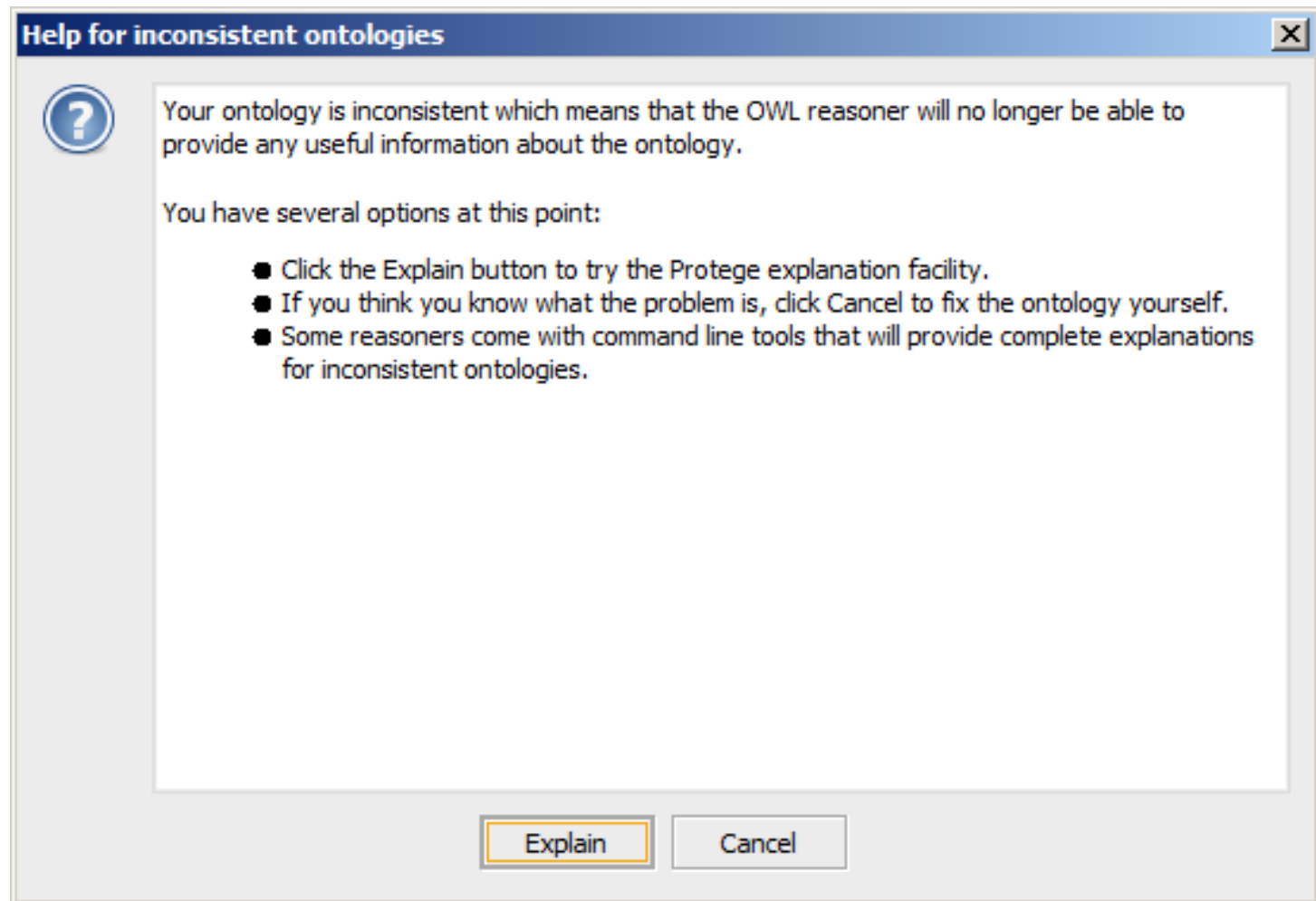
```
:PeopleWhoShaveThemselves owl:equivalentClass [  
  a owl:Class ;  
  owl:intersectionOf (  
    :People  
    [  
      a owl:Restriction ;  
      owl:onProperty :shavedBy ;  
      owl:hasSelf "true"^^xsd:boolean  
    ]  
  )  
] .
```

# OWL2 Example: Russell's Paradox



- Definition of people who do not shave themselves:

```
:PeopleWhoDoNotShaveThemselves owl:equivalentClass [  
  a owl:Class ;  
  owl:intersectionOf (  
    :People  
    [  
      a owl:Restriction  
      owl:onProperty :shavedBy ;  
      owl:allValuesFrom :Barbers  
    ]  
  )  
] .
```

# OWL2 Example: Russell's Paradox



# OWL2 Example: Russell's Paradox

 **Inconsistent ontology explanation** 

☒ Show regular justifications

☐ All justifications

☐ Show laconic justifications

☒ Limit justifications to

1

▲

▼

Explanation 1 ☐ Display laconic explanation

Explanation for: Thing SubClassOf Nothing

1)	PersonsWhoDoNotShaveThemselves(?x) -> shaves(the-barber, ?x)	In 1 other justifications	?
2)	PersonsWhoDoNotShaveThemselves DisjointWith PersonsWhoShaveThemselves	In ALL other justifications	?
3)	Barber SubClassOf Person	In ALL other justifications	?
4)	shaves(?x, ?x) -> PersonsWhoShaveThemselves(?x)	In ALL other justifications	?
5)	shaves(the-barber, ?x) -> PersonsWhoDoNotShaveThemselves(?x)	In 1 other justifications	?
6)	PersonsWhoShaveThemselves(?x) -> shaves(?x, ?x)	In ALL other justifications	?
7)	Person EquivalentTo PersonsWhoDoNotShaveThemselves or PersonsWhoShaveThemselves	ALL other justifications	?
8)	the-barber Type Barber	In ALL other justifications	?

