Web Ontology Language (OWL) Part II

IE650 Knowledge Graphs

Previously on "Knowledge Graphs"

- We have got to know
  - OWL, a more powerful ontology language than RDFS
  - Simple ontologies and some reasoning
  - Sudoku solving

- Today
  - New constructs in OWL2
  - Russell's paradox
  - Reasoning in OWL
  - Complexity of ontologies
  - A peek at rule languages for Knowledge Graphs

Semantic Web Technology Stack

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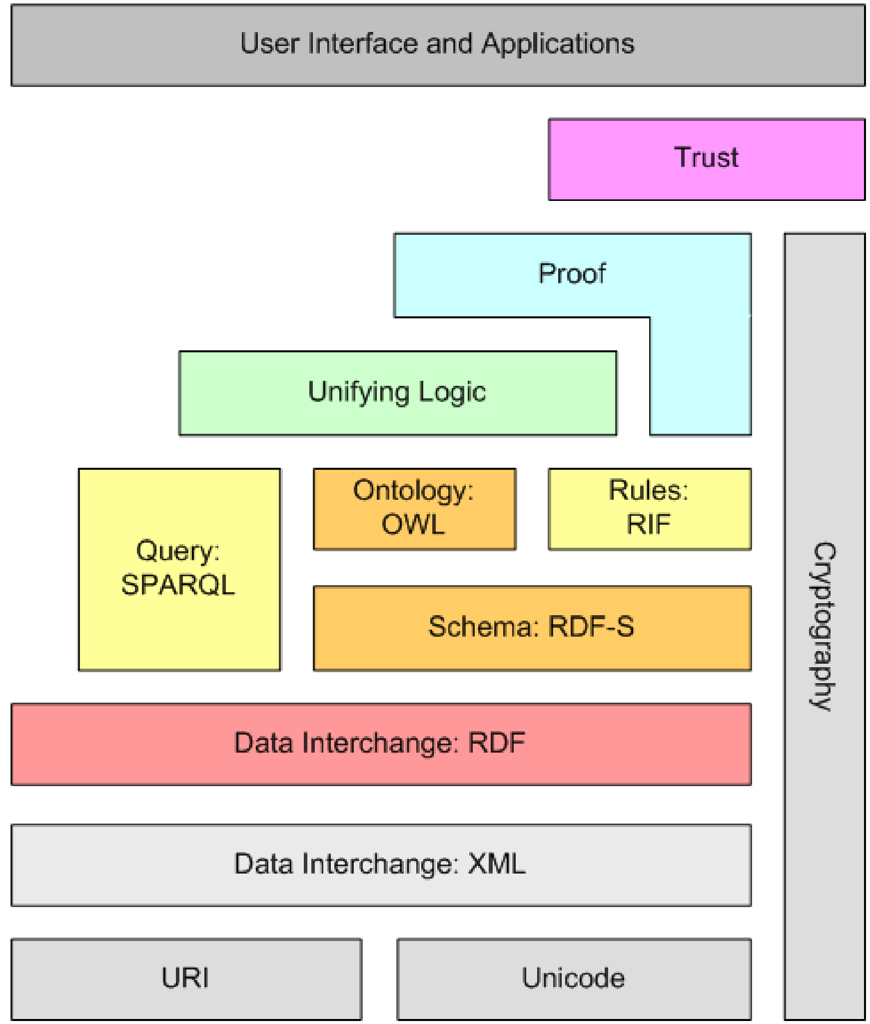
here be dragons...

Knowledge Graphs
(This lecture)

Technical
Foundations



Berners-Lee (2009): *Semantic Web and Linked Data* http://www.w3.org/2009/Talks/0120-campus-party-tbl/

OWL2 – New Constructs and More

- Five years after the first OWL standard
- OWL2: 2009
  - Syntactic sugar
  - New language constructs
  - OWL profiles

- We have already encountered some, e.g.,
  - Qualified restrictions
  - Reflexive, irreflexive, and antisymmetric properties

OWL2: Syntactic Sugar

- Disjoint classes and disjoint unions
  - OWL 1:

```
:Wine owl:equivalentClass [
  a owl:Class ;
  owl:unionOf (:RedWine :RoséWine :WhiteWine) ].
:RedWine  owl:disjointWith :RoséWine, :WhiteWine.
:RoséWine owl:disjointWith :WhiteWine .
```

  - OWL 2:

```
:Wine owl:disjointUnionOf
    (:RedWine :RoséWine :WhiteWine ).
```

  - Also possible:

```
_:x a owl:AllDisjointClasses ;
  owl:members (:RedWine :RoséWine WhiteWine ).
```

OWL2: Syntactic Sugar

- Negative(Object|Data)PropertyAssertion
- Allow negated statements
  - e.g.: Paul is not Peter's father

```
_:x [
    a owl:NegativeObjectPropertyAssertion;
    owl:sourceIndividual :Paul ;
    owl:targetIndividual :Peter ;
    owl:assertionProperty :fatherOf
] .
```

  - If that's syntactic sugar, it must also be possible differently
    - But how?

OWL2: Syntactic Sugar

- Negative(Object|Data)PropertyAssertion
- Allow negated statements
  - Replaces less intuitive set constructs
  - Paul is not Peter's father

```
:Paul a [
   owl:complementOf [
      a owl:Restriction ;
      owl:onProperty :fatherOf ;
      owl:hasValue :Peter
   ]
].
```

OWL2: Reflexive Class Restrictions

- Using `hasSelf`
- Example: defining the set of all autodidacts:

```
:AutoDidact owl:equivalentClass [
   a owl:Restriction ;
   owl:onProperty :teaches ;
   owl:hasSelf "true"^^xsd:boolean
] .
```

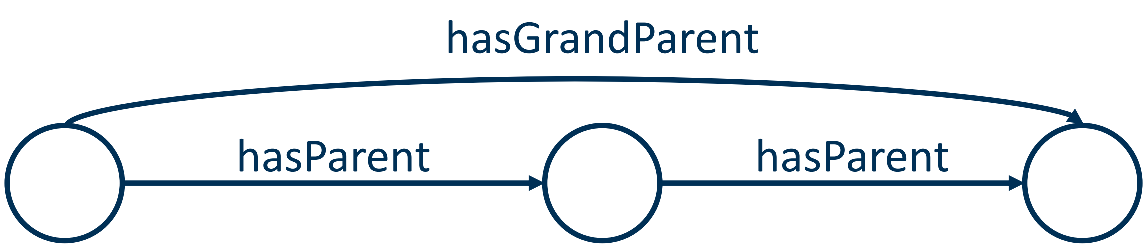
OWL2: Property Chains

- Typically used for defining rule-like constructs, e.g.

```
hasParent(X,Y) and hasParent(Y,Z) →
hasGrandParent(X,Z)
```

- OWL Syntax:

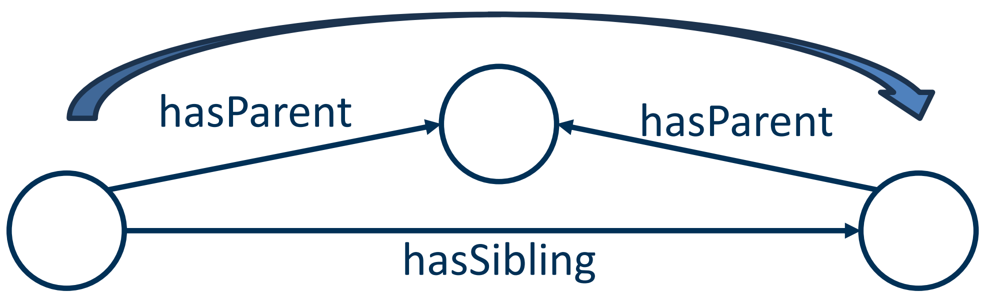
```
:hasGrandparent owl:propertyChainAxiom
    ( :hasParent :hasParent ) .
```



OWL2: Property Chains

`hasParent(X,Y) and hasParent(Z,Y) → hasSibling(X,Z)`

- This is not a proper chain yet, so we have to rephrase it to



OWL2: Property Chains

```
hasParent(X,Y) and hasParent(Z,Y) → hasSibling(X,Z)
```