OK

# Reasoning in OWL DL

- We have seen reasoning for RDFS
  - Forward chaining algorithm
  - Derive axioms from other axioms
- Limitations of forward chaining
  - `:Motorbike owl:intersectionOf (:TwoWheeledVehicle :MotorVehicle).`  
`:x a :Motorbike.`  
→  
`:x a TwoWheeledVehicle, :MotorVehicle .`
  - `:TwoWheeledVehicle owl:unionOf (:Bicycle :Motorbike).`  
`:x a :TwoWheeledVehicle`  
→  
?

# Reasoning in OWL DL

- Reasoning for OWL DL is more difficult
  - Forward chaining may have scalability issues
  - Disjunction (e.g., unionOf) is not supported by forward chaining
    - Same holds for some other constructs
    - No negation
  - Different approach: Tableau Reasoning
  - Underlying idea: find contradictions in ontology
    - i.e., both a statement and its opposite can be derived from the ontology

# Reasoning in OWL DL

- What do we want to know from a reasoner?
  - Subclass relations
    - e.g., Are all birds flying animals?
  - Equivalent classes
    - e.g., Are all birds flying animals *and vice versa*?
  - Disjoint classes
    - e.g., Are there animals that are mammals and birds at the same time?
  - Class consistency
    - e.g., Can there be mammals that lay eggs?
  - Class instantiation
    - e.g., Is Flipper a dolphin?
  - Class enumeration
    - e.g., List all dolphins



# Example: A Simple Contradiction

- Given:

```
:Human a owl:Class .  
:Animal a owl:Class .  
:Human owl:disjointWith :Animal .  
  
:Jimmy a :Animal .  
:Jimmy a :Human .
```

# Example: A Simple Contradiction

- We can derive:
  - $\text{:Human} \cap \text{:Animal} = \emptyset$   
`owl:Nothing owl:intersectionOf (:Human :Animal) .`
  - $\text{:Jimmy} \in (\text{:Human} \cap \text{:Animal})$   
`:Jimmy a [  
 a owl:Class;  
 owl:intersectionOf (:Human :Animal)  
] .`
- i.e.:
  - $\text{:Jimmy} \in \emptyset$   
`:Jimmy a owl:Nothing .`
  - That means: the instance must not exist
  - But it does



# Reasoning Tasks Revisited

- Subclass Relations

$\text{Student} \subseteq \text{Person} \Leftrightarrow \text{„Every student is a person“}$

- Proof method: Reductio ad absurdum

- "Invent" an instance  $i$
- Define  $\text{Student}(i)$  and  $\neg \text{Person}(i)$
- Check for contradictions
  - If there is one:  $\text{Student} \subseteq \text{Person}$  has to hold
  - If there is none:  $\text{Student} \subseteq \text{Person}$  cannot be derived
    - Note: it may still hold!

# Example: Subclass Relations

- **Ontology:**  
`:Student owl:subClassOf :UniversityMember .`  
`:UniversityMember owl:subClassOf :Person .`
- **Invented instance:**  
`:i a :Student .`  
`:i a [ owl:complementOf :Person ] .`
- **We have**  
`:i a :Student .`  
`:Student owl:subClassOf :UniversityMember .`
- **Thus** `:i a :UniversityMember .`
- **And from**  
`:UniversityMember owl:subClassOf :Person .`
- **We further derive that** `:i a Person .`

# Example: Subclass Relations

- Now, we have

```
:i a :Person .
```

```
:i a [ owl:complementOf :Person ] .
```

i.e.,

```
:i a [  
    owl:intersectionOf (  
        :Person  
        [ owl:complementOf :Person ]  
    )  
] .
```

- from which we derive

```
:i a owl:Nothing .
```



# Reasoning Tasks Revisited

- Class equivalence
  - $\text{Person} \equiv \text{Human}$
- Split into
  - $\text{Person} \subseteq \text{Human}$  and
  - $\text{Human} \subseteq \text{Person}$
- i.e., show subclass relation twice
  - We have seen that
- Class disjointness
  - Are C and D disjoint?
  - "Invent" an instance i
  - Define C(i) and D(i)
    - We have done set (the Jimmy example)

# Class Consistency

- Can a class have instances? e.g., married bachelors

```
:Bachelor owl:subClassOf :Man,  
  [ a owl:Restriction;  
    owl:onProperty :marriedTo;  
    owl:cardinality 0 ] .
```

```
:MarriedPerson owl:subClassOf [  
  a owl:Restriction;  
  owl:onProperty :marriedTo;  
  owl:cardinality 1 ] .
```

```
:MarriedBachelor owl:intersectionOf  
  (:Bachelor :MarriedPerson) .
```

- Now: invent an instance of the class
  - And check for contradictions

# Reasoning Tasks Revisited

- Class Instantiation
  - Is Flipper a dolphin?
- Check:
  - Define  $\neg \text{Dolphin(Flipper)}$
  - Check for contradiction
- Class enumeration
  - Repeat class instantiation for all known instances



# Typical Reasoning Tasks Revisited

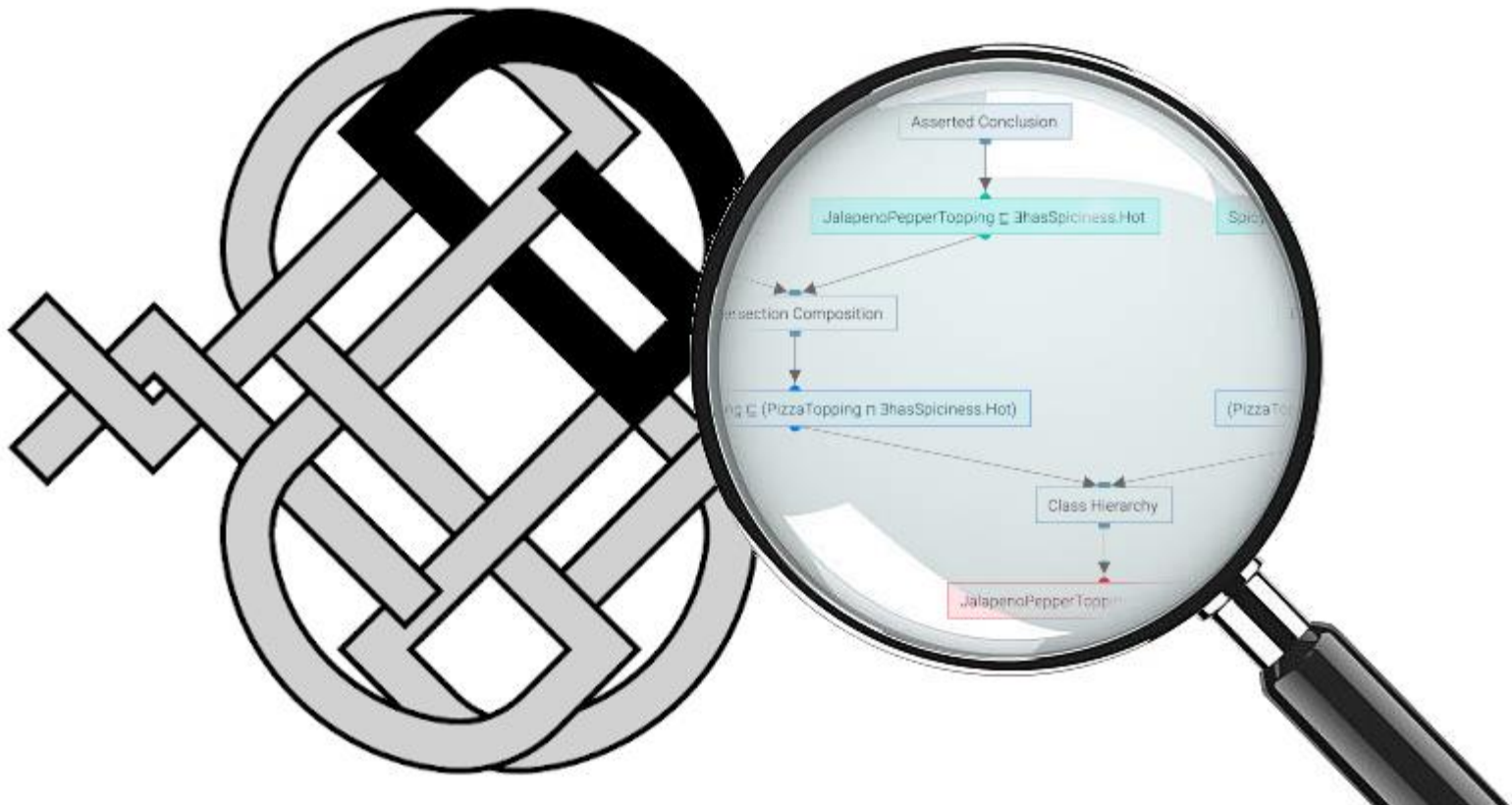
- What do we want to know from a reasoner?
  - Subclass relations
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  - Disjoint classes
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  - Class consistency
    - e.g., Can there be mammals that lay eggs?
  - Class instantiation
    - e.g., Is Flipper a dolphin?
  - Class enumeration
    - e.g., List all dolphins

# Typical Reasoning Tasks Revisited

- We have seen
  - All reasoning tasks can be reduced to the same basic task
  - i.e., consistency checking
- This means:  
for building a reasoner that can solve those tasks,
  - We only need a reasoner capable of consistency checking

# OWL DL

- The DL stands for “Description Logics”
- A logic formalism dating back to the 1980s



# Ontologies in Description Logics

## Notation

- Classes and Instances
  - $C(x) \leftrightarrow x \text{ a } C .$
  - $R(x,y) \leftrightarrow x \text{ R } y .$
  - $C \sqsubseteq D \leftrightarrow C \text{ rdfs:subClassOf } D$
  - $C \equiv D \leftrightarrow C \text{ owl:equivalentClass } D$
  - $C \sqsubseteq \neg D \leftrightarrow C \text{ owl:disjointWith } D$
  - $C \equiv \neg D \leftrightarrow C \text{ owl:complementOf } D$
  - $C \equiv D \sqcap E \leftrightarrow C \text{ owl:intersectionOf } (D \ E) .$
  - $C \equiv D \sqcup E \leftrightarrow C \text{ owl:unionOf } (D \ E) .$
  - $T \leftrightarrow \text{owl:Thing}$
  - $\perp \leftrightarrow \text{owl:Nothing}$

# Ontologies in Description Logics

## Notation

- Domains, ranges, and restrictions
  - $\exists R.T \sqsubseteq C \leftrightarrow R \text{ rdfs:domain } C$  .
  - $\forall R.C \leftrightarrow R \text{ rdfs:range } C$  .
  - $C \sqsubseteq \forall R.D \leftrightarrow C \text{ rdfs:subClassOf } [$   
    `a owl:Restriction;`  
    `owl:onProperty R;`  
    `owl:allValuesFrom D ]` .
  - $C \sqsubseteq \exists R.D \leftrightarrow C \text{ rdfs:subClassOf } [$   
    `a owl:Restriction;`  
    `owl:onProperty R;`  
    `owl:someValuesFrom D ]` .
  - $C \sqsubseteq \geq nR \leftrightarrow C \text{ rdfs:subClassOf } [$   
    `a owl:Restriction;`  
    `owl:onProperty R;`  
    `owl:minCardinality n ]` .

# Global Statements in Description Logic

- So far, we have seen mostly statements about single classes
  - e.g.,  $C \sqsubseteq D$
- In Description Logics, we can also make global statements
  - e.g.,  $D \sqcup E$
  - This means: every single instance is a member of D or E (or both)
- Those global statements are heavily used in the reasoning process

# Negation Normal Form (NNF)

- Transforming ontologies to Negation Normal Form:
  - $\sqsubseteq$  und  $\sqsupseteq$  are not used
  - Negation only for atomic classes and axioms
- A simplified notation of ontologies
- Used by tableau reasoners

# Negation Normal Form (NNF)

- Eliminating  $\sqsubseteq$ :
  - Replace  $C \sqsubseteq D$  by  $\neg C \sqcup D$
  - Note: this is a shorthand notation for  $\forall x: \neg C(x) \vee D(x)$
- Why does this hold?
  - $C \sqsubseteq D$  is equivalent to  $C(x) \rightarrow D(x)$