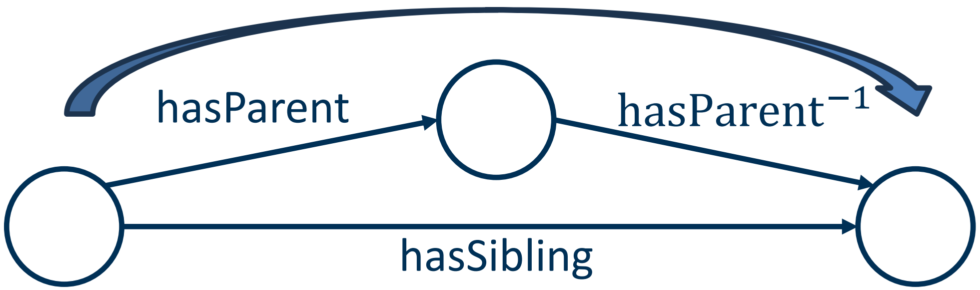
- This is not a proper chain yet, so we have to rephrase it to

  hasParent(X,Y) and hasParent^{-1}(Y,Z) → hasSibling(X,Z)

  – Property Chains can be combined with inverse properties and others

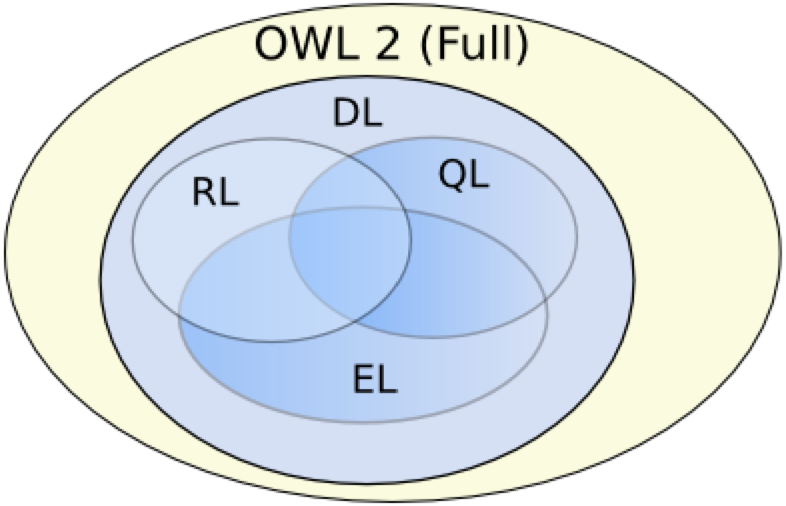
- OWL Syntax:

```
:hasSibling owl:propertyChainAxiom
    ( :hasParent  [ owl:inverseOf :hasParent ] ) .
```



OWL2: Profiles

- Profiles are subsets of OWL2 DL
  - EL, RL und QL
  - Similar to complexity classes
- Different runtime and memory complexity
- Depending on requirements



OWL2 Profile

- OWL2 EL (Expressive Language)
  - Fast reasoning on many standard ontologies
  - Restrictions, e.g.:
    - someValuesFrom, but not allValuesFrom
    - No inverse and symmetric properties
    - No unionOf and complementOf

- OWL2 QL (Query Language)
  - Fast query answering on relational databases
  - Restrictions, e.g.:
    - No unionOf, allValuesFrom, hasSelf, …
    - No cardinalities and functional properties

OWL2 Profile

- OWL2 RL (Rule Language)
  - Subset similar to rule languages such as datalog
    - subClassOf is translated to a rule (Person ← Student)
  - Restrictions, e.g.:
    - Only qualified restrictions with 0 or 1
    - Some restrictions for head and body

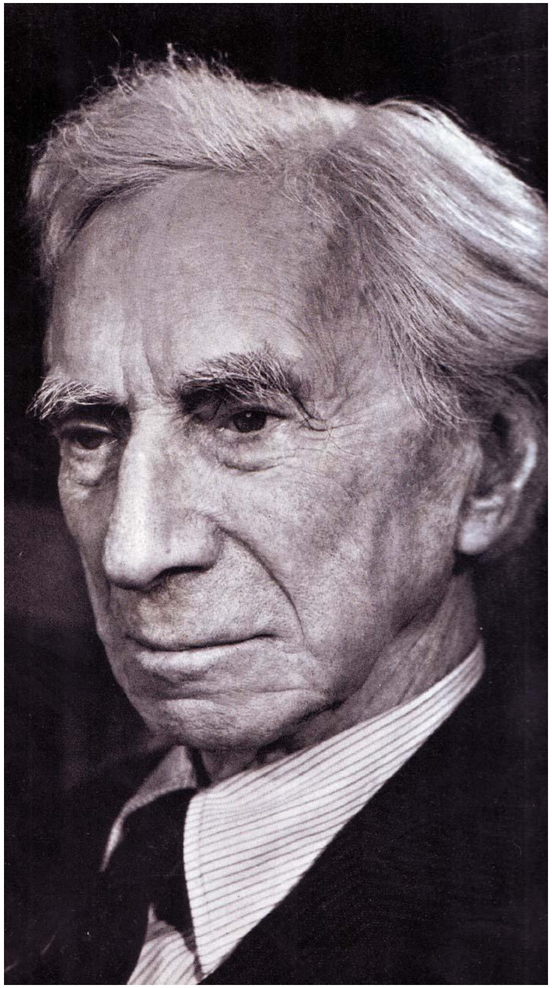
- The following holds for all three profiles:
  - Reasoning can be implemented in polynomial time for each of the three
  - Reasoning on the union of two profiles only possible in exponential time

OWL2 Example: Russell's Paradox

- A classic paradox by
  Bertrand Russell, 1918

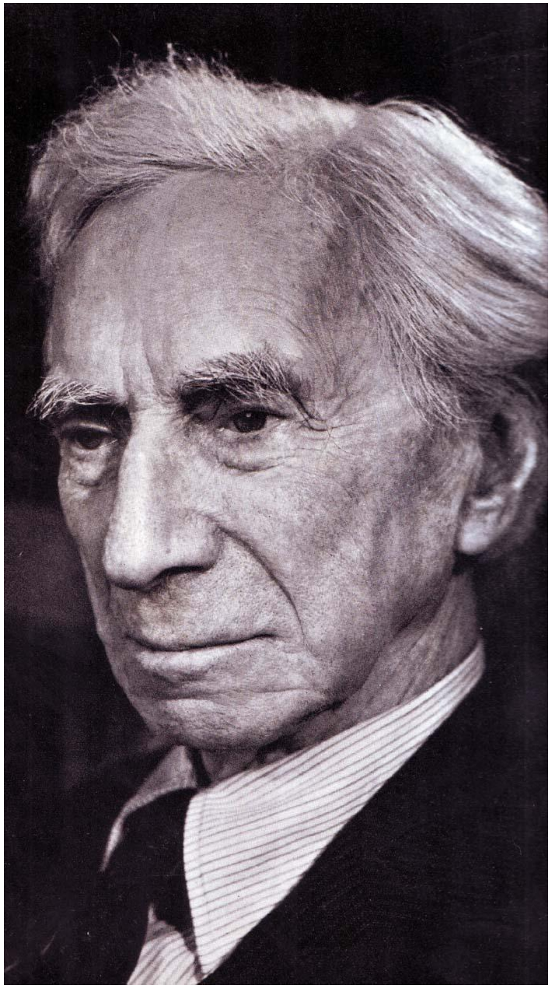
- In a city, there is exactly one barber who shaves everybody who does not shave themselves.

  Who shaves the barber?



OWL2 Example: Russell's Paradox

- In a city, there is exactly one barber who shaves everybody who does not shave themselves.
  Who shaves the barber?
- Assume:
  - The barber **shave himself**:
    - he ceases to be the barber specified because he only shaves those who do not shave themselves
  - The barber **does not** shave himself:
    - then he fits into the group of people who would be shaved by the specified barber, and thus, as that barber, he must shave himself.