$C(x)$	$D(x)$	$C(x) \rightarrow D(x)$	$\neg C(x) \vee D(x)$
true	true	true	true
true	false	false	false
false	true	true	true
false	false	true	true



# Negation Normal Form (NNF)

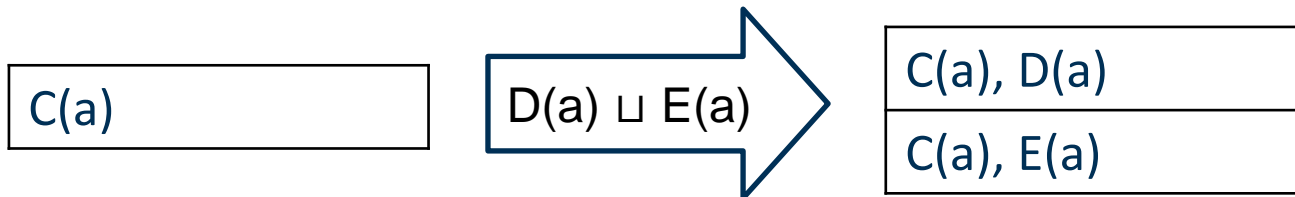
- Eliminating  $\equiv$ 
  - Replace  $C \equiv D$  by  $C \sqsubseteq D$  and  $D \sqsubseteq C$
  - Proceed as before
- i.e.:  $C \equiv D$  becomes
  - $C \sqsubseteq D$
  - $D \sqsubseteq C$
  - and thus
    - $\neg C \sqcup D$
    - $\neg D \sqcup C$

# Negation Normal Form (NNF)

- Further transformation rules
  - $\text{NNF}(C) = C$  (for atomic  $C$ )
  - $\text{NNF}(\neg C) = \neg C$  (for atomic  $C$ )
  - $\text{NNF}(\neg \neg C) = C$
  - $\text{NNF}(C \sqcup D) = \text{NNF}(C) \sqcup \text{NNF}(D)$
  - $\text{NNF}(C \sqcap D) = \text{NNF}(C) \sqcap \text{NNF}(D)$
  - $\text{NNF}(\neg(C \sqcap D)) = \text{NNF}(\neg C) \sqcup \text{NNF}(\neg D)$
  - $\text{NNF}(\neg(C \sqcup D)) = \text{NNF}(\neg C) \sqcap \text{NNF}(\neg D)$
  - $\text{NNF}(\forall R.C) = \forall R.\text{NNF}(C)$
  - $\text{NNF}(\exists R.C) = \exists R.\text{NNF}(C)$
  - $\text{NNF}(\neg \forall R.C) = \exists R.\text{NNF}(\neg C)$
  - $\text{NNF}(\neg \exists R.C) = \forall R.\text{NNF}(\neg C)$

# The Basic Tableau Algorithm

- Tableau: Collection of derived axioms
  - Is subsequently extended
  - As for forward chaining
- In case of conjunction
  - Split the tableau



# When is an Ontology Free of Contradictions?

- Tableau is continuously extended and split
- Free of contradictions if...
  - No further axioms can be created
  - At least one partial tableau is free of contradictions
  - A partial tableau has a contradiction if it contains both an axiom and its negation
    - e.g.  $\text{Person}(\text{Peter})$  und  $\neg \text{Person}(\text{Peter})$
    - The partial tableau is then called *closed*

# The Basic Tableau Algorithm

- Given: an ontology  $O$  in NNF
- While not all partial tableaux are closed
  - \* Choose a non-closed partial tableau  $T$  and an  $A \in O \cup T$ 
    - If  $A$  is not contained in  $T$ 
      - If  $A$  is an atomic statement
        - add  $A$  to  $T$
        - back to \*
      - If  $A$  is a non-atomic statement
        - Choose an individual  $i \in O \cup T$
        - Add  $A(i)$  to  $T$
        - back to \*
    - else
      - Extend the tableau with consequences from  $A$
      - back to \*

# The Basic Tableau Algorithm

- Extending a tableau with consequences

Nr	Axiom	Action
1	$C(a)$	Add $C(a)$
2	$R(a,b)$	Add $R(a,b)$
3	$C$	Choose an individual $a$ , add $C(a)$
4	$(C \sqcap D)(a)$	Add $C(a)$ and $D(a)$
5	$(C \sqcup D)(a)$	Split tableau into T1 and T2. Add $C(a)$ to T1, $D(a)$ to T2
6	$(\exists R.C)(a)$	Add $R(a,b)$ and $C(b)$ for a <i>new</i> Individual $b$
7	$(\forall R.C)(a)$	For all $b$ with $R(a,b) \in T$ : add $C(b)$

# A Simple Example

- Given the following ontology:

```
:Animal owl:disjointWith :Human .  
:Animal owl:unionOf (:Mammal :Bird :Fish :Insect :Reptile) .  
:Seth a :Human .  
:Seth a :Insect .
```

- Is this knowledge graph consistent?

# A Simple Example

- Given the following ontology:

```
:Animal owl:disjointWith :Human .  
:Animal owl:unionOf (:Mammal :Bird :Fish :Insect :Reptile) .  
:Seth a :Human .  
:Seth a :Insect .
```

- The same ontology in DL-NNF:

$\neg \text{Animal} \sqcup \neg \text{Human}$

$\text{Animal} \sqcup (\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect} \sqcap \neg \text{Reptile})$

$\neg \text{Animal} \sqcup (\text{Mammal} \sqcup \text{Bird} \sqcup \text{Fish} \sqcup \text{Insect} \sqcup \text{Reptile})$

$\text{Human}(\text{Seth})$

$\text{Insect}(\text{Seth})$

- Let's try how reasoning works now!



# A Simple Example

1d, 1e

Human(Seth), Insect(Seth)

Nr	Axiom	Action
1	<b>C(a)</b>	<b>Add C(a)</b>
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C $\sqcap$ D)(a)	Add C(a) and D(a)
5	(C $\sqcup$ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	( $\exists$ R.C)(a)	Add R(a,b) and C(b) for a <i>new</i> Individual b
7	( $\forall$ R.C)(a)	For all b with R(a,b) $\in$ T: add C(b)

a)  $\neg \text{Animal} \sqcup \neg \text{Human}$   
b)  $\text{Animal} \sqcup (\neg \text{Mammal} \sqcap \neg \text{Bird} \sqcap \neg \text{Fish} \sqcap \neg \text{Insect} \sqcap \neg \text{Reptile})$   
c)  $\neg \text{Animal} \sqcup (\text{Mammal} \sqcup \text{Bird} \sqcup \text{Fish} \sqcup \text{Insect} \sqcup \text{Reptile})$   
**d) Human(Seth)**  
**e) Insect(Seth)**

# A Simple Example

1d, 1e

Human(Seth), Insect(Seth)

3a

Human(Seth), Insect(Seth),  
( $\neg$ Animal  $\sqcup$   $\neg$ Human)(Seth)

Nr	Axiom	Action	
1	$C(a)$	Add $C(a)$	<b>a) <math>\neg</math>Animal <math>\sqcup</math> <math>\neg</math>Human</b> b) Animal $\sqcup$ ( $\neg$ Mammal $\sqcap$ $\neg$ Bird $\sqcap$ $\neg$ Fish $\sqcap$ $\neg$ Insect $\sqcap$ $\neg$ Reptile) c) $\neg$ Animal $\sqcup$ (Mammal $\sqcup$ Bird $\sqcup$ Fish $\sqcup$ Insect $\sqcup$ Reptile) d) Human(Seth) e) Insect(Seth)
2	$R(a,b)$	Add $R(a,b)$	
3	<b>C</b>	<b>Choose an individual a, add C(a)</b>	
4	$(C \sqcap D)(a)$	Add $C(a)$ and $D(a)$	
5	$(C \sqcup D)(a)$	Split tableau into T1 and T2. Add $C(a)$ to T1, $D(a)$ to T2	
6	$(\exists R.C)(a)$	Add $R(a,b)$ and $C(b)$ for a <i>new</i> Individual b	
7	$(\forall R.C)(a)$	For all b with $R(a,b) \in T$ : add $C(b)$	

# A Simple Example



Human(Seth), Insect(Seth)



Human(Seth), Insect(Seth),  
( $\neg$ Animal  $\sqcup$   $\neg$ Human)(Seth)



Human(Seth), Insect(Seth),  
 $\neg$ Animal(Seth)

Human(Seth), Insect(Seth),  
 $\neg$ Human(Seth)

Nr	Axiom	Action	
1	$C(a)$	Add $C(a)$	a) $\neg$ Animal $\sqcup$ $\neg$ Human
2	$R(a,b)$	Add $R(a,b)$	b) Animal $\sqcup$ ( $\neg$ Mammal $\sqcap$ $\neg$ Bird $\sqcap$ $\neg$ Fish $\sqcap$ $\neg$ Insect $\sqcap$ $\neg$ Reptile)
3	$C$	Choose an individual $a$ , add $C(a)$	c) $\neg$ Animal $\sqcup$ (Mammal $\sqcup$ Bird $\sqcup$ Fish $\sqcup$ Insect $\sqcup$ Reptile)
4	$(C \sqcap D)(a)$	Add $C(a)$ and $D(a)$	d) Human(Seth)
5	$(C \sqcup D)(a)$	<b>Split tableau into T1 and T2. Add <math>C(a)</math> to T1, <math>D(a)</math> to T2</b>	e) Insect(Seth)
6	$(\exists R.C)(a)$	Add $R(a,b)$ and $C(b)$ for a <i>new</i> Individual $b$	
7	$(\forall R.C)(a)$	For all $b$ with $R(a,b) \in T$ : add $C(b)$	

# A Simple Example



Human(Seth), Insect(Seth), $\neg$ Animal(Seth)	Human(Seth), Insect(Seth), $\neg$ Human(Seth)
---	--



Human(Seth), Insect(Seth), $\neg$ Animal(Seth) Animal $\sqcup$ ( $\neg$ Mammal $\sqcap$ $\neg$ Bird $\sqcap$ $\neg$ Fish $\sqcap$ $\neg$ Insect)(Seth)
--

Nr	Axiom	Action	
1	C(a)	Add C(a)	a) $\neg$ Animal $\sqcup$ $\neg$ Human
2	R(a,b)	Add R(a,b)	b) Animal $\sqcup$ ( $\neg$ Mammal $\sqcap$ $\neg$ Bird $\sqcap$ $\neg$ Fish $\sqcap$ $\neg$ Insect $\sqcap$ $\neg$ Reptile)
3	<b>C</b>	<b>Choose an individual a, add C(a)</b>	c) $\neg$ Animal $\sqcup$ (Mammal $\sqcup$ Bird $\sqcup$ Fish $\sqcup$ Insect $\sqcup$ Reptile)
4	(C $\sqcap$ D)(a)	Add C(a) and D(a)	d) Human(Seth)
5	(C $\sqcup$ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2	e) Insect(Seth)
6	( $\exists$ R.C)(a)	Add R(a,b) and C(b) for a <i>new</i> Individual b	
7	( $\forall$ R.C)(a)	For all b with R(a,b) $\in$ T: add C(b)	

# A Simple Example



Human(Seth), Insect(Seth), $\neg$ Animal(Seth)	Human(Seth), Insect(Seth), $\neg$ Human(Seth)
---	--



Human(Seth), Insect(Seth), $\neg$ Animal(Seth) Animal $\sqcup$ ( $\neg$ Mammal $\sqcap$ $\neg$ Bird $\sqcap$ $\neg$ Fish $\sqcap$ $\neg$ Insect)(Seth)
--



Human(Seth), Insect(Seth), $\neg$ Animal(Seth) Animal(Seth)	Human(Seth), Insect(Seth), $\neg$ Animal(Seth) $\neg$ Mammal $\sqcap$ $\neg$ Bird $\sqcap$ $\neg$ Fish $\sqcap$ $\neg$ Insect)(Seth)
---	--