OWL2 Example: Russell's Paradox

- Class definitions

```
:People owl:disjointUnionOf
    (:PeopleWhoShaveThemselves
     :PeopleWhoDoNotShaveThemselves ) .
```

- Relation definitions:

```
:shavedBy rdfs:domain :People .
:shavedBy rdfs:range :People .
:shaves owl:inverseOf :shavedBy .
```

- Every person is shaved by exactly one person:

```
:People rdfs:subClassOf [
    a owl:Restriction ;
    owl:onProperty :shavedBy ;
    owl:cardinality "1"^^xsd:integer
] .
```

OWL2 Example: Russell's Paradox

- Then, we define the barber:

```
:Barbers rdfs:subClassOf :People ;
         owl:equivalentClass [
             rdf:type owl:Class ;
             owl:oneOf ( :theBarber )
         ] .
```

OWL2 Example: Russell's Paradox

- Definition of people shaving themselves:

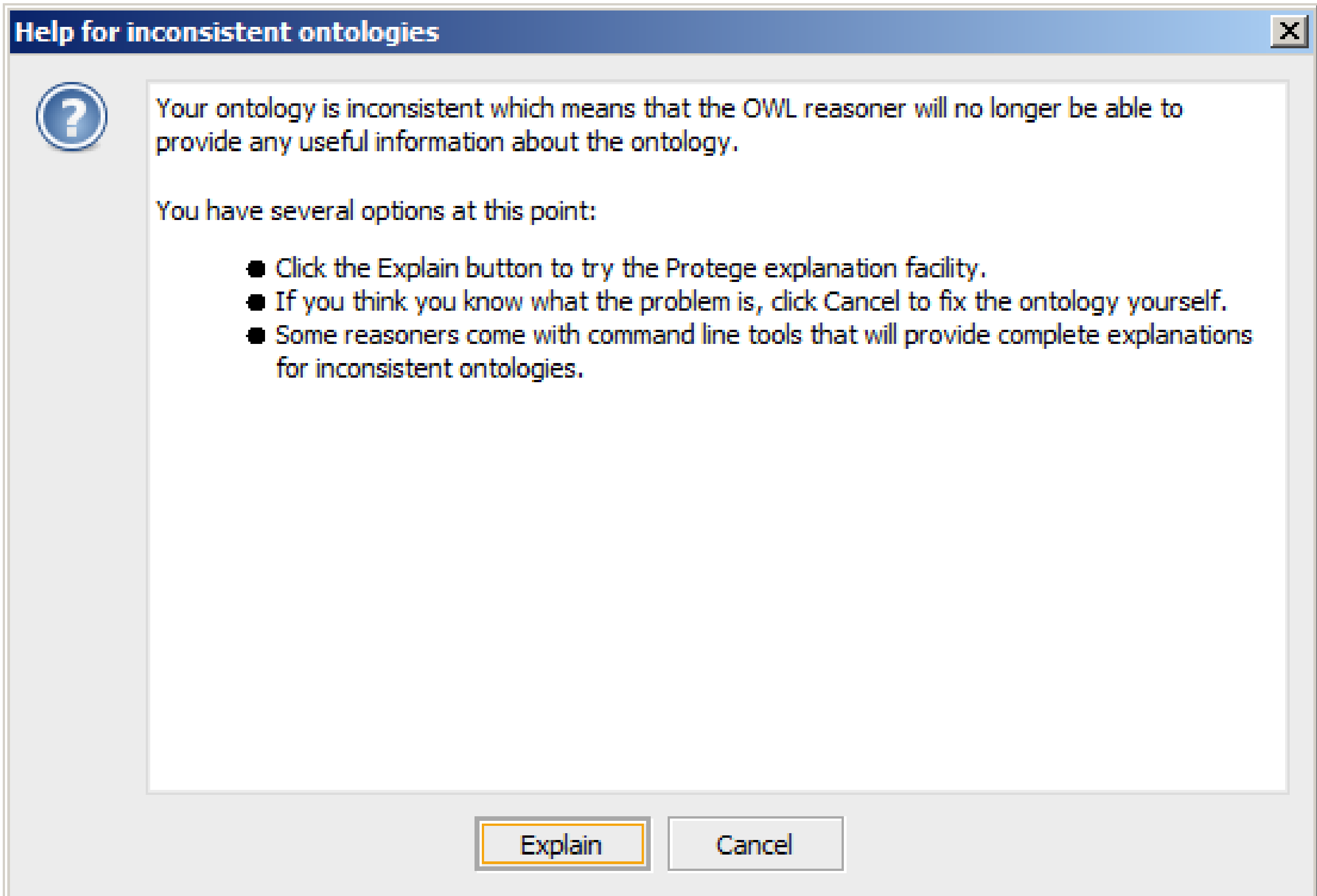
```
:PeopleWhoShaveThemselves owl:equivalentClass [
   a owl:Class ;
   owl:intersectionOf (
      :People
      [
          a owl:Restriction ;
          owl:onProperty :shavedBy ;
          owl:hasSelf "true"^^xsd:boolean
      ]
   )
] .
```

OWL2 Example: Russell's Paradox

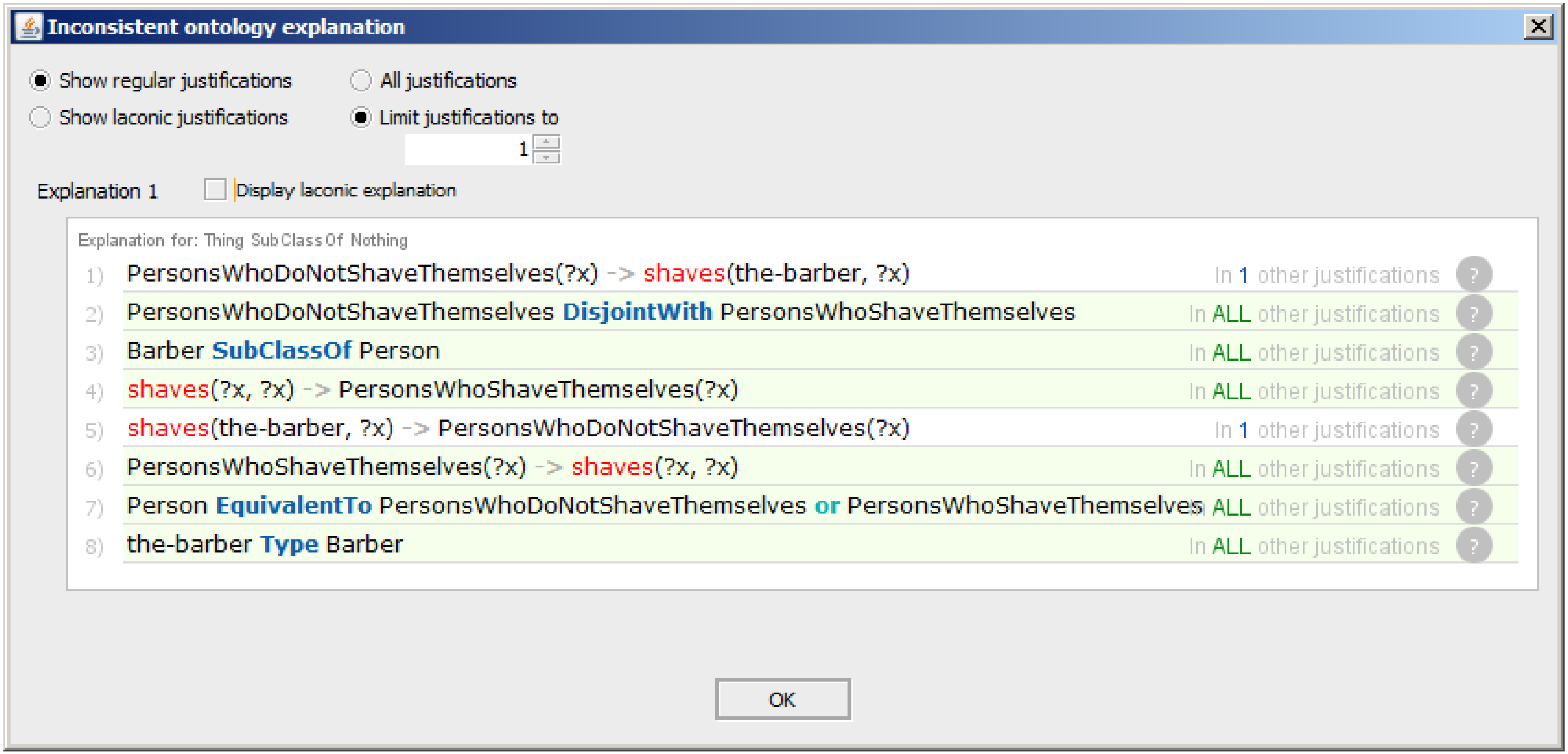
- Definition of people who do not shave themselves:

```
:PeopleWhoDoNotShaveThemselves owl:equivalentClass [
   a owl:Class ;
   owl:intersectionOf (
      :People
      [
         a owl:Restriction
         owl:onProperty :shavedBy ;
         owl:allValuesFrom :Barbers
      ]
   )
] .
```

OWL2 Example: Russell's Paradox



OWL2 Example: Russell's Paradox



Reasoning in OWL DL

- We have seen reasoning for RDFS
  - Forward chaining algorithm
  - Derive axioms from other axioms
- Limitations of forward chaining
  - ```
    :Motorbike owl:intersectionOf
       (:TwoWheeledVehicle :MotorVehicle).
    :x a :Motorbike.
    →
    :x a TwoWheeledVehicle, :MotorVehicle .
    ```
  - ```
    :TwoWheeledVehicle owl:unionOf (:Bicycle :Motorbike).
    :x a :TwoWheeledVehicle
    →
    ?
    ```

Reasoning in OWL DL

- Reasoning for OWL DL is more difficult
  - Forward chaining may have scalability issues
  - Disjunction (e.g., unionOf) is not supported by forward chaining
    - Same holds for some other constructs
    - No negation
  - Different approach: Tableau Reasoning
  - Underlying idea: find contradictions in ontology
    - i.e., both a statement and its opposite
      can be derived from the ontology

Reasoning in OWL DL

- What do we want to know from a reasoner?
  - Subclass relations
    - e.g., Are all birds flying animals?
  - Equivalent classes
    - e.g., Are all birds flying animals *and vice versa*?
  - Disjoint classes
    - e.g., Are there animals that are mammals and birds at the same time?
  - Class consistency
    - e.g., Can there be mammals that lay eggs?
  - Class instantiation
    - e.g., Is Flipper a dolphin?
  - Class enumeration
    - e.g., List all dolphins

Example: A Simple Contradiction

- Given:

```
:Human a owl:Class .
:Animal a owl:Class .
:Human owl:disjointWith :Animal .

:Jimmy a :Animal .
:Jimmy a :Human .
```

Example: A Simple Contradiction

- We can derive:
  - :Human ∩ :Animal = ∅

    ```
    owl:Nothing owl:intersectionOf (:Human :Animal).
    ```
  - :Jimmy ∈ (:Human ∩ :Animal)

    ```
    :Jimmy a [
        a owl:Class;
        owl:intersectionOf (:Human :Animal)
    ] .
    ```
- i.e.:
  - :Jimmy ∈ ∅

    ```
    :Jimmy a owl:Nothing .
    ```
  - That means: the instance must not exist
  - But it does

Reasoning Tasks Revisited

- Subclass Relations

  Student $\subseteq$ Person $\Leftrightarrow$ „Every student is a person“
- Proof method: Reductio ad absurdum
  - "Invent" an instance i
  - Define Student(i) and $\neg$ Person(i)
  - Check for contradictions
    - If there is one: Student $\subseteq$ Person has to hold
    - If there is none: Student $\subseteq$ Person cannot be derived
      - Note: it may still hold!

Example: Subclass Relations

- Ontology:

```
:Student owl:subClassOf :UniversityMember .
:UniversityMember owl:subClassOf :Person .
```

- Invented instance:

```
:i a :Student .
:i a [ owl:complementOf :Person ] .
```

- We have

```
:i a :Student .
:Student owl:subClassOf :UniversityMember .
```

- Thus `:i a :UniversityMember .`
- And from

```
:UniversityMember owl:subClassOf :Person .
```

- We further derive that `:i a Person .`

Example: Subclass Relations

- Now, we have

```
:i a :Person .
:i a [ owl:complementOf :Person ] .
```

i.e.,

```
:i a [
   owl:intersectionOf (
       :Person
       [ owl:complementOf :Person ]
   )
] .
```

- from which we derive

```
:i a owl:Nothing .
```

Reasoning Tasks Revisited

- Class equivalence
  - Person $\equiv$ Human
- Split into
  - Person $\subseteq$ Human and
  - Human $\subseteq$ Person
- i.e., show subclass relation twice
  - We have seen that
- Class disjointness
  - Are C and D disjoint?
  - "Invent" an instance i
  - Define C(i) and D(i)
    - We have done set (the Jimmy example)

Class Consistency

- Can a class have instances? e.g., married bachelors

```
:Bachelor owl:subClassOf :Man,
    [ a owl:Restriction;
      owl:onProperty :marriedTo;
      owl:cardinality 0 ] .
:MarriedPerson owl:subClassOf [
    a owl:Restriction;
    owl:onProperty :marriedTo;
    owl:cardinality 1 ] .

:MarriedBachelor owl:intersectionOf
  (:Bachelor :MarriedPerson) .
```

- Now: invent an instance of the class
  - And check for contradictions

Reasoning Tasks Revisited

- Class Instantiation
  - Is Flipper a dolphin?
- Check:
  - Define $\neg$ Dolphin(Flipper)
  - Check for contradiction
- Class enumeration
  - Repeat class instantiation for all known instances

Typical Reasoning Tasks Revisited

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- What do we want to know from a reasoner?
  - Subclass relations
    - e.g., Are all birds flying animals?
  - Equivalent classes
    - e.g., Are all birds flying animals *and vice versa*?
  - Disjoint classes
    - e.g., Are there animals that are mammals and birds at the same time?
  - Class consistency
    - e.g., Can there be mammals that lay eggs?
  - Class instantiation
    - e.g., Is Flipper a dolphin?
  - Class enumeration
    - e.g., List all dolphins