Nr	Axiom	Action
1	$C(a)$	Add $C(a)$
2	$R(a,b)$	Add $R(a,b)$
3	$C$	Choose an individual $a$ , add $C(a)$
4	$(C \sqcap D)(a)$	Add $C(a)$ and $D(a)$
5	$(C \sqcup D)(a)$	Split tableau into T1 and T2. Add $C(a)$ to T1, $D(a)$ to T2
6	$(\exists R.C)(a)$	Add $R(a,b)$ and $C(b)$ for a new individual $b$
7	$(\forall R.C)(a)$	For all $b$ with $R(a,b) \in T$ : add $C(b)$

- a)  $\neg$ Animal  $\sqcup$   $\neg$ Human
- b) Animal  $\sqcup$  ( $\neg$ Mammal  $\sqcap$   $\neg$  Bird  $\sqcap$   $\neg$  Fish  $\sqcap$   $\neg$  Insect  $\sqcap$   $\neg$  Reptile)
- c)  $\neg$  Animal  $\sqcup$  (Mammal  $\sqcup$  Bird  $\sqcup$  Fish  $\sqcup$  Insect  $\sqcup$  Reptile)
- d) Human(Seth)
- e) Insect(Seth)

# A Simple Example

5

Human(Seth), Insect(Seth),  
 $\neg$ Animal(Seth)  
 Animal(Seth)

Human(Seth), Insect(Seth),  
 $\neg$ Animal(Seth)  
 $(\neg$ Mammal  $\sqcap$   $\neg$ Bird  $\sqcap$   $\neg$ Fish  $\sqcap$   $\neg$ Insect)(Seth)

4

Human(Seth), Insect(Seth),  
 $\neg$ Animal(Seth)  
 $\neg$ Mammal(Seth)  
 $\neg$ Bird(Seth)  
 $\neg$ Fish(Seth)  
 $\neg$ Insect(Seth)

Nr	Axiom	Action	
1	$C(a)$	Add $C(a)$	a) $\neg$ Animal $\sqcup$ $\neg$ Human
2	$R(a,b)$	Add $R(a,b)$	b) Animal $\sqcup$ ( $\neg$ Mammal $\sqcap$ $\neg$ Bird $\sqcap$ $\neg$ Fish $\sqcap$ $\neg$ Insect $\sqcap$ $\neg$ Reptile)
3	$C$	Choose an individual $a$ , add $C(a)$	c) $\neg$ Animal $\sqcup$ (Mammal $\sqcup$ Bird $\sqcup$ Fish $\sqcup$ Insect $\sqcup$ Reptile)
4	$(C \sqcap D)(a)$	<b>Add <math>C(a)</math> and <math>D(a)</math></b>	d) Human(Seth)
5	$(C \sqcup D)(a)$	Split tableau into T1 and T2. Add $C(a)$ to T1, $D(a)$ to T2	e) Insect(Seth)
6	$(\exists R.C)(a)$	Add $R(a,b)$ and $C(b)$ for a <i>new</i> Individual $b$	
7	$(\forall R.C)(a)$	For all $b$ with $R(a,b) \in T$ : add $C(b)$	

# Another Example

- Again, a simple ontology:  

```
:Woman rdfs:subClassOf [  
        a owl:Restriction;  
        owl:onProperty :hasMother;  
        owl:someValuesFrom :Woman  
    ].  
:Jane a :Woman.
```

# Another Example

- Again, a simple ontology:  

```
:Woman rdfs:subClassOf [  
  a owl:Restriction;  
  owl:onProperty :hasMother;  
  owl:someValuesFrom :Woman  
].  
:Jane a :Woman.
```
- in DL NNF:  
 $\neg \text{Woman} \sqcup \exists \text{hasMother.Woman}$   
**Woman(Jane)**



# Another Example

1b, 3a

Woman(Jane),  $\neg \text{Woman} \sqcup \exists \text{hasMother.Woman(Jane)}$

Nr	Axiom	Action
1	<b>C(a)</b>	<b>Add C(a)</b>
2	R(a,b)	Add R(a,b)
3	<b>C</b>	<b>Choose an individual a, add C(a)</b>
4	(C $\sqcap$ D)(a)	Add C(a) and D(a)
5	(C $\sqcup$ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	( $\exists$ R.C)(a)	Add R(a,b) and C(b) for a <i>new</i> Individual b
7	( $\forall$ R.C)(a)	For all b with R(a,b) $\in$ T: add C(b)

a)  $\neg \text{Woman} \sqcup \exists \text{hasMother.Woman}$   
b) Woman(Jane)

# Another Example

1b, 3a

Woman(Jane),  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman(Jane)

5

Woman(Jane),  $\neg$  Woman (Jane)

Woman(Jane),  $\exists$ hasMother.Woman(Jane)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C $\sqcap$ D)(a)	Add C(a) and D(a)
5	<b>(C <math>\sqcup</math> D)(a)</b>	<b>Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2</b>
6	( $\exists$ R.C)(a)	Add R(a,b) and C(b) for a <i>new</i> Individual b
7	( $\forall$ R.C)(a)	For all b with R(a,b) $\in$ T: add C(b)

a)  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman  
b) Woman(Jane)

# Another Example

1b, 3a

Woman(Jane),  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman(Jane)

5

Woman(Jane),  $\neg$  Woman (Jane)

Woman(Jane),  $\exists$ hasMother.Woman(Jane)

6

Woman(Jane),  $\exists$ hasMother.Woman(Jane)  
hasMother(Jane, b0), Woman(b0)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C $\sqcap$ D)(a)	Add C(a) and D(a)
5	(C $\sqcup$ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	( $\exists$ R.C)(a)	<b>Add R(a,b) and C(b) for a new Individual b</b>
7	( $\forall$ R.C)(a)	For all b with R(a,b) $\in$ T: add C(b)

a)  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman  
b) Woman(Jane)

# Another Example

1b, 3a

Woman(Jane),  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman(Jane)

5

Woman(Jane),  $\neg$  Woman (Jane)

Woman(Jane),  $\exists$ hasMother.Woman(Jane)

6

Woman(Jane),  $\exists$ hasMother.Woman(Jane)  
hasMother(Jane, b0), Woman(b0)

6

Woman(Jane),  $\exists$ hasMother.Woman(Jane)  
hasMother(Jane, b0), Woman(b0)  
hasMother(Jane, b1), Woman(b1)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C $\sqcap$ D)(a)	Add C(a) and D(a)
5	(C $\sqcup$ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	( $\exists$ R.C)(a)	<b>Add R(a,b) and C(b) for a new Individual b</b>
7	( $\forall$ R.C)(a)	For all b with R(a,b) $\in$ T: add C(b)

a)  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman  
b) Woman(Jane)

# Another Example

1b, 3a

Woman(Jane),  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman(Jane)

5

Woman(Jane),  $\neg$  Woman (Jane)

Woman(Jane),  $\exists$ hasMother.Woman(Jane)

6

Woman(Jane),  $\exists$ hasMother.Woman(Jane)  
hasMother(Jane, b0), Woman(b0)

6

Woman(Jane),  $\exists$ hasMother.Woman(Jane)  
hasMother(Jane, b0), Woman(b0)  
hasMother(Jane, b1), Woman(b1)

6

...

a)  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman  
b) Woman(Jane)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C $\sqcap$ D)(a)	Add C(a) and D(a)
5	(C $\sqcup$ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	( $\exists$ R.C)(a)	<b>Add R(a,b) and C(b) for a new Individual b</b>
7	( $\forall$ R.C)(a)	For all b with R(a,b) $\in$ T: add C(b)

# Introducing Rule Blocking

- Observation
  - The tableau algorithm does not necessarily terminate
  - We can add arbitrarily many new axioms

Nr	Axiom	Action
6	$(\exists R.C)(a)$	Add $R(a,b)$ und $C(b)$ for a <i>new</i> Individual $b$

- Idea: avoid rule 6 if no new information is created
  - i.e., if we already created one instance  $b_a$  for instance  $a$ , then block using rule 6 for  $a$ .

# Tableau Algorithm with Rule Blocking

- Given: an ontology  $O$  in NNF
- While not all partial tableaux are closed  
and further axioms can be created
  - \* Choose a non-closed partial tableau  $T$  and a non-blocked  $A \in O \cup T$ 
    - If  $A$  is not contained in  $T$ 
      - If  $A$  is an atomic statement
        - add  $A$  to  $T$
        - back to \*
      - If  $A$  is a non-atomic statement
        - Choose an individual  $i \in O \cup T$
        - Add  $A(i)$  to  $T$
        - back to \*
    - else
      - Extend the tableau with consequences from  $A$
      - If rule 6 was used, block  $A$  for  $T$
      - back to \*

# Example with Rule Blocking

1b, 3a

Woman(Jane),  $\neg$  Woman  $\sqcup$   $\exists$ hasMother.Woman(Jane)

5

Woman(Jane),  $\neg$  Woman (Jane)

Woman(Jane),  $\exists$ hasMother.Woman(Jane)

6

Woman(Jane),  $\exists$ hasMother.Woman(Jane)  
hasMother(Jane, b0), Woman(b0)

Block Rule 6 for Jane

now it will terminate  
ultimately

Nr	Axiom	Action
6	$(\exists R.C)(a)$	Add R(a,b) und C(b) for a <i>new</i> Individual b, <b>block rule 6 for a</b>



# Tableau Algorithm: Wrap Up

- An algorithm for description logic based ontologies
  - Works for OWL Lite and DL
- We have seen examples for some OWL expressions
  - Other OWL DL expressions can be “translated” to DL as well
  - And they come with their own expansion rules
  - Reasoning may become more difficult
    - e.g., dynamic blocking and unblocking

# Optimizing Tableau Reasoners

- Given: an ontology  $O$  in NNF
- While not all partial tableaux are closed  
and further axioms can be created
  - \* Choose a non-closed partial tableau  $T$  and a non-blocked  $A \in O \cup T$ 
    - If  $A$  is not contained in  $T$ 
      - If  $A$  is an atomic statement
        - add  $A$  to  $T$
        - back to \*
      - If  $A$  is a non-atomic statement
        - Choose an individual  $i \in O \cup T$
        - Add  $A(i)$  to  $T$
        - back to \*
    - else
      - Extend the tableau with consequences from  $A$
      - If rule 6 was used, block  $A$  for  $T$
      - back to \*

# OWL Lite vs DL Revisited

- Recap: OWL Lite has some restrictions
  - Those are meant to allow for faster reasoning
- Restrictions only with cardinalities 0 and 1
  - Higher cardinalities make blocking more complex
- `unionOf`, `disjointWith`, `complementOf`, closed classes, ...
  - They all introduce more disjunctions
  - i.e., more splitting operations

# Complexity of Ontologies

- Reasoning is usually expensive
- Reasoning performance depends on ontology complexity
  - Rule of thumb: the more complexity, the more costly
- Most useful ontologies are in OWL DL
  - But there are differences
  - In detail: complexity classes

# Simple Ontologies: ALC

- ALC: Attribute Language with Complement
- Allowed:
  - subClassOf, equivalentClass
  - unionOf, complementOf, disjointWith
  - Restrictions: allValuesFrom, someValuesFrom
  - domain, range
  - Definition of individuals

- Complexity classes are noted as letter sequences
- Using
  - S = ALC plus transitive properties (basis for most ontologies)
  - H = Property hierarchies (subPropertyOf)
  - O = closed classes (oneOf)
  - I = inverse properties (inversePropertyOf)
  - N = numeric restrictions (min/maxCardinality)
  - F = functional properties
  - Q = qualified numerical restrictions (OWL2)
  - (D) = Usage of datatype properties

# Some Tableau Reasoners

- Fact
  - University of Manchester, free
  - SHIQ
- Fact++/JFact
  - Extension of Fact, free
  - SHOIQ(and a little D), OWL-DL + OWL2
- Pellet
  - Clark & Parsia, free for academic use
  - SHOIN(D), OWL-DL + OWL2
- RacerPro
  - Racer Systems, commercial
  - SHIQ(D)

# Sudoku Revisited

- Recap: we used a closed class
  - Plus some disjointness
- Resulting complexity: SO
- Which reasoners do support that?
  - Fact: SHIQ :-(
    - RacerPro: SHIQ(D) :-(
      - Pellet: SHOIN(D) :-)
      - HermiT: SHOIQ :-)