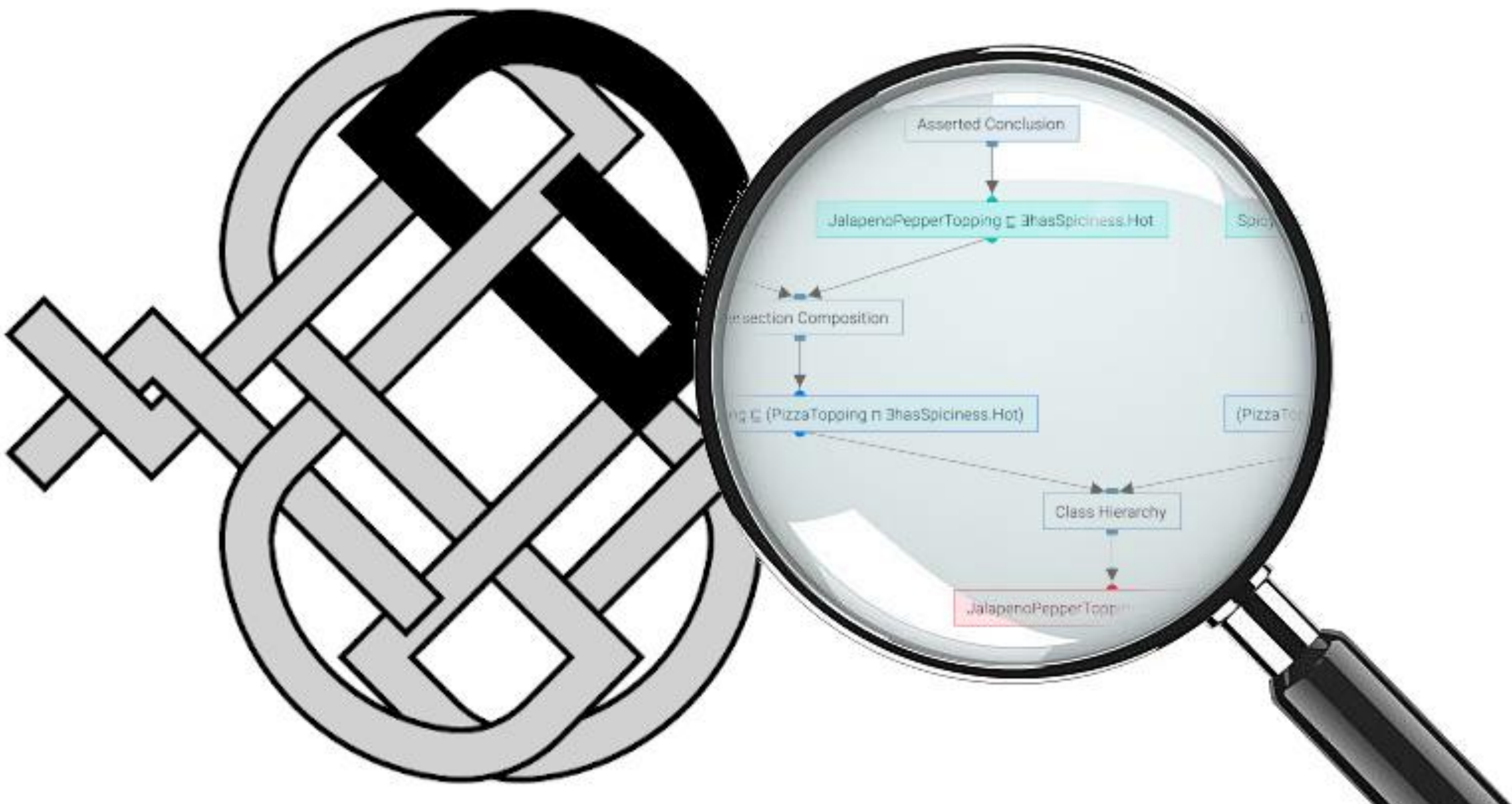
Typical Reasoning Tasks Revisited

- We have seen
  - All reasoning tasks can be reduced to the same basic task
  - i.e., consistency checking

- This means:
  for building a reasoner that can solve those tasks,
  - We only need a reasoner capable of consistency checking

OWL DL

- The DL stands for "Description Logics"
- A logic formalism dating back to the 1980s



Ontologies in Description Logics
Notation

- Classes and Instances
  - C(x) ↔ `x a C .`
  - R(x,y) ↔ `x R y .`
  - $C \sqsubseteq D$ ↔ `C rdfs:subClassOf D`
  - $C \equiv D$ ↔ `C owl:equivalentClass D`
  - $C \sqsubseteq \neg D$ ↔ `C owl:disjointWith D`
  - $C \equiv \neg D$ ↔ `C owl:complementOf D`
  - $C \equiv D \sqcap E$ ↔ `C owl:intersectionOf (D E) .`
  - $C \equiv D \sqcup E$ ↔ `C owl:unionOf (D E) .`
  - $\top$ ↔ `owl:Thing`
  - $\bot$ ↔ `owl:Nothing`

Ontologies in Description Logics Notation

- Domains, ranges, and restrictions
  - $\exists R.\top \sqsubseteq C$ ↔ `R rdfs:domain C .`
  - $\forall R.C$ ↔ `R rdfs:range C .`
  - $C \sqsubseteq \forall R.D$ ↔
```
C rdfs:subClassOf [
        a owl:Restriction;
        owl:onProperty R;
        owl:allValuesFrom D ] .
```
  - $C \sqsubseteq \exists R.D$ ↔
```
C rdfs:subClassOf [
        a owl:Restriction;
        owl:onProperty R;
        owl:someValuesFrom D ] .
```
  - $C \sqsubseteq \geq nR$ ↔
```
C rdfs:subClassOf [
        a owl:Restriction;
        owl:onProperty R;
        owl:minCardinality n ] .
```

Global Statements in Description Logic

- So far, we have seen mostly statements about single classes
  - e.g., $C \sqsubseteq D$
- In Description Logics, we can also make global statements
  - e.g., $D \sqcup E$
  - This means: every single instance is a member of D or E (or both)
- Those global statements are heavily used in the reasoning process

Negation Normal Form (NNF)

- Transforming ontologies to Negation Normal Form:
  - $\sqsubseteq$ und $\equiv$ are not used
  - Negation only for atomic classes and axioms

- A simplified notation of ontologies
- Used by tableau reasoners

Negation Normal Form (NNF)

- Eliminating $\sqsubseteq$:
  - Replace $C \sqsubseteq D$ by $\neg C \sqcup D$
  - Note: this is a shorthand notation for $\forall x: \neg C(x) \vee D(x)$

- Why does this hold?
  - $C \sqsubseteq D$ is equivalent to $C(x) \rightarrow D(x)$

C(x)	D(x)	C(x) → D(x)	¬C(x) v D(x)
true	true	true	true
true	false	false	false
false	true	true	true
false	false	true	true

Negation Normal Form (NNF)

- Eliminating $\equiv$
  - Replace $C \equiv D$ by $C \sqsubseteq D$ and $D \sqsubseteq C$
  - Proceed as before

- i.e.: $C \equiv D$ becomes

  $C \sqsubseteq D$

  $D \sqsubseteq C$

  - and thus

  $\neg C \sqcup D$

  $\neg D \sqcup C$

Negation Normal Form (NNF)

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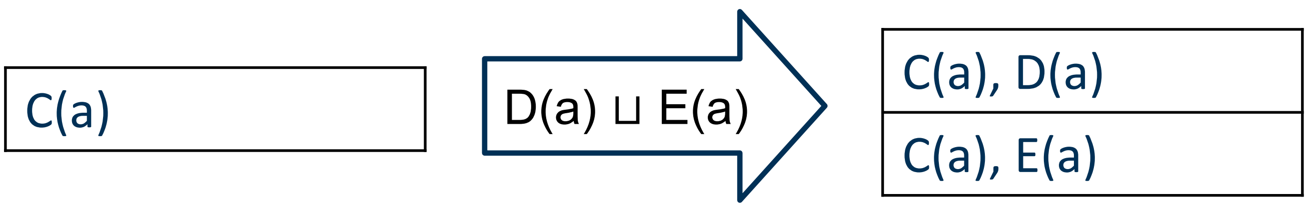
- Further transformation rules
  - $\mathrm{NNF}(C) = C$ (for atomic C)
  - $\mathrm{NNF}(\neg C) = \neg C$ (for atomic C)
  - $\mathrm{NNF}(\neg\neg C) = C$
  - $\mathrm{NNF}(C \sqcup D) = \mathrm{NNF}(C) \sqcup \mathrm{NNF}(D)$
  - $\mathrm{NNF}(C \sqcap D) = \mathrm{NNF}(C) \sqcap \mathrm{NNF}(D)$
  - $\mathrm{NNF}(\neg(C \sqcap D)) = \mathrm{NNF}(\neg C) \sqcup \mathrm{NNF}(\neg D)$
  - $\mathrm{NNF}(\neg(C \sqcup D)) = \mathrm{NNF}(\neg C) \sqcap \mathrm{NNF}(\neg D)$
  - $\mathrm{NNF}(\forall R.C) = \forall R.\mathrm{NNF}(C)$
  - $\mathrm{NNF}(\exists R.C) = \exists R.\mathrm{NNF}(C)$
  - $\mathrm{NNF}(\neg \forall R.C) = \exists R.\mathrm{NNF}(\neg C)$
  - $\mathrm{NNF}(\neg \exists R.C) = \forall R.\mathrm{NNF}(\neg C)$

The Basic Tableau Algorithm

- Tableau: Collection of derived axioms
  - Is subsequently extended
  - As for forward chaining
- In case of conjunction
  - Split the tableau



When is an Ontology Free of Contradictions?