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9



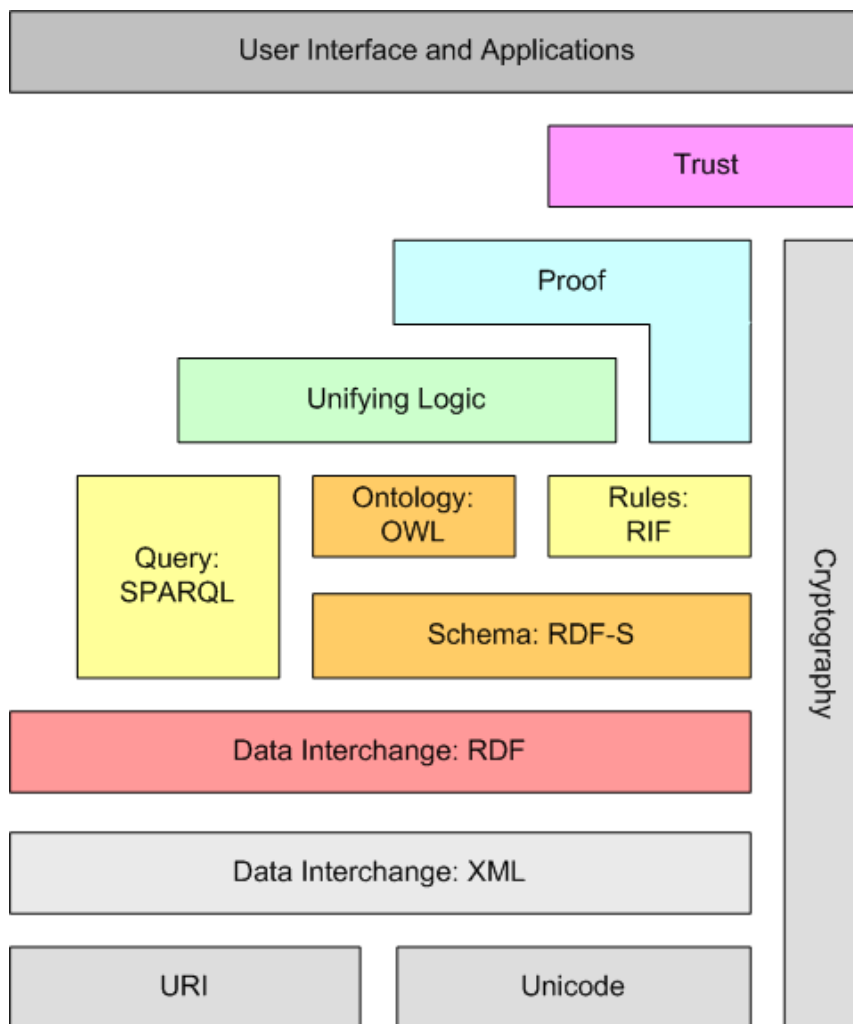
# Rules: Beyond OWL



here be dragons...

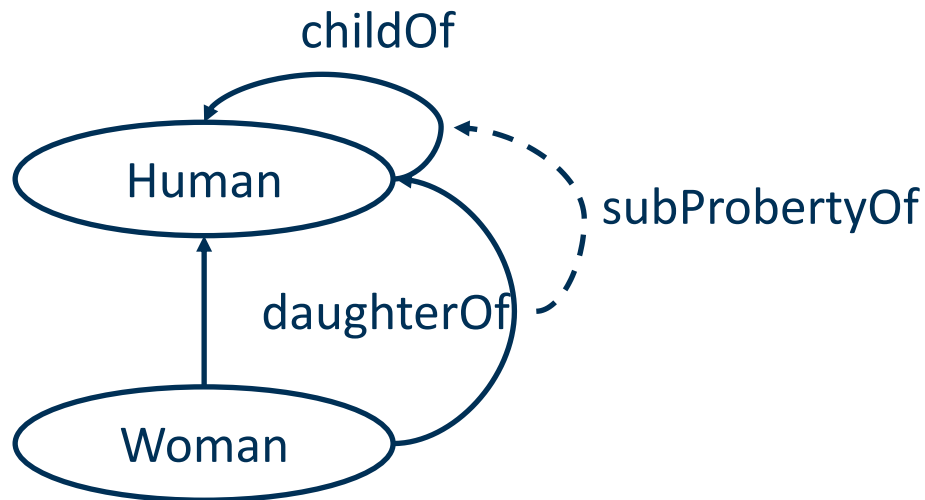
Semantic Web Technologies  
(This lecture)

Technical  
Foundations



# Limitations of OWL

- Some things are hard or impossible to express in OWL
- Example:
  - If A is a woman and the child of B  
then A is the daughter of B



# Limitations of OWL

- Let's try this in OWL:

```
:Woman rdfs:subClassOf :Human .  
:childOf      a owl:ObjectProperty ;  
               rdfs:domain :Human ;  
               rdfs:range  :Human .  
:daughterOf a owl:ObjectProperty ;  
            rdfs:subPropertyOf :childOf ;  
            rdfs:domain :Woman .
```

# Limitations of OWL

- What can a reasoner conclude with this ontology?
- Example:  
    :Julia :daughterOf :Peter .  
    → :Julia a :Woman .
- What we would like to have instead:  
    :Julia :childOf :Peter .  
    :Julia a :Woman .  
    → :Julia :daughterOf :Peter .

# Limitations of OWL

- What we would like to have:  
 $\text{daughterOf}(X,Y) \leftarrow \text{childOf}(X,Y) \wedge \text{Woman}(X) .$
- Rules are flexible
- There are rules in the Semantic Web, e.g.
  - Semantic Web Rule Language (SWRL)
  - Rule Interchange Format (RIF)
  - See lecture in a few weeks
- Some reasoners do (partly) support rules

# Wrap Up

- OWL comes in many flavours
  - OWL Lite, OWL DL, OWL Full
  - Detailed complexity classes of OWL DL
  - Additions and profiles from OWL2
  - However, there are still some things that cannot be expressed...
- Reasoning is typically done using the Tableau algorithm

# Questions?