- Tableau is continuously extended and split
- Free of contradictions if...
  - No further axioms can be created
  - At least one partial tableau is free of contradictions
  - A partial tableau has a contradiction if it contains both an axiom and its negation
    - e.g. Person(Peter) und $\neg$Person(Peter)
    - The partial tableau is then called *closed*

The Basic Tableau Algorithm

- Given: an ontology O in NNF
- While not all partial tableaus are closed

```
* Choose a non-closed partial tableau T and an A ∈ O ∪ T
  If A is not contained in T
        If A is an atomic statement
                add A to T
                back to *
        If A is a non-atomic statement
                Choose an individual i ∈ O ∪ T
                Add A(i) to T
                back to *
  else
        Extend the tableau with consequences from A
        back to *
```

The Basic Tableau Algorithm

- Extending a tableau with consequences

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	$(C \sqcap D)(a)$	Add C(a) and D(a)
5	$(C \sqcup D)(a)$	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	$(\exists R.C)(a)$	Add R(a,b) and C(b) for a *new* Individual b
7	$(\forall R.C)(a)$	For all b with R(a,b) ∊ T: add C(b)

A Simple Example

- Given the following ontology:

```
:Animal owl:disjointWith :Human .
:Animal owl:unionOf (:Mammal :Bird :Fish :Insect :Reptile).
:Seth a :Human .
:Seth a :Insect .
```

- Is this knowledge graph consistent?

A Simple Example

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- Given the following ontology:

```
:Animal owl:disjointWith :Human .
:Animal owl:unionOf (:Mammal :Bird :Fish :Insect :Reptile).
:Seth a :Human .
:Seth a :Insect .
```

- The same ontology in DL-NNF:

$\neg$Animal $\sqcup$ $\neg$Human
Animal $\sqcup$ ($\neg$Mammal $\sqcap$ $\neg$ Bird $\sqcap$ $\neg$ Fish $\sqcap$ $\neg$ Insect $\sqcap$ $\neg$ Reptile)
$\neg$ Animal $\sqcup$ (Mammal $\sqcup$ Bird $\sqcup$ Fish $\sqcup$ Insect $\sqcup$ Reptile)
Human(Seth)
Insect(Seth)

- Let's try how reasoning works now!

A Simple Example

1d, 1e

Human(Seth), Insect(Seth)

Nr	Axiom	Action
1	**C(a)**	**Add C(a)**
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	$(C \sqcap D)(a)$	Add C(a) and D(a)
5	$(C \sqcup D)(a)$	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	$(\exists R.C)(a)$	Add R(a,b) and C(b) for a *new* Individual b
7	$(\forall R.C)(a)$	For all b with R(a,b) ∊ T: add C(b)

a) ¬Animal ⊔ ¬Human
b) Animal ⊔ (¬Mammal ⊓ ¬ Bird ⊓ ¬ Fish ⊓ ¬ Insect ⊓ ¬ Reptile)
c) ¬ Animal ⊔ (Mammal ⊔ Bird ⊔ Fish ⊔ Insect ⊔ Reptile)
d) Human(Seth)
e) Insect(Seth)

A Simple Example

1d, 1e

Human(Seth), Insect(Seth)

3a

Human(Seth), Insect(Seth),
(¬Animal ⊔ ¬Human)(Seth)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	**C**	**Choose an individual a, add C(a)**
4	(C ⊓ D)(a)	Add C(a) and D(a)
5	(C ⊔ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	(∃R.C)(a)	Add R(a,b) and C(b) for a *new* Individual b
7	(∀R.C)(a)	For all b with R(a,b) ∈ T: add C(b)

a) ¬Animal ⊔ ¬Human
b) Animal ⊔ (¬Mammal ⊓ ¬ Bird ⊓ ¬ Fish ⊓ ¬ Insect ⊓ ¬ Reptile)
c) ¬ Animal ⊔ (Mammal ⊔ Bird ⊔ Fish ⊔ Insect ⊔ Reptile)
d) Human(Seth)
e) Insect(Seth)

A Simple Example

1d, 1e

Human(Seth), Insect(Seth)

3a

Human(Seth), Insect(Seth),
(¬Animal ⊔ ¬Human)(Seth)

5

Human(Seth), Insect(Seth), ¬Animal(Seth)	Human(Seth), Insect(Seth), ¬Human(Seth)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C ⊓ D)(a)	Add C(a) and D(a)
5	**(C ⊔ D)(a)**	**Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2**
6	(∃R.C)(a)	Add R(a,b) and C(b) for a *new* Individual b
7	(∀R.C)(a)	For all b with R(a,b) ∊ T: add C(b)

a) ¬Animal ⊔ ¬Human
b) Animal ⊔ (¬Mammal ⊓ ¬ Bird ⊓ ¬ Fish ⊓ ¬ Insect ⊓ ¬ Reptile)
c) ¬ Animal ⊔ (Mammal ⊔ Bird ⊔ Fish ⊔ Insect ⊔ Reptile)
d) Human(Seth)
e) Insect(Seth)

A Simple Example

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5

Human(Seth), Insect(Seth), ¬Animal(Seth)	Human(Seth), Insect(Seth), ¬Human(Seth)

3b

Human(Seth), Insect(Seth),
¬Animal(Seth)
Animal ⊔ (¬Mammal ⊓ ¬Bird ⊓ ¬Fish ⊓ ¬Insect)(Seth)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	**C**	**Choose an individual a, add C(a)**
4	(C ⊓ D)(a)	Add C(a) and D(a)
5	(C ⊔ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	(∃R.C)(a)	Add R(a,b) and C(b) for a *new* Individual b
7	(∀R.C)(a)	For all b with R(a,b) ∈ T: add C(b)

a) ¬Animal ⊔ ¬Human
b) Animal ⊔ (¬Mammal ⊓ ¬ Bird ⊓ ¬ Fish ⊓ ¬ Insect ⊓ ¬ Reptile)
c) ¬ Animal ⊔ (Mammal ⊔ Bird ⊔ Fish ⊔ Insect ⊔ Reptile)
d) Human(Seth)
e) Insect(Seth)

A Simple Example

5

Human(Seth), Insect(Seth), ¬Animal(Seth)	Human(Seth), Insect(Seth), ¬Human(Seth)

3b

Human(Seth), Insect(Seth),
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Animal ⊔ (¬Mammal ⊓ ¬Bird ⊓ ¬Fish ⊓ ¬Insect)(Seth)

5

Human(Seth), Insect(Seth), ¬Animal(Seth) Animal(Seth)	Human(Seth), Insect(Seth), ¬Animal(Seth) ¬Mammal ⊓ ¬Bird ⊓ ¬Fish ⊓ ¬Insect)(Seth)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C ⊓ D)(a)	Add C(a) and D(a)
5	**(C ⊔ D)(a)**	**Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2**
6	(∃R.C)(a)	Add R(a,b) and C(b) for a *new* Individual b
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a) ¬Animal ⊔ ¬Human
b) Animal ⊔ (¬Mammal ⊓ ¬ Bird ⊓ ¬ Fish ⊓ ¬ Insect ⊓ ¬ Reptile)
c) ¬ Animal ⊔ (Mammal ⊔ Bird ⊔ Fish ⊔ Insect ⊔ Reptile)
d) Human(Seth)
e) Insect(Seth)

A Simple Example

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5

Human(Seth), Insect(Seth), ¬Animal(Seth) Animal(Seth)	Human(Seth), Insect(Seth), ¬Animal(Seth) (¬Mammal ⊓ ¬Bird ⊓ ¬Fish ⊓ ¬Insect)(Seth)

4

Human(Seth), Insect(Seth),
¬Animal(Seth)
¬Mammal(Seth)
¬Bird(Seth)
¬Fish(Seth)
¬Insect(Seth)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	**(C ⊓ D)(a)**	**Add C(a) and D(a)**
5	(C ⊔ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	(∃R.C)(a)	Add R(a,b) and C(b) for a *new* Individual b
7	(∀R.C)(a)	For all b with R(a,b) ∊ T: add C(b)

a) ¬Animal ⊔ ¬Human
b) Animal ⊔ (¬Mammal ⊓ ¬ Bird ⊓ ¬ Fish ⊓ ¬ Insect ⊓ ¬ Reptile)
c) ¬ Animal ⊔ (Mammal ⊔ Bird ⊔ Fish ⊔ Insect ⊔ Reptile)
d) Human(Seth)
e) Insect(Seth)

A Simple Example

5

Human(Seth), Insect(Seth),
¬Animal(Seth)
Animal(Seth)

Human(Seth), Insect(Seth),
¬Animal(Seth)
(¬Mammal ⊓ ¬Bird ⊓ ¬Fish ⊓ ¬Insect)(Seth)

4

Human(Seth), Insect(Seth),
¬Animal(Seth)
¬Mammal(Seth)
¬Bird(Seth)
¬Fish(Seth)
¬Insect(Seth)

- All partial tableaus have a contradiction
  - > Knowledge Graph is inconsistent

Another Example

- Again, a simple ontology:

```
:Woman rdfs:subClassOf [
    a owl:Restriction;
    owl:onProperty :hasMother;
    owl:someValuesFrom :Woman
].
:Jane a :Woman.
```

Another Example

- Again, a simple ontology:

```
:Woman rdfs:subClassOf [
    a owl:Restriction;
    owl:onProperty :hasMother;
    owl:someValuesFrom :Woman
].
:Jane a :Woman.
```

- in DL NNF:

  $\neg$ Woman $\sqcup$ $\exists$hasMother.Woman

  Woman(Jane)

Another Example

1b, 3a

Woman(Jane), ¬ Woman ⊔ ∃hasMother.Woman(Jane)

Nr	Axiom	Action
1	**C(a)**	**Add C(a)**
2	R(a,b)	Add R(a,b)
3	**C**	**Choose an individual a, add C(a)**
4	(C ⊓ D)(a)	Add C(a) and D(a)
5	(C ⊔ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	(∃R.C)(a)	Add R(a,b) and C(b) for a *new* Individual b
7	(∀R.C)(a)	For all b with R(a,b) ∈ T: add C(b)

a) ¬ Woman ⊔ ∃hasMother.Woman
b) Woman(Jane)

Another Example

1b, 3a

Woman(Jane), ¬ Woman ⊔ ∃hasMother.Woman(Jane)

5

Woman(Jane), ¬ Woman (Jane)	Woman(Jane), ∃hasMother.Woman(Jane)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C ⊓ D)(a)	Add C(a) and D(a)
5	**(C ⊔ D)(a)**	**Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2**
6	(∃R.C)(a)	Add R(a,b) and C(b) for a *new* Individual b
7	(∀R.C)(a)	For all b with R(a,b) ∈ T: add C(b)

a) ¬ Woman ⊔ ∃hasMother.Woman
b) Woman(Jane)

Another Example

1b, 3a

Woman(Jane), $\neg$ Woman $\sqcup$ $\exists$hasMother.Woman(Jane)

5

Woman(Jane), $\neg$ Woman (Jane)	Woman(Jane), $\exists$hasMother.Woman(Jane)

6

Woman(Jane), $\exists$hasMother.Woman(Jane)
hasMother(Jane, b0), Woman(b0)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	$(C \sqcap D)(a)$	Add C(a) and D(a)
5	$(C \sqcup D)(a)$	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	**$(\exists R.C)(a)$**	**Add R(a,b) and C(b) for a *new* Individual b**
7	$(\forall R.C)(a)$	For all b with R(a,b) $\in$ T: add C(b)

a) $\neg$ Woman $\sqcup$ $\exists$hasMother.Woman
b) Woman(Jane)

Another Example

1b, 3a

Woman(Jane), ¬ Woman ⊔ ∃hasMother.Woman(Jane)

5

Woman(Jane), ¬ Woman (Jane)	Woman(Jane), ∃hasMother.Woman(Jane)

6

Woman(Jane), ∃hasMother.Woman(Jane)
hasMother(Jane, b0), Woman(b0)

6

Woman(Jane), ∃hasMother.Woman(Jane)
hasMother(Jane, b0), Woman(b0)
hasMother(Jane, b1), Woman(b1)

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C ⊓ D)(a)	Add C(a) and D(a)
5	(C ⊔ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	**(∃R.C)(a)**	**Add R(a,b) and C(b) for a *new* Individual b**
7	(∀R.C)(a)	For all b with R(a,b) ∈ T: add C(b)

a) ¬ Woman ⊔ ∃hasMother.Woman
b) Woman(Jane)

Another Example

1b, 3a

Woman(Jane), ¬ Woman ⊔ ∃hasMother.Woman(Jane)

5

Woman(Jane), ¬ Woman (Jane)	Woman(Jane), ∃hasMother.Woman(Jane)

6

Woman(Jane), ∃hasMother.Woman(Jane)
hasMother(Jane, b0), Woman(b0)

6

Woman(Jane), ∃hasMother.Woman(Jane)
hasMother(Jane, b0), Woman(b0)
hasMother(Jane, b1), Woman(b1)

6

…

Nr	Axiom	Action
1	C(a)	Add C(a)
2	R(a,b)	Add R(a,b)
3	C	Choose an individual a, add C(a)
4	(C ⊓ D)(a)	Add C(a) and D(a)
5	(C ⊔ D)(a)	Split tableau into T1 and T2. Add C(a) to T1, D(a) to T2
6	**(∃R.C)(a)**	**Add R(a,b) and C(b) for a *new* Individual b**
7	(∀R.C)(a)	For all b with R(a,b) ∈ T: add C(b)

a) ¬ Woman ⊔ ∃hasMother.Woman
b) Woman(Jane)

Introducing Rule Blocking

- Observation
  - The tableau algorithm does not necessarily terminate
  - We can add arbitrarily many new axioms

Nr	Axiom	Action
6	(∃R.C)(a)	Add R(a,b) und C(b) for a *new* Individual b

- Idea: avoid rule 6 if no new information is created
  - i.e., if we already created one instance b_a for instance a, then block using rule 6 for a.

Tableau Algorithm with Rule Blocking

- Given: an ontology O in NNF
- While not all partial tableaus are closed
  and further axioms can be created

  * Choose a non-closed partial tableau T and a non-blocked A ∈ O ∪ T

  If A is not contained in T

      If A is an atomic statement

          add A to T

          back to *

      If A is a non-atomic statement

          Choose an individual i ∈ O ∪ T

          Add A(i) to T

          back to *

  else

      Extend the tableau with consequences from A

      If rule 6 was used, block A for T

      back to *

Example with Rule Blocking

1b, 3a

Woman(Jane), ¬ Woman ⊔ ∃hasMother.Woman(Jane)

5

Woman(Jane), ¬ Woman (Jane)	Woman(Jane), ∃hasMother.Woman(Jane)

6

Woman(Jane), ∃hasMother.Woman(Jane) hasMother(Jane, b0), Woman(b0)

Block Rule 6 for Jane

now it will terminate ultimately

Nr	Axiom	Action
6	(∃R.C)(a)	Add R(a,b) und C(b) for a *new* Individual b, **block rule 6 for a**

Tableau Algorithm: Wrap Up

- An algorithm for description logic based ontologies
  – Works for OWL Lite and DL
- We have seen examples for some OWL expressions
  – Other OWL DL expressions can be “translated” to DL as well
  – And they come with their own expansion rules
  – Reasoning may become more difficult
    - e.g., dynamic blocking and unblocking

Optimizing Tableau Reasoners

- Given: an ontology O in NNF
- While not all partial tableaus are closed
  and further axioms can be created

  * Choose a non-closed partial tableau T and a non-blocked A ∈ O ∪ T

  If A is not contained in T

      If A is an atomic statement

          add A to T

          back to *

      If A is a non-atomic statement

          Choose an individual i ∈ O ∪ T

          Add A(i) to T

          back to *

  else

      Extend the tableau with consequences from A

      If rule 6 was used, block A for T

      back to *

OWL Lite vs DL Revisited

- Recap: OWL Lite has some restrictions
  - Those are meant to allow for faster reasoning
- Restrictions only with cardinalities 0 and 1
  - Higher cardinalities make blocking more complex
- unionOf, disjointWith, complementOf, closed classes, ...
  - They all introduce more disjunctions
  - i.e., more splitting operations

Complexity of Ontologies

- Reasoning is usually expensive

- Reasoning performance depends on ontology complexity
  - Rule of thumb: the more complexity, the more costly
- Most useful ontologies are in OWL DL
  - But there are differences
  - In detail: complexity classes

Simple Ontologies: ALC

- ALC: Attribute Language with Complement

- Allowed:
  - subClassOf, equivalentClass
  - unionOf, complementOf, disjointWith
  - Restrictions: allValuesFrom, someValuesFrom
  - domain, range
  - Definition of individuals

SHIQ, SHOIN & co

- Complexity classes are noted as letter sequences
- Using
  - S = ALC plus transitive properties (basis for most ontologies)
  - H = Property hierarchies (subPropertyOf)
  - O = closed classes (oneOf)
  - I = inverse properties (inversePropertyOf)
  - N = numeric restrictions (min/maxCardinality)
  - F = functional properties
  - Q = qualified numerical restrictions (OWL2)
  - (D) = Usage of datatype properties

Some Tableau Reasoners

- Fact
  - University of Manchester, free
  - SHIQ
- Fact++/JFact
  - Extension of Fact, free
  - SHOIQ(and a little D), OWL-DL + OWL2
- Pellet
  - Clark & Parsia, free for academic use
  - SHOIN(D), OWL-DL + OWL2
- RacerPro
  - Racer Systems, commercial
  - SHIQ(D)

Sudoku Revisited

- Recap: we used a closed class
  - Plus some disjointness
- Resulting complexity: SO
- Which reasoners do support that?
  - Fact: SHIQ :-(
  - RacerPro: SHIQ(D) :-(
  - Pellet: SHOIN(D) :-)
  - HermiT: SHOIQ :-)

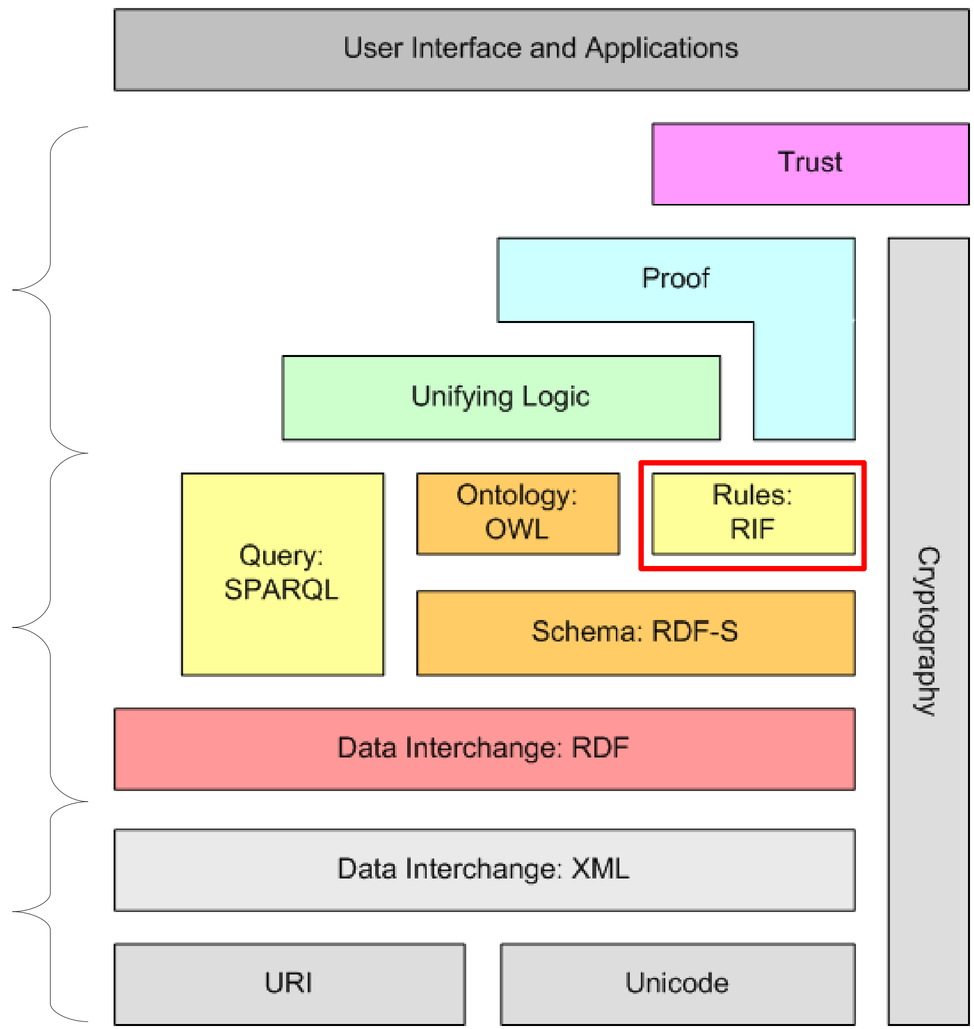
5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Rules: Beyond OWL

here be dragons...

Knowledge Graphs
(This lecture)

Technical
Foundations



Berners-Lee (2009): *Semantic Web and Linked Data* http://www.w3.org/2009/Talks/0120-campus-party-tbl/

Limitations of OWL

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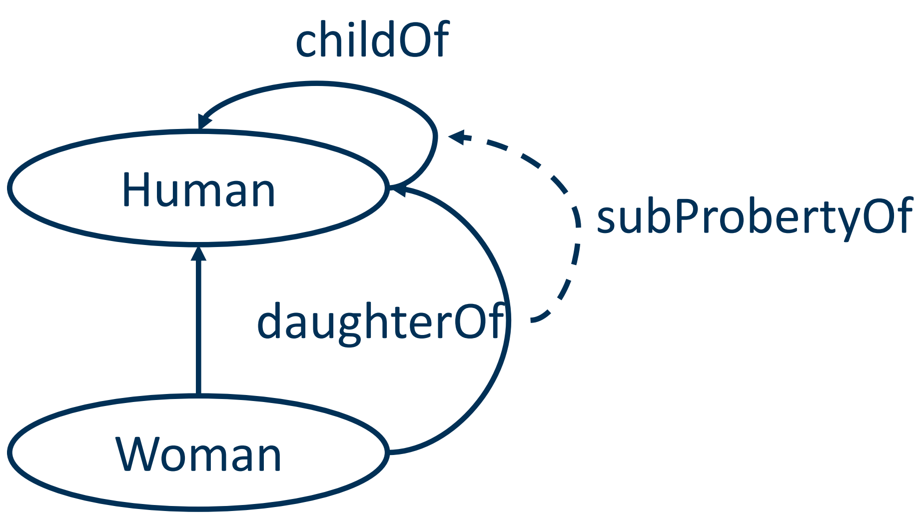
- Some things are hard or impossible to express in OWL
- Example:
  - If A is a woman and the child of B
    then A is the daughter of B



Limitations of OWL

- Let's try this in OWL:

```
:Woman rdfs:subClassOf :Human .
:childOf    a owl:ObjectProperty ;
            rdfs:domain :Human ;
            rdfs:range :Human .
:daughterOf a owl:ObjectProperty ;
            rdfs:subPropertyOf :childOf ;
            rdfs:domain :Woman .
```

Limitations of OWL

- What can a reasoner conclude with this ontology?
- Example:

```
:Julia :daughterOf :Peter .
→ :Julia a :Woman .
```

- What we would like to have instead:

```
:Julia :childOf :Peter .
:Julia a :Woman .
→ :Julia :daughterOf :Peter .
```

Limitations of OWL

- What we would like to have:

  daughterOf(X,Y) ← childOf(X,Y) ∧ Woman(X) .

- Rules are flexible
- There are rules in the Semantic Web, e.g.
  - Semantic Web Rule Language (SWRL)
  - Rule Interchange Format (RIF)
  - See lecture in a few weeks
- Some reasoners do (partly) support rules

Wrap Up

- OWL comes in many flavours
  - OWL Lite, OWL DL, OWL Full
  - Detailed complexity classes of OWL DL
  - Additions and profiles from OWL2
  - However, there are still some things that cannot be expressed...
- Reasoning is typically done using the Tableau algorithm

Questions?